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Innovative Look at the 'General Method' of Assessing Buckling Resistance of Steel Structures

Abstract: Stability checking is an essential element of the dimensioning of steel frame structures. One of the stability checking methods allowed by EN 1993-1-1 is the so-called general method of assessing structural stability, based on buckling curves and relative structure slenderness $\overline{\lambda}_{op}$ usually determined through numerical analyses. But this method is not widely used because of the limited computing capabilities of the engineering programs dedicated to static load analyses and difficulties in interpreting the results of the computations. The commonly used computer programs enable one to determine the shape of buckling and critical load amplifier α_{cr}, from which one cannot directly determine the risk of buckling of a real structure. This paper presents a modified and innovative approach to the general method of assessing structural stability, which uses only three parameters, that is, the type of cross section, cross-section strength utilisation and α_{cr}, to determine a member's/structure's bearing capacity mobilisation from the stability condition. The problem solution is presented in the form of simple formulas and legible diagrams. Finally, synthetic conclusions are formulated.

Keywords: steel structures; stability; buckling; general method.

1 Introduction

In order to evaluate the ultimate limit state (ULS) of steel members in compression or/and bending, it is necessary to determine their buckling resistance (Boissonnade et al., 2006; Sedlacek & Naumes, 2009; Davison & Owens, 2012; Stachura & Giżejowski, 2015; Hajdu & Papp, 2018). According to EN 1993-1-1:2005, one can use one of the following four methods to assess the ULS of a structure or its members with regard to global buckling:

1) The classical method based on buckling lengths, buckling curves and the standard ULS criteria for compressed and flexural members (sect. 6.3.1-6.3.3 [EN 1993-1-1:2005])

2) The general method of evaluating stability, using buckling curves and global relative structural slenderness $\overline{\lambda}_{op}$ determined on the basis of the critical load amplifiers for system elastic buckling (sect. 6.3.4 [EN 1993-1-1:2005]) (Bijlaard et al., 2010). The amplifiers are usually determined through a numerical analysis

3) A geometrically non-linear elastic analysis (GNA) of a structure with initial bow imperfections of bars and sway imperfections of frames, which can be replaced with equivalent forces (sect. 5.3.2 [EN 1993-1-1:2005])

4) A GNA of a structure with initial equivalent imperfections corresponding to the scaled shape of elastic buckling mode (sect. 5.3.2 (11) [EN 1993-1-1:2005])

The classical method 1 is most commonly used by designers owing to its simplicity, reliability and computation speed. Methods 3 and 4 are rarely used because of the required long computing time, the difficult choice of proper imperfections for a large number of bars and load combinations and the fact that they have not been widely implemented in engineering computer programs. Also, method 2 is not commonly used due to insufficient knowledge as to its proper use, difficulties in interpreting its results and the lack of software that would enable a comprehensive analysis of structures in different computing systems. Currently, in engineering programs (e.g. RFEM by Dlubal, SCIA Engineer by Nemetschek, Advance Design by Graitec, Idea StatiCa Member by IDEA StatiCa, AxisVM by Inter-CAD), there are additional functions that try to fill this gap. The Hungarian ConSteel software seems to be the most advanced in this matter.

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The method 2, 3, 4 of the calculation of steel structures was implemented to ConSteel software in cooperation with Szalai and Papp (2011), Szalai (2017, 2018), Szalai and Papp (2019) and Papp et al. (2019).

The aim of this study was to simplify the general method of stability assessment (sect. 6.3.4 [EN 1993-1-1:2005]) and popularise it in the design of steel structures. Moreover, this paper provides an answer to the question regarding the safe value of α_{cr} (the critical load amplifier for elastic buckling) yielded by numerical computations. Moreover, the paper presents an innovative modification of the general stability assessment method, resulting in its unification and simplification. It should be noted that the proposed modification does not affect the standard level of safety of the structure being designed (EN 1993-1-1:2005) and is in accordance with the standard recommendations (EN 1993-1-1:2005).

Since denotations consistent with the nomenclature of EN 1993-1-1 (EN 1993-1-1:2005) are used in this paper, some of the symbols are not explained here as they are defined in the standard.

