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The influence of mass parameters and gear ratio on the speed and energy expenditure of a cyclist

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The wavelength of moment of active forces (driving forces) for a full cycle while pedaling with platform pedals was determined. There was defined the value of moment of passive forces, depending on drag, rolling resistance and grade of surface. Kinematic motion parameters were determined from the equation of motion of the machine, which was solved numerically. In numerical example, there were determined and compared the temporal courses of bicycle speed for possible gear ratios for the two different waveforms of the driving torque – the determined, the time-varying and the constant ones. There were compared extreme values of active and passive forces, the kinetic energy of the bike and work expended by the rider at a specified time.

Key words: *bicycle, dynamics, kinematics, platform pedal, theoretical model*

1. Introduction

The first pedal-powered bikes with chain box and wheels with wire spokes were designed in the second half of the 19th century. From that time there has been noted much publishing activity in the topic, which can be divided into three groups: experimental works, theoretical and experimental-theoretical. The problem is considered in two aspects: the analysis of the man-machine system and the analysis of a bicycle design, while the range of topics is enormous.

Tests have been carried out on the influence of the bike dimensions on selected quantities such as those of handlebar on bike stability at high speeds (Prince and Jumaily [18]), a different frame geometry on the crank load (Gregor et al. [11]), pedal-crank systems of variable length or independent arms of bicycle drive cranks on pedaling functionality (Rankin and Neptune [19]; Höchtl et al. [12]; Bini et al. [4]). There were measured the waveforms of the angular changes (Cockcroft [6]) and loads in the hip, knee and ankle joints for a constant pedaling force using EMG analysis (Park [17]). The geometry of movement of the lower limb and muscle load in terms of reducing fatigue or increasing sports performance was analyzed (Bini et al. [3]; Abiss and Laursen [1]; Ricard et al. [21]; Wanich et al. [23]).

In the study of Diefenthaeler et al. [10], using a camera, body motion was studied to assess the aerodynamic loads. Debraux et al. [8] and Defraeye et al. [9] describe and evaluate the effectiveness of the techniques that determine the aerodynamic drag during road cycling and wind tunnel conditions. The work of Moore et al. [15] experimentally confirmed the hypothesis about the role of minor movements of the upper body of a cyclist in motion control of bike–rider system. It was found that the dominant trunk movements in stabilizing bicycle path, such as tilting, bending, torsion, are related to the speed of pedaling force. In addition, at low speeds, lateral movements of the knees were observed in cyclists that are likely to be more effective in the control of bicycle ride than the upper body movements. In the work of Stone and Hull [22], the

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hypothesis that there is a direct relationship between the mass of cyclists and the loads induced by the elements of the bike was verified. Since the load induced by the rider varies considerably depending on the circumstances, tests were carried out under various riding conditions. There was pointed out a possibility of using the results for the optimization of bicycle frames with a minimum mass and acceptable structural reliability.

The study resulted in developing a heuristic procedure for determining the minimum number of gear ratios and their most efficient values (Chang et al. [5]). Using the results of ergonomic research, a new planetary gear has been developed by Lee et al. [14], with four outer gears, the prototype of which successfully passed the tests in mass-produced bicycles. In the work of Bertuccia et al. [2], the hypothesis was confirmed that with the pedaling frequency of 60–100 cycles per minute, while riding on the flat surface, there are other profiles of the crank torque than on the uphill. There was also measured the generated power, pulse and the level of lactic acid in isokinetic conditions (Koninckx et al. [13]), and the adaptation of muscle power at varying speeds (Neptune and Herzog [16]).

The research of Dahmen et al. [7], Rasmussen [20] confirmed the effectiveness of mathematical models in evaluating the effectiveness of riding for the road cycling. The models allow for determining the power required to ride on the flat surface and the inclines and the speed of the cyclist based on power on chain drive. The authors perceive the usefulness of the system for the development of effective tactics of training, and optimize the preparation of individual riders to race in unknown terrain conditions.

