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Parameter selection of an adaptive PI state observer for an induction motor

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Abstract. The paper discusses problems connected with the parameters selection of the proportional-integral observer, designed for reconstruction of magnetic fluxes and angular speed of an induction motor. The selection is performed in several stages that are focused on different criteria. The first stage consists in selecting observer's gains and provides desired dynamical properties, taking into consideration immunity to disturbances and parameter variations of observed system. The second stage prevents an observer from DC-offset cumulation and instability. The last stage consists in setting the parameters of a speed adaptation mechanism. The impact of different settings on the properties of an observer is illustrated with experimental results, obtained in the multiscalar control system of an induction motor.

Key words: Luenberger observer, induction motor, multiscalar control system.

1. Introduction

PI observer has been rarely applied in control systems of induction motors, mostly for the rotor temperature estimation [1, 2] and fault detection [3]. A full-order PI observer for the magnetic fluxes reconstruction is presented in [4]. However, the observer presented in [4] is based on a mathematical model of an induction motor that uses the rotor current oriented d-q transform and treats the angular speed of rotor current phasor as an input quantity. The resulting need of the rotor current angular speed estimation decreases practical usability of this solution. A reduced-order PI observer for the rotor flux components reconstruction is described in [5]. Reconstructing the two of the four state variables of an induction motor, the observer described in [5] cannot cooperate with a speed adaptation mechanism.

A PI observer [6, 7] provides better reconstruction error attenuation in comparison with proportional (P) observer however, its application is difficult because of a complicated gain selection process. Another difficulty is the fact, that there exists a class of observed systems, for which the PI observer is always unstable, independently of its gains. The dependence of stability on the numbers of outputs and state variables is stated in [8]. The induction motor is the exemplary system that belongs to this class. The method that provides for stability in every possible case is proposed in [9]. The method is based on replacing of the integral unit of the observer with the inertia with a properly set time constant. The similar solution is applied in rotor flux estimators described in [10].

Due to these facts, the parameter selection of the observer is performed in several stages. First, the gains of the proportional and the integral unit should be set, in order to provide desirable dynamical properties. Second stage consists in setting the inertia time constants.

The discussed observer is designed for reconstruction of magnetic fluxes coupled with stator and rotor windings ($\psi_{s\alpha}$, $\psi_{s\beta}$, $\psi_{r\alpha}$ and $\psi_{r\beta}$) [9, 11], additionally it is equipped with the adaptation mechanism for the reconstruction of the rotor angular speed ω. The last stage of the parameters selection consists in setting adaptation mechanism gains.

2. Mathematical model of the induction motor

Equations of the motor are the basis for formation the state space model of the motor. An induction motor can be described in the steady Cartesian coordinate system $\alpha-\beta$ with four differential equations [12, 13]:

$$\begin{cases} \dfrac{\mathrm{d}\psi_{s\alpha}}{\mathrm{d}t} + R_s i_{s\alpha} = u_{s\alpha} \\ \dfrac{\mathrm{d}\psi_{s\beta}}{\mathrm{d}t} + R_s i_{s\beta} = u_{s\beta} \\ \dfrac{\mathrm{d}\psi_{r\alpha}}{\mathrm{d}t} + \omega\psi_{r\beta} + R_r i_{r\alpha} = 0 \\ \dfrac{d\psi_{r\beta}}{dt} - \omega\psi_{r\alpha} + R_r i_{r\beta} = 0 \end{cases}, \tag{1}$$

where $\psi_{s\alpha}$, $\psi_{s\beta}$ – stator axial magnetic fluxes, $\psi_{r\alpha}$, $\psi_{r\beta}$ – rotor axial magnetic fluxes, $u_{s\alpha}$, $u_{s\beta}$ – stator winding axial voltages, $i_{s\alpha}$, $i_{s\beta}$, $i_{r\alpha}$, $i_{r\beta}$ – stator and rotor windings axial currents, R_s, R_r, – stator and rotor windings resistances, ω – electrical angular speed of the motor, t – time. All quantities are represented in p.u. relative units (per-unit system) normalizing quantities to a common base [14].

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Four algebraic equations describe relations between magnetic fluxes and currents:

$$\begin{cases} \psi_{s\alpha} = L_s i_{s\alpha} + L_m i_{r\alpha} \\ \psi_{s\beta} = L_s i_{s\beta} + L_m i_{r\beta} \\ \psi_{r\alpha} = L_r i_{r\alpha} + L_m i_{s\alpha} \\ \psi_{r\beta} = L_r i_{r\beta} + L_m i_{s\beta} \end{cases}, \tag{2}$$

where L_s, L_r, L_m – parameters of an equivalent circuit of the motor [11].