2 Assessment of Buckling Resistance of Compressed Members

A well-known condition, proposed in EN 1993-1-1:2005, for the ULS of a compressed member is described by equation (6.46) (EN 1993-1-1:2005), where the χ buckling coefficient is determined on the basis of the relative slenderness $\overline{\lambda}$ (6.49) (EN 1993-1-1:2005), which depends on N_{cr} (elastic critical force for the relevant buckling mode based on the gross cross-sectional properties).

The critical force N_{cr} is very often determined using computer programs. However, such programs usually do not specify critical force values for the bars, providing only the critical load amplifier $\alpha_{cr,N}$ and the shape of the buckling of the whole analysed structure. On the other hand, the computer programs used should be able to determine N_{cr} or $\alpha_{cr,N}$ in accordance with the requirements of the standard (EN 1993-1-1:2005).

In order to facilitate interpretation of the results of numerical structural buckling computations and to reduce possible errors, it is proposed to modify the method of checking the stability of compressed members presented in EN 1993-1-1:2005. For this purpose, the relative slenderness of the structure under compression (EN 1993-1-1:2005) is modified as follows:

$$\overline{\lambda} = \sqrt{\frac{Af_y}{N_{cr}}} = \sqrt{\frac{Af_y}{\alpha_{cr,N}N_{Ed}}} = \sqrt{\frac{1}{\alpha_{cr,N}U_{k,N}}} \qquad (1)$$

where $\alpha_{cr,N}$ is the critical load amplifier for elastic (flexural weak or strong axis, torsional or flexural–torsional) buckling of the member, determined for the gross cross-section characteristics and $U_{k,N}$ is the level of utilisation of the cross section's characteristic strength, calculated from the formula:

$$U_{k,N} = \frac{N_{Ed}}{N_{Rk}} \leq 1.0 \qquad (2)$$

The cross section's characteristic compressive strength N_{Rk} (the strength with no coefficient γ_{M0} taken into account) is determined in accordance with the procedures specified in the standard EN 1993-1-1:2005, depending on the cross-section class. When γ_{M0}=1.0, then N_{Rk}=$N_{c,Rd}$.

Moreover, the load capacity condition for the compressed member is written as follows:

$$U_{b,N} = \frac{\gamma_{M1}U_{k,N}}{\chi} \leq 1.0 \qquad (3)$$

The proposed procedure for evaluating the stability of a member in compression is valid for structures/members satisfying the requirements of the standard EN 1993-1-1:2005 and for any form of buckling (flexural, torsional and flexural–torsional). The key to a correct assessment of the stability of a member is to find a buckling form with a minimal critical load value.

For the so-defined condition in Eq. (3), both buckling reduction factor χ and member resistance utilisation $U_{b,N}$ depend on only three variables: the kind of cross section (and hence, the associated buckling curve), cross-section strength utilisation $U_{k,N}$ (2) and the critical load amplifier $\alpha_{cr,N}$. Thanks to this modification, easy-to-interpret nomographs or a spreadsheet for the quick assessment of the stability of a member can be developed.

3 Assessment of Flexural Buckling Resistance of Members

A well-known condition, proposed in EN 1993-1-1:2005, for the ULS of lateral torsional buckling of a member in bending has the form (6.54) (EN 1993-1-1:2005), where the

χ_{LT} reduction factor for lateral torsional buckling for the general case (sect. 6.3.2.2 [EN 1993-1-1:2005]) is determined on the basis of relative slenderness $\overline{\lambda}_{LT}$ (6.56) (EN 1993-1-1:2005), which depends on M_{cr} (elastic critical moment for lateral torsional buckling based on gross cross-sectional properties).

The critical moment M_{cr} is very often determined using computer programs. However, such programs usually do not specify critical moment values for the beams, providing only the critical load amplifier and the form of the buckling of the whole analysed structure. On the other hand, the computer programs used should be able to determine M_{cr} or $\alpha_{cr,M}$ in accordance with the requirements of the standard EN 1993-1-1:2005.