Through experimental and theoretical research, bicycles are still being improved and demands placed on today are more sophisticated than a few years back. The measure of the quality of the bikes manufactured at present is fatigue strength of their components, resistance to impact strength, functionality of components such as brakes, shock absorbers, reliability, and low mass. The heaviest urban bikes are made of structural steel, the mass of which reaches 25 kg. The lighter by 2 to 8 kg, but less durable ones, shall be racing bikes, ATB bikes, (All Terrain Bicycles), adjusted to cycling in all possible terrain conditions, fitness bikes with frames made of thin-walled tubes, fork, seat posts, handlebar, saddle, crank and rims made of aluminum alloys. The lightest bikes weighing about 3 kg are made of composite materials, scandium, magnesium, titanium and technologies of aerospace industry. Here, we can mention the frequently asked questions on Internet forums, where is the line between "race" to reduce mass and adding additional improvements and what impact these activities have on the broadly understood effectiveness of riding. Second frequently asked question concerns the relevance of using latching systems, such as the SPD, (Shimano Pedaling Dynamics) introduced by Shimano, connecting footwear with pedals to each other, to change the way of pedaling and capable of producing a fairly constant drive torque.

To answer these questions, calculation algorithm and simulation program were developed that related geometric parameters (limbs and bike parts size, gear ratio), kinematic parameters (acceleration, speed, distance traveled), dynamic parameters (the course of active and passive forces in the drive system, the expended energy). The algorithm used simplifications and omitted many important phenomena discussed by the authors of the cited works. The influence of cyclist body position geometry, terrain gradient and endurance adaptation of muscles on the frequency of pedaling and the driving torque course, were not accounted for. By using authorship software, there were designated and compared the temporal courses of the bicycle speed with possible gear ratios while acting on crank with variable (platform pedal) and fixed (SPD system) driving torque. Extreme values of power of active and passive forces, the kinetic energy of the bike and work expended by the rider at a specified time, were compared.

2. Materials and methods

2.1. Geometry of movement

The first problem that arises when making the description of the geometry of limb movement is inability to determine from geometrical relationships the course of changes in the ankle joint angle $\beta_p(\alpha_k)$ as a function of crank rotation angle α_k. To determine the course $\beta_p(\alpha_k)$, natural expansion and contraction of muscles can be used. When the force generated by the muscle is greater than the load, as is the case while riding a bicycle, then it is contracted, i.e., the ratio of joint "torsion elasticity" k_s increases with increasing maximum load. Assuming linear dependence of the movement in the ankle joint as a function of ankle loading torque, in simplification, we can assume that