An additional equation describes mechanical properties of the motor:

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t} = p_b\left(t_e - t_l\right), \tag{3}$$

where t_e, t_l – electromagnetic torque and load torque, J – moment of inertia, p_b – number of poles of the motor.

For the purpose of magnetic fluxes reconstruction, a mathematical model of the motor consists only of electromagnetic Eqs. (1) and (2). The mechanical equation can be omitted, assuming that the angular speed ω changes much more slowly than magnetic fluxes and currents. Therefore, an angular speed can be treated as a parameter [12] and excluded from a state vector. Following forms of state vector $\boldsymbol{x}$, input vector $\boldsymbol{u}$ and output vector $\boldsymbol{y}$ have been assumed to be equal to:

$$\begin{gathered} \boldsymbol{x} = \left[\begin{array}{cccc} \psi_{s\alpha} & \psi_{s\beta} & \psi_{r\alpha} & \psi_{r\beta} \end{array}\right]^T, \\ \boldsymbol{u} = \left[\begin{array}{cc} u_{s\alpha} & u_{s\beta} \end{array}\right]^T, \\ \boldsymbol{y} = \left[\begin{array}{cc} i_{s\alpha} & i_{s\beta} \end{array}\right]^T. \end{gathered} \tag{4}$$

3. Mathematical model of the PI observer

An induction motor, as a linear system with input vector $\boldsymbol{u}\in\mathrm{R}^p$, state vector $\boldsymbol{x}\in\mathrm{R}^n$ and output vector $\boldsymbol{y}\in\mathrm{R}^q$, is described with the matrix differential state equation and the matrix algebraic output equation [9, 11, 15, 16]:

$$\begin{cases} \dfrac{\mathrm{d}\boldsymbol{x}}{\mathrm{d}t} = \boldsymbol{A}\left(\omega\right)\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \end{cases}, \tag{5}$$

where $\boldsymbol{A}(\omega) \in R^{n\times n}$, $\boldsymbol{B} \in R^{n\times p}$, $\boldsymbol{C} \in R^{q\times n}$ – real matrices dependent on parameters values of the equivalent circuit of the motor, and in case of $\boldsymbol{A}$, on the angular speed ω treated as an parameter. For the induction motor described with (4) and (5) following vectors' sizes are given $n = 4$, $p = 2$, $q = 2$.

State vector $\boldsymbol{x}$ of the observed system described with (5), can be reconstructed with use of the PI observer described with the matrix differential-integral equation:

$$\begin{aligned} \frac{\mathrm{d}\widehat{\boldsymbol{x}}}{\mathrm{d}t} &= \boldsymbol{A}\left(\widehat{\omega}\right)\widehat{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{K}_P\left(\widehat{\omega}\right)\left(\boldsymbol{C}\widehat{\boldsymbol{x}} - \boldsymbol{y}\right) \\ &+ \boldsymbol{K}_I\left(\widehat{\omega}\right)\int_0^t\left(\boldsymbol{C}\widehat{\boldsymbol{x}} - \boldsymbol{y}\right)d\tau, \end{aligned} \tag{6}$$

where $\widehat{\boldsymbol{x}} \in \mathrm{R}^{n\times n}$ – reconstructed state vector, $\boldsymbol{K}_P(\widehat{\omega}) \in \mathrm{R}^{n\times q}$ and $\boldsymbol{K}_I(\widehat{\omega}) \in \mathrm{R}^{n\times q}$ – real gain matrices of the proportional and the integral unit respectively, $\widehat{\omega}$ – reconstructed angular speed of the motor. Gain matrices can be either constant or parametrically dependent on reconstructed angular speed.

Additionally, the mathematical model of the observer is extended with the adaptation mechanism for angular speed reconstruction, proposed by Kubota and Matsuse [12]. The adaptation mechanism assumes shape of PI regulator:

$$\widehat{\omega} = k_P\varepsilon + k_I\int_0^t\varepsilon\,\mathrm{d}\tau \tag{7}$$

where k_P, k_I – regulator's gains. Tuning signal ε is defined as:

$$\varepsilon = \left(i_{s\alpha} - \widehat{i}_{s\alpha}\right)\widehat{\psi}_{r\beta} - \left(i_{s\beta} - \widehat{i}_{s\beta}\right)\widehat{\psi}_{r\alpha}. \tag{8}$$

Reconstructed stator currents $\widehat{i}_{s\alpha}$ and $\widehat{i}_{s\beta}$are to be calculated from an output equation (5), basing on reconstructed space vector $\widehat{\mathbf{x}}$.