In order to facilitate interpretation of the results of numerical structural stability computations with regard to lateral torsional buckling and to reduce the possible errors, it is proposed to modify the method of checking the stability of beams in bending. For this purpose, the relative slenderness (6.56) (EN 1993-1-1:2005) of the structure under bending only is modified as follows:

$$\overline{\lambda}_{LT} = \sqrt{\frac{W_y f_y}{M_{cr}}} = \sqrt{\frac{W_y f_y}{\alpha_{cr,M} M_{y,Ed}}} = \sqrt{\frac{1}{\alpha_{cr,M} U_{k,M}}} \qquad (4)$$

where $\alpha_{cr,M}$ is the critical load amplifier for the elastic lateral torsional buckling of the member, determined for the gross cross-section characteristics, and $U_{k,M}$ is the level of utilisation of the cross section's characteristic strength, calculated from the formula:

$$U_{k,M} = \frac{M_{y,Ed}}{M_{y,Rk}} \le 1.0 \qquad (5)$$

The cross section's characteristic resistance to bending $M_{y,Rk}$ (the resistance with no coefficient γ_{M0} taken into account) is determined in accordance with the procedures specified in the standard EN 1993-1-1:2005, depending on the cross-section class. When $\gamma_{M0} = 1.0$, then $M_{y,Rk} = M_{c,Rd}$.

Moreover, the load capacity condition of the member in bending is written as follows:

$$U_{b,M} = \frac{\gamma_{M1} U_{k,M}}{\chi_{LT}} \le 1.0 \qquad (6)$$

For the so-defined buckling condition in Eq. (6), both the reduction factor for lateral torsional buckling χ_{LT} and member resistance utilisation $U_{b,M}$ depend only on three variables: the kind of cross section (and hence, the associated buckling curve), cross-section strength utilisation $U_{k,M}$ and critical load amplifier $\alpha_{cr,M}$. Thanks

to this modification, easy-to-interpret nomographs or a spreadsheet for the quick assessment of the stability of a member can be developed.

4 Assessment of Stability of Members Being Simultaneously in Compression and In-Plane Bending

It is proposed to modify the general method described in sect. 6.3.4 of standard EN 1993-1-1:2005 to assess the stability of members being simultaneously in compression (N_{Ed}) and in-plane bending relative to the axis of greater stiffness ($M_{y,Ed}$). According to the standard recommendations, the general method of evaluating structural buckling resistance can be applied to the following:

1) single members or built-up (sect. 6.3.4 (1) [EN 1993-1-1:2005]),
2) members with uniform cross section or not (sect. 6.3.4 (1) [EN 1993-1-1:2005]),
3) members with complex support conditions (sect. 6.3.4 (1) [EN 1993-1-1:2005]),
4) members subjected to compression and/or mono-axial bending in the system plane (sect. 6.3.4 (1) [EN 1993-1-1:2005]) and
5) members losing elastic stability only out of the system plane (see the standard's definition of parameters $\alpha_{cr,op}$ [sect. 6.3.4 (3) {EN 1993-1-1:2005}] and χ_{op} [sect. 6.3.4 (2) {EN 1993-1-1:2005}]).

Under the above standard assumptions, the structural stability condition (6.63) (EN 1993-1-1:2005) takes the form:

$$\frac{\chi_{op} \alpha_{ult,k}}{\gamma_{M1}} \ge 1.0 \qquad (7)$$

where $\alpha_{ult,k}$ is the minimal design load amplifier at which the critical cross section reaches the characteristic strength in plain strain conditions with proper geometric imperfections taken into account and χ_{op} is the reduction factor for non-dimensional slenderness $\overline{\lambda}_{op}$ (6.64) (EN 1993-1-1:2005) corresponding to lateral and lateral torsional buckling at

$$\overline{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} \qquad (8)$$

where $\alpha_{cr,op}$ is the minimum amplifier for the in-plane design loads to reach the elastic critical resistance of the structural component with regards to lateral or lateral

torsional buckling without accounting for the in-plane flexural buckling.