$$\beta_p(\alpha_k) = \beta_0 + \frac{M_{R3}(\alpha_k)}{k_s}, \qquad (1)$$

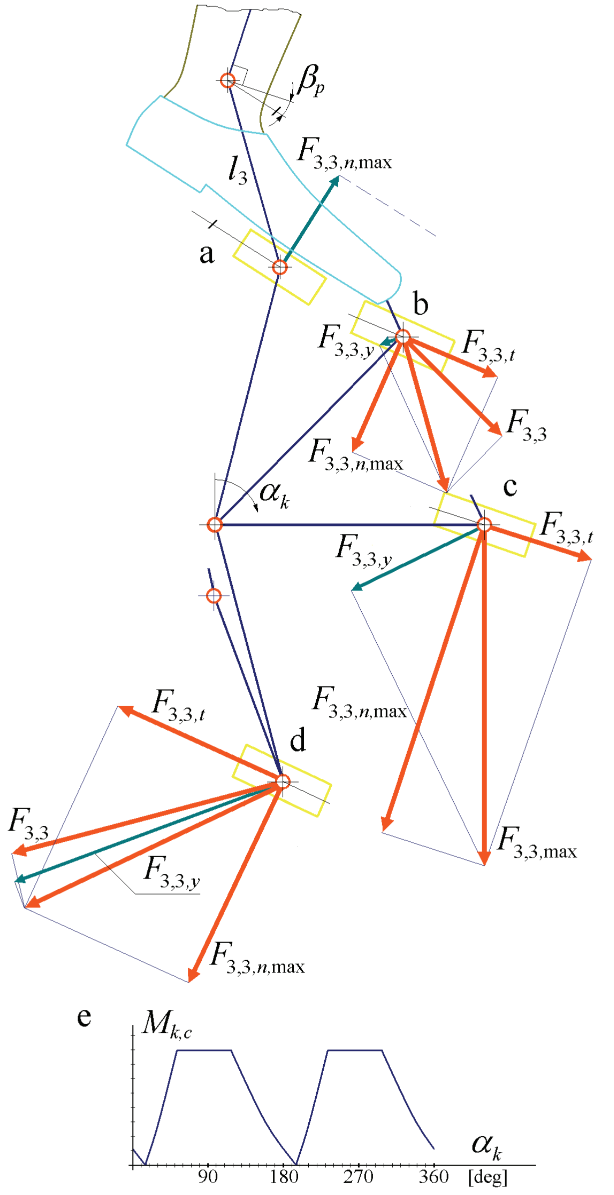

Fig. 1. The wavelength of the moment of active forces for a complete cycle while pedaling with the platform pedals

where the torque loading ankle joint is

$$M_{R3}(\alpha_k) = F_{3,3,y}(\alpha_k) l_3 , \tag{2}$$

and the "torsion elasticity" coefficient of the ankle joint is assumed as linearly-dependent on the module of maximum force of muscle stimulation occurring during single cycle

$$k_s = k_p \,|\, F_{3,3,\max} \,| , \tag{3}$$

where

- $F_{3,3,\max}$ – maximal value of active force perpendicular to crank during a cycle (index 3,3 means the third part of lower limb – the foot) [N],
- $F_{3,3,y}$ – perpendicular force to direction joining the centers of rotating couples of ankle and platform pedal [N],
- k_p – the value of muscle contraction coefficient [m·rad^{-1}],
- k_s – the value of "torsion elasticity" coefficient in ankle joint [N·m·rad^{-1}],
- M_{R3} – the torque loading ankle joint [N·m],
- l_3 – distance between the axes of centers of rotating couples of ankle and pedal [m],
- α_k – angular displacement of crank measured from the top position clockwise [rad],
- β_0 – angle in the ankle joint with no load [rad],
- β_p – angular displacement in the ankle joint [rad] (Fig. 1).

2.2. Moment of passive forces

The moment of passive forces loading pair of rotating crank $M_{k,b}$ depends primarily on the gear ratio, shape, and nature of the surface and the drag as a function of bicycle speed. Also the mass of bike and of the cyclist, rolling resistance of tires on the ground, resistance to motion of the driving system (chain, bearings) and even the pedaling technique influence the load of cranks. The moment of passive forces can be described as the relationship

$$M_{k,b}(i_p, \omega_k, \alpha_t) = M_A + M_F + M_T + M_{c,b} , \tag{4}$$

where

- M_A – the torque loading the rotating pair of crank depending on drag [N·m],
- $M_{c,b}$ – the torque of constant friction forces in the driving system (bearings, transmission [N·m],
- M_F – the torque loading the rotating pair of crank depending on rolling resistance [N·m],
- $M_{k,b}$ – total torque of passive forces loading the crank rotating pair [N·m],
- M_T – the torque loading the crank rotating pair depending on the surface gradient [N·m],

while

$$M_A(i_p, \omega_k) = 0.5 c_x \rho_p A \omega_k^{\,2} (i_p R_o)^3 , \tag{5}$$

$$M_F(i_p, \alpha_t) = m_r g f i_p \cos\alpha_t , \tag{6}$$

$$M_T(i_p, \alpha_t) = m_r g R_o i_p \sin\alpha_t , \tag{7}$$

where

A – the frontal area of a cyclist [m^2],
c_x – drag coefficient,
f – coefficient of rolling friction [m],
g – gravity [m·s^{-2}],
m_r – total mass of bike and cyclist [kg],
R_o – radius of tire tread [m],
α_t – the surface gradient [rad],
ρ_p – air density [kg·m^{-3}],
ω_k – crank angular velocity [rad·s^{-1}],