4. Parameter selection of the PI observer

The first stage of the parameter selection is a selection of element values of matrices $\boldsymbol{K}_P$ and $\boldsymbol{K}_I$, which should be chosen so that the observer was stable and had desired dynamical properties [7, 9, 15]. For this purpose, eigenvalues of the observer λ, should be properly placed on the complex plane. The eigenvalues of the PI observer are the roots of characteristic polynomial [9, 15]:

$$\phi(\lambda) = \det\left(\begin{bmatrix} \boldsymbol{A}\left(\widehat{\omega}\right) + \boldsymbol{K}_P\left(\widehat{\omega}\right)\boldsymbol{C} - \lambda\boldsymbol{1}_n & \boldsymbol{1}_n \\ \boldsymbol{K}_I\left(\widehat{\omega}\right)\boldsymbol{C} & -\lambda\boldsymbol{1}_n \end{bmatrix}\right), \tag{9}$$

where $\boldsymbol{1}_n$ denotes the identity matrix of size n. Eigenvalues are connected with the time constants of estimation errors attenuation. At the same time, in order to provide immunity to disturbances and parameter variations of the motor, value of the matrix amplification index $||\boldsymbol{K}||_w$ of the following block matrix should be minimized:

$$\boldsymbol{K}\left(\widehat{\omega}\right) = \begin{bmatrix} \boldsymbol{K}_P\left(\widehat{\omega}\right) \\ \boldsymbol{K}_I\left(\widehat{\omega}\right) \end{bmatrix}. \tag{10}$$

The matrix amplification index is a quantity introduced by the authors, and for the considered PI observer is defined as:

$$\left\|\boldsymbol{K}\left(\widehat{\omega}\right)\right\|_w = \frac{1}{2n}\sum_{i=1}^{2n}\sqrt{\sum_{j=1}^{q}\boldsymbol{K}\left(\widehat{\omega}\right)_{[i,j]}^2}, \tag{11}$$

where $\boldsymbol{K}(\widehat{\omega})_{[i,j]}$ denotes the element of matrix $\boldsymbol{K}(\widehat{\omega})$, placed in i-th row and j-th column. Applications and mathematical properties of matrix amplification index are described in [15]. In [15] it is also proved that the lesser matrix amplification index, the more immune is the observer to parameter variations of observed system.

The values of gains matrices were set with the use of a method based on optimization, using the genetic algorithm [15, 16]. Each of gain matrices has $n \times q = 8$ elements, therefore, the number of elements that are to be optimised equals to 16. Moreover, each of elements can be an arbitrary function of the reconstructed angular speed. The number of unknowns can be decreased by assuming fixed relations between elements. In the case of the considered observer, following shapes were assumed:

$$\boldsymbol{K}_P\left(\widehat{\omega}\right) = \begin{bmatrix} a\boldsymbol{1}_q \\ b\boldsymbol{1}_q \end{bmatrix} + \widehat{\omega} \begin{bmatrix} c\boldsymbol{J} \\ d\boldsymbol{J} \end{bmatrix}, \tag{12}$$

$$\boldsymbol{K}_I\left(\widehat{\omega}\right) = \begin{bmatrix} e\boldsymbol{1}_q \\ f\boldsymbol{1}_q \end{bmatrix} + \widehat{\omega} \begin{bmatrix} g\boldsymbol{J} \\ h\boldsymbol{J} \end{bmatrix}, \tag{13}$$

where: a, b, c, d, e, f, g, h – unknowns obtained during parameter selection of the observer. Matrix $\boldsymbol{J}$ is give as:

$$\boldsymbol{J} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}. \tag{14}$$

It can be proven, that dynamical properties of the observer with gain matrices (12) and (16) are independent on direction (sign) of the rotor angular speed. It comes from the form of the characteristic polynomial (8), in which the angular speed $\widehat{\omega}$ occurs only in even powers in this case.

The selection was performed twice, and resulted with two different sets of $\boldsymbol{K}_P(\widehat{\omega})$ and $\boldsymbol{K}_I(\widehat{\omega})$ values. Observers equipped with these sets of parameters hereinafter are referred to as observers PI 1 and PI 2. In the case of PI 1 observer, all the unknowns in (12) and (13) were optimised. In case of PI 2 observer, in order to decrease number of optimised variables, unknowns c and g were set to 0.