Reduction factor $\underline{\chi_{op}}$ can be evaluated on the basis of relative slenderness $\overline{\lambda}_{op}$ as the lower of the factors χ or χ_{LT} determined from the standard buckling curves. Load amplifier $\alpha_{ult,k}$ is calculated from the characteristic cross-section strength condition:

$$\frac{1}{\alpha_{ult,k}} = \frac{N_{Ed}}{N_{Rk}} + \frac{M_{y,Ed}}{M_{y,Rk}} \qquad (9)$$

and then the stability condition (6.65) (EN 1993-1-1:2005) assumes the form:

$$\frac{N_{Ed}}{N_{Rk}/\gamma_{M1}} + \frac{M_{y,Ed}}{M_{y,Rk}/\gamma_{M1}} \le \chi_{op} \qquad (10)$$

In this paper, a modification of the formula of general method is proposed. For this purpose, the relative slenderness of members simultaneously compressed and bent is defined as follows:

$$\overline{\lambda}_{op} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{cr,op}}} = \sqrt{\frac{1}{\alpha_{cr,NM} U_{k,NM}}} \qquad (11)$$

where $\alpha_{cr,NM}$ is the critical load amplifier for the elastic buckling of a member being simultaneously in compression and mono-axial bending, determined for the gross cross-section characteristics. As opposed to its counterpart ($\alpha_{cr,op}$) in the standard EN 1993-1-1:2005, this coefficient takes into account the effects of in-system plane and out-of-system plane buckling. $U_{k,NM}$ is the level of characteristic cross-section strength utilisation calculated from the formula:

$$U_{k,NM} = \frac{N_{Ed}}{N_{Rk}} + \frac{M_{y,Ed}}{M_{y,Rk}} \le 1.0 \qquad (12)$$

Resistances N_{Rk} and M_{Rk} are determined according to the cross-section class, taking into account the plastic resistance of cross sections of class 1, 2 or effective resistance of cross sections of class 4 (Boissonnade et al., 2006; Sedlacek & Naumes, 2009).

If resistance M_{Rk} is determined only in the elastic state (e.g. for cross-section class 3), then the level of characteristic cross-section strength utilisation can be calculated from the relation:

$$U_{k,NM,el} = \frac{\sigma_N + \sigma_{My}}{f_y} \le 1.0 \qquad (13)$$

where σ_N denotes the normal stresses in the cross section, produced by compressive force N_{Ed}, and σ_{My} denotes the normal stresses in the cross section, produced by bending moment $M_{y,Ed}$.

Stresses above can be determined according to the first- or second-order analysis as defined in sect. 5.2.1 (3) (EN 1993-1-1:2005). If the structure has a small deformation, second-order analysis gives the same internal forces and stresses as first-order analysis.

The values of $U_{k,NM}$ determined according to Eqs (12) and (13) are identical if the characteristic bending resistance of the cross section $M_{y,Rk}$ determined on the basis of the elastic bending section modulus $W_{y,el}$ is inserted into Eq. (12) as for the section of class 3.

In the modified method, the load capacity condition for a member being simultaneously in compression and in-plane bending can be defined in two ways:

$$U_{b,NM,1} = \frac{\gamma_{M1} U_{k,NM}}{min(\chi_{NM}; \chi_{LT,NM})} \le 1.0 \qquad (14)$$

or

$$U_{b,NM,2} = \frac{\gamma_{M1} U_{k,NM}}{\chi_{NM}} \frac{U_{k,N}}{U_{k,NM}} + \frac{\gamma_{M1} U_{k,NM}}{\chi_{LT,NM}} \frac{U_{k,M}}{U_{k,NM}} =$$
$$= U_{b,NM,\chi} \frac{U_{k,N}}{U_{k,NM}} + U_{b,NM,\chi LT} \frac{U_{k,M}}{U_{k,NM}} \le 1,0 \qquad (15)$$

where χ_{NM} and $\chi_{LT,NM}$ are the buckling and lateral torsional buckling reduction factors, respectively, determined for the proper buckling curve, at cross-section strength utilisation $U_{k,NM}$ and $\alpha_{cr,NM}$.