while gear rate of driving system is equal to

$$i_p = \frac{z_{p,i}}{z_{t,i}}, \qquad (8)$$

where

$z_{p,i}$ – the number of teeth of the chainring,
$z_{t,i}$ – the number of teeth of the rear sprocket.

The value of a passive force $R_{3,3}$ perpendicular to the crank is defined by the relationship

$$R_{3,3}(\alpha_k, \alpha_t) = \frac{M_{k,b}(\omega_k, \alpha_t)}{R_k}, \qquad (9)$$

where R_k – length of the crank [m].

2.3. The moment of active forces

Analyzing the pedaling cycle using the platform pedals assuming initially that the most advantageous pedaling technique is when exerting a constant force $F_{3,3,\max}$ perpendicular to the crank, five stages resulting from the distribution of forces are obtained, Fig. 1.

1. Rotation of the pedal is difficult, because the direction of the normal component of the active force $F_{3,3,n,\max}$ is directed from the plane of the pedal, Fig. 1a.
2. It is necessary to decrease the value of the active force $F_{3,3,\max}$ to the value of $F_{3,3,}$, because the tangential component $F_{3,3,t,\max}$ exceeds the frictional force occurring between the sole and the pedal specified by the product of friction coefficient and the force $F_{3,3,n,\max}$, Fig. 1b.
3. Slipless frictional contact between the sole and the pedal is provided, $F_{3,3} = F_{3,3,\max}$, Fig. 1c.
4. It is necessary to change the value and the direction of active force $F_{3,3,\max}$ to the $F_{3,3}$, as in the second stage, Fig. 1d.
5. Rotation of the pedal is not possible, because the pedal is at the stage of lifting upward, the second limb is at the stage specified by position “b”.

Based on the distributions of forces described in the individual phases of pedaling, the active force $F_{3,3}$ perpendicular to the crank is determined from the geometrical relationships (relationships determining the values of the force $F_{3,3}$ are omitted in this paper), then the value of the active force torque is determined.

The value of the moment of active force $M_{k,c}$ is determined by the relationship

$$M_{k,c}(\alpha_k) = F_{3,3} R_k . \qquad (10)$$

The course of active torque $M_{k.c}$ defined this way depends only on the angular position of the crank (Fig. 1e); it does not account for the impact of individual characteristics of the cyclist, among others stress muscle fatigue or the impact of the geometry of the body position.

2.4. Dynamics of bike movement

Kinematic and dynamic parameters of the bicycle movement are included in the equation of motion

$$M_{k,c}(\alpha_k) - M_{k,b}(\omega_k, \alpha_t) = J_r \frac{d\omega_k}{dt}, \qquad (11)$$

where

J_r – mass moment of inertia reduced to the axis of rotation of the crank [kg·m^2],
t – time [s].

Mass moment of inertia of the bike with the cyclist, reduced to the axis of rotation of the crank with disregarding elements that have a little influence on the value is determined by the relationship

$$J_r(i_p) = i_p^2 m_r R_0^2 + m_o i_p^2 [(R_o - r_o)^2 + 0{,}75 r_o^2], \qquad (12)$$

where

m_o – mass of both wheels of the bike ($m_{op} + m_{ot}$) [kg],
r_o – mean radius of tire section (profile) [m],
R_o – the radius of the tire tread [m].