In Fig. 1. dynamical properties of both observers are compared with the properties of proportional (P) observer described in [12, 16]. The lower attenuation and lesser immunity to disturbances characterizes the P observer, resulting from its structure and the applied method for parameter selection [16]. Experimental results, acquired in the multiscalar control system [17, 18] of the induction motor rated at 3.5 kW, are shown in Fig. 2. The P observer provides worse estimation quality than PI observers, manifesting itself in disturbances overlaying the waveforms of reconstructed angular speed $\widehat{\omega}$ and multiscalar state variable x_{12}, proportional to the electromagnetic torque of the motor [17, 18]. These disturbances are shown in the magnified parts of plots in Fig. 2. A slightly better operation of PI 1 observer in comparison with PI 2, visible in the magnified parts of plots, results from the lower value of the matrix amplification index $||\boldsymbol{K}(\widehat{\omega})||_w$ of observer PI 1.

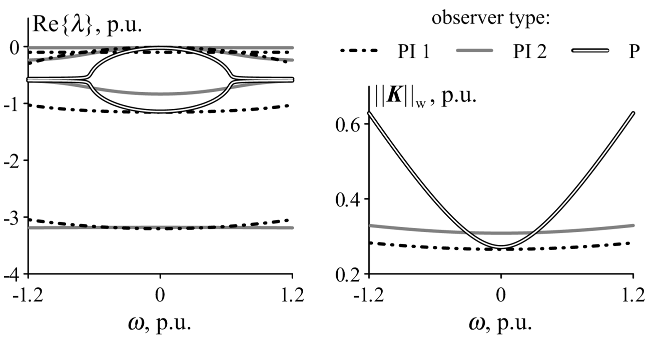

Fig. 1. Dynamical properties comparison of examined observers

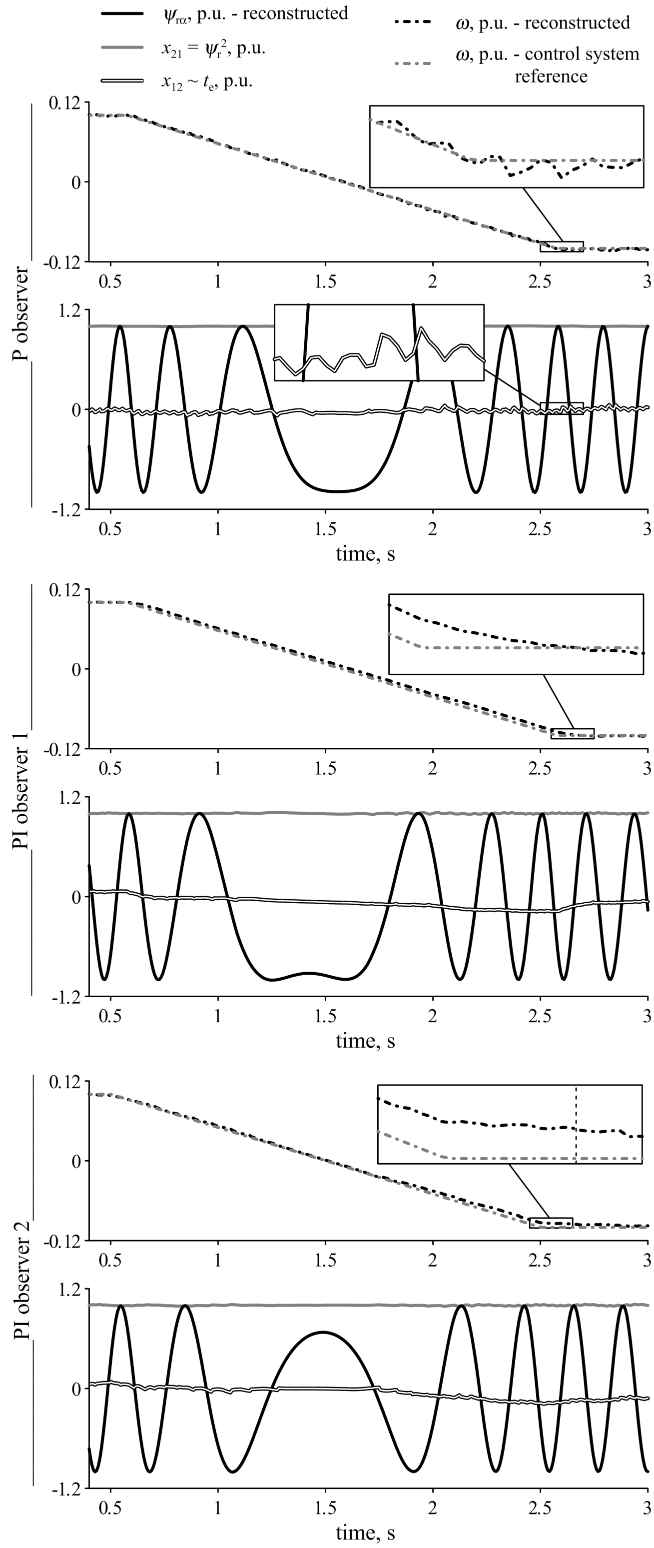

Fig. 2. Experimental results acquired in multiscalar control system for proportional (P) observer and proportional-integral observers PI 1 and PI 2

The second stage of the parameter selection of PI observers is setting of the ω_c parameter of inertial unit replacing integration in observer's feedback [9]. The state equation of the PI observer with modified integration unit, transformed to a set of differential equations, assumes the following form:

$$\begin{cases} \dfrac{\mathrm{d}\widehat{\boldsymbol{x}}}{\mathrm{d}t} = \boldsymbol{A}(\omega)\,\widehat{\boldsymbol{x}} + \boldsymbol{B}\boldsymbol{u} + \boldsymbol{K}_P(\widehat{\omega})\,(\boldsymbol{C}\widehat{\boldsymbol{x}} - \boldsymbol{y}) + \boldsymbol{w} \\ \dfrac{\mathrm{d}\boldsymbol{w}}{\mathrm{d}t} = \boldsymbol{K}_I(\widehat{\omega})\,(\boldsymbol{C}\widehat{\boldsymbol{x}} - \boldsymbol{y}) - \omega_c \boldsymbol{1}_n \boldsymbol{w} \end{cases}, \quad (15)$$