Equation (11), taking into account (4)–(7) can be written in the form

$$M_{k,c}(\alpha_k) - C_1 - C_2\omega_k^2 = J_r \frac{d\omega_k}{dt}, \qquad (13)$$

where C_1 and C_2 are constants defined in the following way

$$C_1 = M_{c,b} + m_r g i_p (R_o \sin\alpha_t + f \cos\alpha_t),$$

$$C_2 = 0.5 c_x \rho_p A (i_p R_o)^3 . \qquad (14)$$

After description of the coordinates of the state

$$Y_1 = \omega_k, \quad Y_2 = \alpha_k, \qquad (15)$$

the following equations are obtained

$$\frac{dY_1}{dt} = \varepsilon_k = \frac{1}{J_r}[M_{k,c}(Y_2) - C_1 - C_2 Y_1^2], \quad (16)$$

$$\frac{dY_2}{dt} = Y_1, \quad (17)$$

where

$\frac{dY_1}{dt} = \varepsilon_k$ – angular acceleration of the crank [rad·s^{-2}],

while the initial conditions are the following

$$Y_1(0) = 0, \quad Y_2(0) = 0.5\pi. \quad (18)$$

In order to solve the set of equations (16) and (17) the method of Runge–Kutta fourth order was used. As a result of solving the equations, there are obtained values of displacement $\alpha_k(t)$, velocity $\omega_k(t)$ and the acceleration of the crank $\varepsilon_k(t)$ as a function of time. Using the obtained values, the displacement $s_r(t)$, velocity $v_r(t)$ and acceleration $a_r(t)$, of the bike can be determined

$$s_r(t) = \alpha_k(t) i_p R_o, \quad (19)$$

$$v_r(t) = \omega_k(t) i_p R_o, \quad (20)$$

$$a_r(t) = \varepsilon_k(t) i_p R_o, \quad (21)$$

as well as the momentary kinetic energy of the bicycle $E_{kr}(t)$, temporary power of active $N_c(t)$ and passive $N_b(t)$ forces

$$E_{kr}(t) = 0.5 m_r v_r(t)^2 \quad \text{or} \quad E_{kr}(t) = 0.5 J_r \omega_k(t)^2, \quad (22)$$

$$N_c(t) = M_{k,c}\omega_k(t) \quad \text{or} \quad N_c(t) = \frac{M_{k,c}}{R_k i_p} v_r(t), \quad (23)$$

$$N_b(t) = M_{k,b}\omega_k(t). \quad (24)$$

The work done by the rider $W_r(t)$ while cycling at a given time can be determined according to the relationship

$$W_r(t) = \int_{\alpha_{kp}}^{\alpha_{kk}} M_{k,c}(t) d\alpha_k, \quad (25)$$

where

α_{kp}, α_{kk} – initial and final position of the crank [rad].

2.5. Numerical example

Major data for this calculation was adopted as follows: the distance between the axes of rotating couples of pedal and ankle joint l_3 = 0.18 m, the maximum value of the active force $F_{3,3,\max}$ = 120 N, the value of the moment of constant friction forces $M_{c,b}$ = 0.05 N·m, the values of coefficients: k_p = 0.4 m·rad^{-1}, c_x = 0.98, μ_{bp} = 0.8, the values of the angles: β_0 = –5°, α_t = 0.5°, the values of the masses: $m_r = m_c$+70 kg = 82.65 kg, the values of constants g = 9.81 m·s^{-2}, ρ_p = 1.226 kg·m^{-3}, f = 0.005 m and A = 0.46 m^2. The mass values and the dimensions of the bicycle elements are shown in Table 1.

Table 1. The masses and dimensions of bicycle elements considered in calculations

Name of element	Designation	Name of parameter	Value	Unit
Front wheel	m_{op}	Mass	2.10	kg
Rear wheel	m_{ot}	Mass	2.55	kg
Tire 28 × 2.0	R_o	tread radius	0.364	m
	r_o	radius of tire section	0.025	m
Cranks with pedals	R_k	length	0.175	m
Chainring	$m_{z1}/m_{z2}/m_{z3}$	Mass	19/41/71	×10^{-3} kg
	$r_{z1}/r_{z2}/r_{z3}$	radius	44/64/84	×10^{-3} m
	$z_{p1}/z_{p2}/z_{p3}$	number of teeth	22/32/42	number
Cassette	$z_{t1}/z_{t2}/z_{t3}/z_{t4}/z_{t5}/z_{t6}$	number of teeth	14/18/22/26/30/34	number
Bicycle	m_c	total mass	12.65	kg

3. Results

In order to demonstrate the impact of gears and bicycle mass relative to the total mass which is in motion on acceleration of the crank, the relationships (12) and (16) can be used. Figure 2 shows the

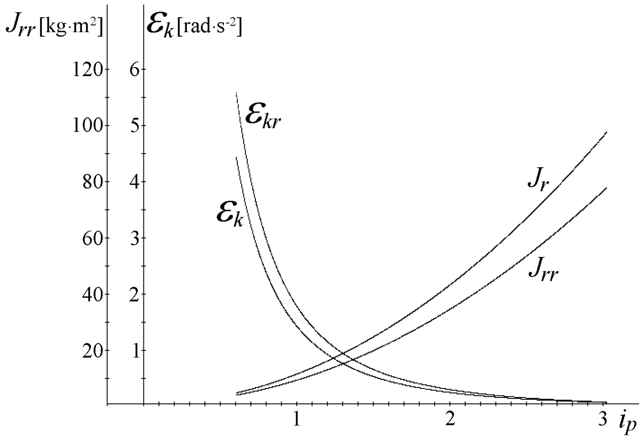

Fig. 2. Waveforms of mass moment of inertia reduced to the axis of rotation of the crank and crank acceleration as a function of gear ratio; J_r, ε_k – bike with a cyclist, J_{rr}, ε_{kr} – a cyclist

relationships between mass moment of inertia reduced to the axis of rotation of the crank and acceleration of the crank as a function of gear ratios.

In order to determine the kinematic parameters there was calculated speed, to which a bike can be accelerated on the following gear ratios using platform pedals and SPD system. For SPD system, an idealized assumption was adopted that the driving torque is constant during a full cycle of crank rotation, which in practice is difficult to meet. Angular velocity of the crank was limited to 9 rad·s^{-1}, and time of cycling using a chosen gear – to 120 s. The calculations are finished when one of the limits is exceeded. The results of calculations as temporal waveforms of bicycle speed are shown in Fig. 3. Table 2 shows the distance traveled, the number of crank rotations, maximum speed, temporary powers