where $\boldsymbol{w} \in \mathrm{R}^n$ denotes the additional state vector of integral unit of the observer. The value of ω_c should be great enough to prevent the observer from effects of DC-offset cumulation. However, increasing of the ω_c value causes stronger attenuation of the signals of the observer's integral unit, weakening attenuation of reconstruction errors. The influence of ω_c value on operation of the PI 1 observer is illustrated by experimental results shown in Fig. 3. During tests, the optimal value of $\omega_c = 0.1$ was obtained with trial-and-error method. For lower value ($\omega_c = 0.032$), the effects connected with insufficient attenuation of DC-offset caused deterioration of operation of the control system in transient states, resulting in longer duration of reversal (Fig. 3). In opposite case, for the value grater than optimal ($\omega_c = 0.32$), increased reconstruction errors, having an impact on stabilization of the value of state variable x_{21}, result from weakened correction.

The modification of the integral unit of the observer has an impact on its eigenvalues, shown in Fig. 4 for different values of ω_c. Figure 4 shows that variations of eigenvalues are small, therefore their impact on dynamical properties of the observer is insignificant.

The last stage of the parameter selection of PI observers is the selection of PI regulator gains, being the part of the angular speed adaptation mechanism [16, 12], described with (7). On increasing gains, the time lag between actual and reconstructed angular speed decreases, but the sensitivity of the mechanism to disturbances and reconstruction errors of magnetic fluxes rises. Moreover, the time constant of the regulator should be at last one order of magnitude greater than the smallest time constant of the observer, resulting from $\boldsymbol{K}_P(\widehat{\omega})$ and $\boldsymbol{K}_I(\widehat{\omega})$ selection.

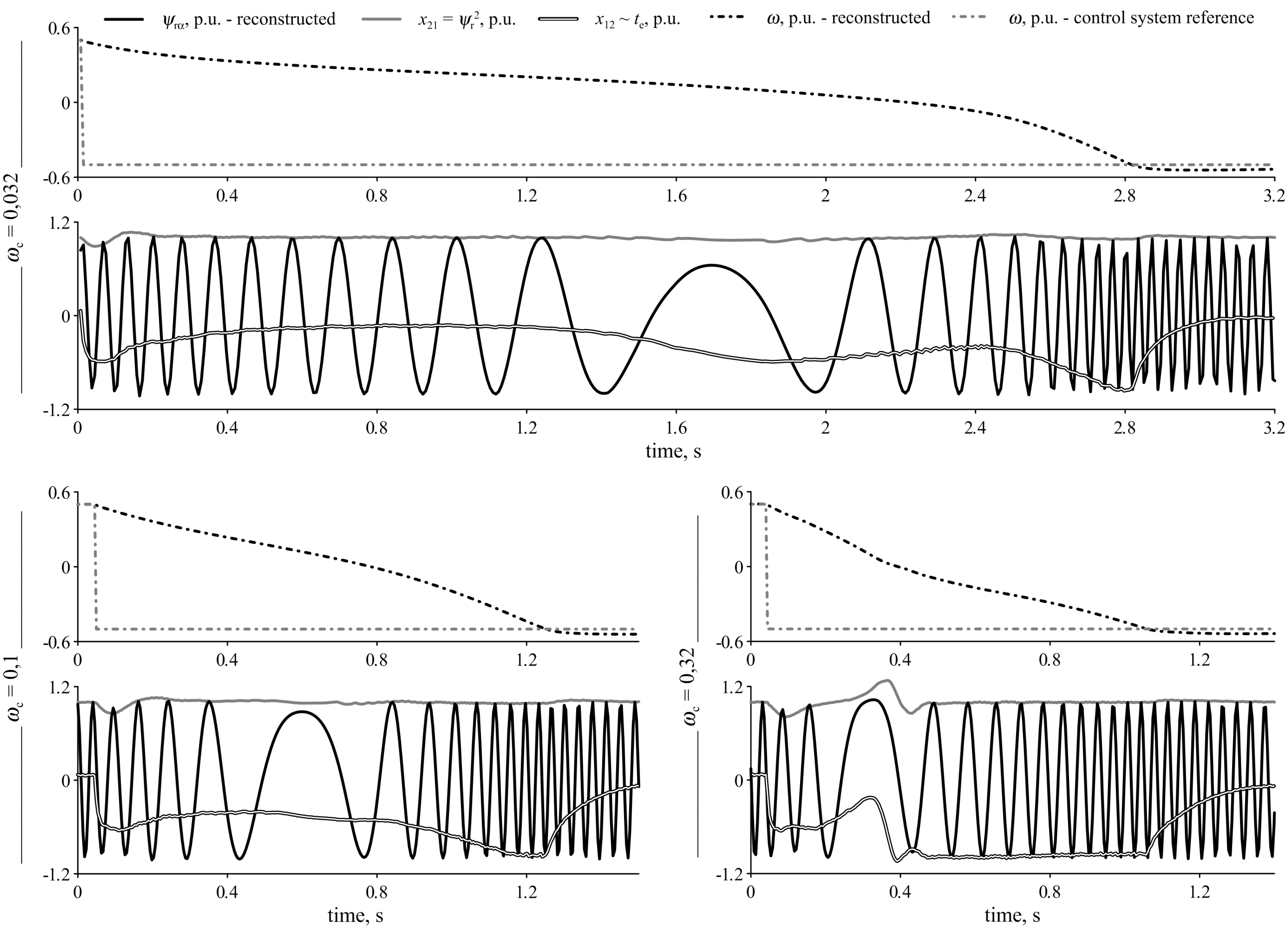

Fig. 3. Experimental results acquired in multiscalar control system for PI 1 observer and different settings of modified integral unit

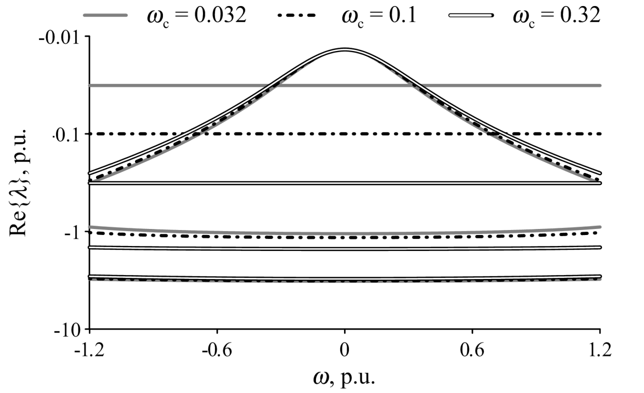

Fig. 4. The impact of the settings of modified integral unit on dynamical properties of PI 1 observer

5. Conclusions

The parameters selection of PI observers is a complicated problem, both from mathematical point of view, as well as taking into consideration a great number of criteria that must be met. A significant level of difficulty results from the complicated structure of the observer, that additionally requires modifications aimed at providing stability for the observer and avoiding the problems with DC-offset cumulation. However, the PI observer provides good conditions for the reconstruction errors attenuation, and when it is applied in a multiscalar control system of an induction motor it provides the better control quality than a proportional observer.

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