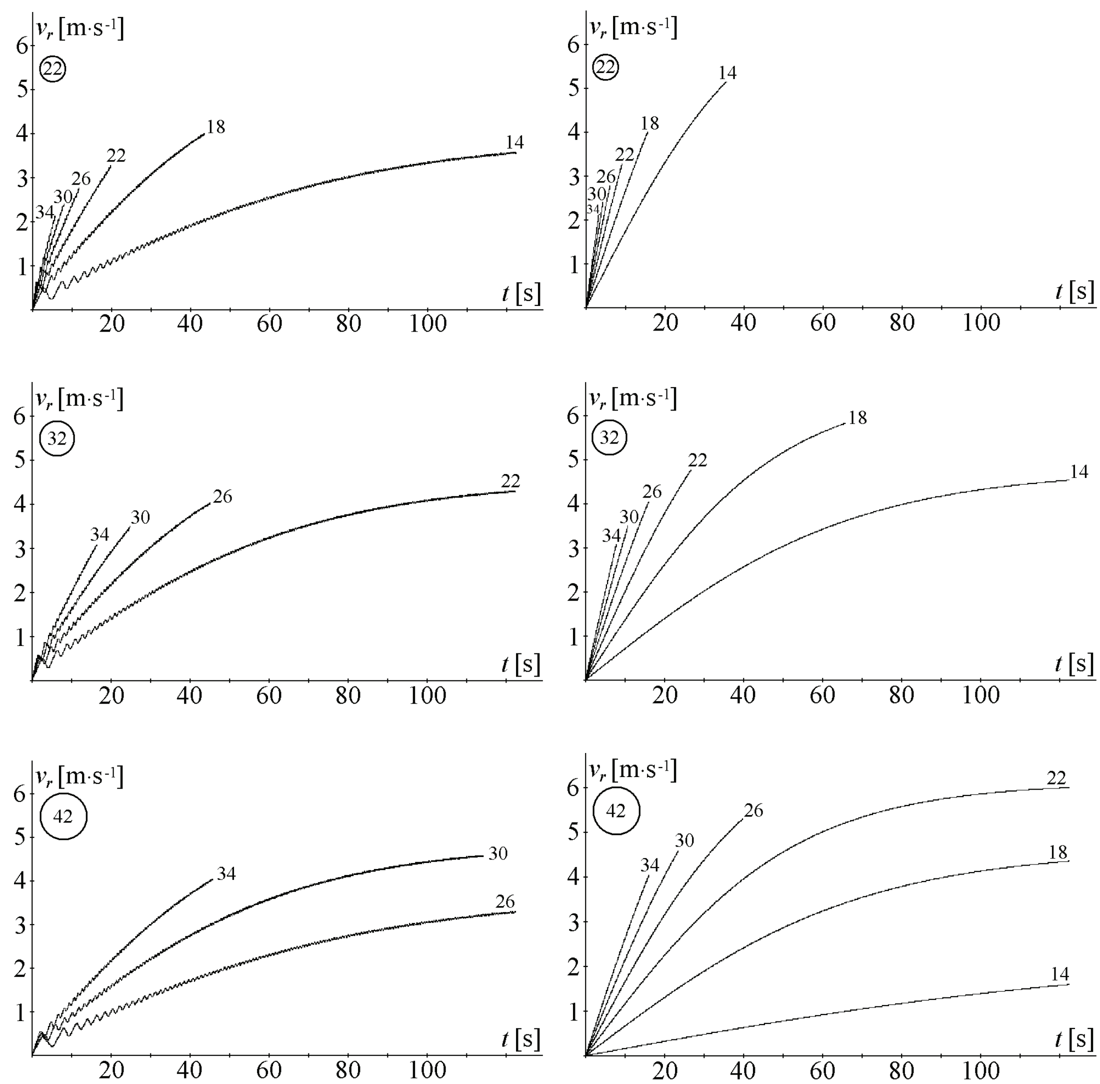

Fig. 3. Temporary waveforms of bicycle speed at the following gear ratios; without the latch system (left side), with the latch system (on the right)

Table 2. Kinematic and dynamic parameters; without the latch system (top value), with the latch system (lower value)

Gear ratio	Time [s]	Distance [m]	Number of crank rotations	Speed [m·s^{-1}]	Active forces power [W]	Passive forces power [W]	Work done [J]
22/34 0.647	5.7 3.2	7.0 3.8	5 3	2.12 2.12	189 189	69 69	405 303
22/30 0.733	7.9 4.3	10.6 5.6	6 3	2.40 2.40	189 189	74 74	557 404
22/26 0.846	12 6	18 9	9 5	2.77 2.77	189 189	81 81	830 569
32/34 0.941	16 8	28 12.6	13 6	3.08 3.08	189 189	87 87	1150 740
22/22 1.000	20 9	35 15.6	15 7	3.27 3.27	189 189	92 92	1396 864
32/30 1.067	25 11	48 20	20 8	3.49 3.49	189 189	97 97	1782 1026
22/18 1.222	43 15	99 32	35 12	4.00 4.00	189 189	110 110	3214 1508
32/26 1.231	45 16	104 34	37 12	4.03 4.03	189 189	111 111	3348 1539
42/34 1.235	45 16	106 34	37 12	4.05 4.05	189 189	112 112	3400 1557
42/30 1.400	113 23	350 57	109 18	4.58 4.58	189 189	127 127	9990 2332
32/22 1.455	120 27	353 68	106 21	4.29 4.76	170 189	115 133	9687 2676
22/14 1.571	120 35	283 101	79 28	3.56 5.14	131 189	88 146	7166 3662
42/26 1.615	120 39	257 117	70 32	3.28 5.29	118 188	78 151	6327 4165
32/18 1.778	– 65	– 234	– 58	– 5.82	– 189	– 172	– 7600
42/22 1.909	– 120	– 526	– 121	– 6.00	– 181	– 179	– 15887
32/14 2.286	– 120	– 369	– 71	– 4.53	– 114	– 111	– 9293
42/18 2.333	– 120	– 351	– 66	– 4.35	– 107	– 105	– 8647
42/14 3.000	– 120	– 109	– 16	– 1.59	– 30	– 30	– 2064

of passive and active forces, the kinetic energy of the bike and the work done by the rider.

4. Discussion

The influence of increased mass of bike and rider on the value of the mass moment of inertia is the greatest for higher gear ratios, at which the possibility of accelerating declines. For lower gear ratios, the changes in mass have little effect on the value of the mass moment of inertia, and the largest influence on acceleration value. Increasing the total moving mass of 12.65 kg (Fig. 2) reduces the possibility of bike acceleration of about 1 rad·s^{-2} at the lowest gear ratio and increases the mass moment of inertia by 30 kg·m^2 at the highest gear ratio.

During recreational cycling with an average speed 4.3–5.3 m·s^{-1} (15.3–19.0 km/h), with platform pedals on flat terrain, the use of gear ratios 22/14, 32/22, 42/26, is recommended. For these gear ratios the work done by the cyclist for 120 s ranges from 6.3 to 9.7 kJ from the time of starting the acceleration of the bicycle. The highest speed of the bicycle when acting on the crank with constant driving torque is achieved with the gear ratio 42/22 (Table 2). The work done by the cyclist during 120 s is the greatest and equals 15.9 kJ.

Assuming an action with a predetermined maximum force on the pedal, the largest acceleration of the bicycle motion, with the least work done by the rider is always obtained for low values of the gear ratios (Table 2). Then, in a short time, the upper value of crank angular velocity is reached at which the bike has a slow speed and you need to change gear into a higher one. In practice, on flat terrain, such gear ratios are not applied and they are used when riding up the hill.

5. Conclusions

The mass of the cyclist has the biggest impact on reduced mass moment of inertia and in consequence on bike motion. With most commonly used gear ratios (above 1.5), the effect of increasing mass of the bicycle is negligible.

Formulation of the final conclusion requires the determination of the destination of bike. If you use the bicycle for locomotion and tourism on roads suitable for cycling, investing in the bicycle with reduced mass seems pointless. The consequence of increasing the mass is a slight increase in the frictional resistance (rolling wheels and bearings) as well as decreasing ability of acceleration. In this case, the more important is selecting the right gear ratios, especially uphill.

Racing bike designed for professional and high-performance acrobatic riding should meet very different needs. Any desire to reduce mass is important as the achievements in sport (long distance cycling) depend on fractions of a second, which are affected by the fractions of mass values. In this case, the bike should be as light as possible, the number of gear ratios kept to a minimum. It would be advisable to use only one control handle enabling fast gear changes, which would have only one sprocket of crank connected with the 4-5 gears of cassette. It requires solving the problem of lateral overload of the chain at the extreme gear ratios. It seems that every quest, now sounding fantastic, aimed at the development of the automatic transmission in the bike is the pursuit at the right direction.

The desirability of SPD system application also should be determined by the destination of the bike use. The system allows the athletes gaining higher accelerations at the expense of unnatural work of limbs, not anatomically suited for stretching. It deprives the legs of rest during lifting phase, which requires a proper training. The use of the SPD during recreational cycling can lead to excessive muscle fatigue and without the training – to damage of joints.

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