Eq. (15) enables linear interpolation (recommended in sect. 6.3.4 (4) b) [EN 1993-1-1:2005]) between the buckling and lateral torsional buckling reduction factors in the case where they are determined from various buckling curves. Eqs (14) and (15) yield identical $U_{b,NM}$ values if the cross section's buckling curve and lateral torsional curve are identical ($\chi_{NM}=\chi_{LT,NM}$), and the level of utilisation of the cross-section capacity is determined consistently from the following equations: $U_{k,N}$ Eq. (2), $U_{k,M}$ Eq. (5), $U_{k,NM}$ Eq. (12). In the other cases, Eq. (14) yields higher $U_{b,NM}$ values. Of course, if the element is protected against strong or weak axis flexural buckling, torsional buckling, flexural–torsional buckling or lateral torsional buckling, the appropriate coefficients $\chi_{NM}=1$ or $\chi_{LT,NM}=1$ should be adopted

in Eq. (15). On the basis of the presented nomographs (Figs 1–6), one can easily determine structural stability from Eq. (14). When structural buckling resistance is evaluated according to Eq. (15), one can determine $U_{b,NM,\chi}$ and $U_{b,NM\chi LT}$ using nomographs (Figs 1–6) and then, they should be multiplied like in Eq. (15).

It should be noted that the general method (EN 1993-1-1:2005) can be used for analysing the buckling resistance of a structure not only out of the system's plane, but also in its plane for the critical factor $\alpha_{cr,NM} \geq 10$ (but the cross sections of the structure's components should be so positioned that the plane of greater stiffness and the bending plane coincide with the system's plane). Moreover, Polish Annexe to EN 1993-1-1:2005 states thus: 'first-order analysis without taking into account geometrical imperfections can be used for non-sway systems as well as for single-storey sway systems'. It means that for typical single-story steel frame systems, general method could be used for the assessment of in-plane buckling resistance. In the proposed method, when only a compressive force or a bending moment acts, parameters $U_{k,NM}$ Eq. (12) and $U_{b,NM}$ Eq. (15) coherently reduce their values to $U_{k,N}$ Eq. (2), $U_{b,N}$ Eq. (3) or $U_{k,M}$ Eq. (5), and $U_{b,M}$ Eq. (6), respectively. Because of assumption 5, such a coherent transition would not be possible in the standard's general method.

In the case of difficulties in interpreting the obtained form of a member's buckling, a safe estimate of its stability will be made assuming the more disadvantageous buckling curve (out-of-system plane buckling).

5 Results and Discussion

Thanks to the presented unified approach to the assessment of the stability of members in compression or/ and bending, the following simple procedure for stability assessment can be proposed:

1) Determine the cross section's strength utilisation: $U_{k,N}$ Eq. (2), $U_{k,M}$ Eq. (5), $U_{k,NM}$ Eq. (12). In the light of the formulas proposed above, the most stressed cross section is representative of the member's stability.

2) Determine the minimal critical load amplifier for the elastic buckling ($\alpha_{cr,N}$, $\alpha_{cr,M}$, $\alpha_{cr,NM}$) of a member or the whole structure. Make sure that the software is able to reliably determine the load amplifiers for all the forms of global buckling (flexural, torsional, flexural–torsional, lateral torsional), disregarding the local buckling of the walls of class 4 cross sections. For example, the popular Autodesk ROBOT software, when modelling a structure by means of beams finite elements, determines only the flexural forms of buckling. The neglect of the other buckling forms can lead to an incorrect result.

3) Identify the member's buckling curve/curves (a_0, a, b, c and d) on the basis of the geometry of its cross section and direction of buckling.

4) Determine the level of utilisation of the structure's resistance from the stability condition: U_b Eqs (3, 6, 14, 15) on the basis of the nomographs (Figs 1–6 – intermediate values can be interpolated) or the proposed formulas for the level of utilisation of the member's resistance.

5) Repeat steps 1–4 for all the load combinations. Each time, resistance utilisation U_b in the analysed cross sections of the structure should be stored.

6) Generate an envelope of structure resistance utilisation U_b stemming from the general buckling condition.

In the graphs (Figs 1–6), the critical load amplifier (α_{cr}) values are marked on the horizonal axis while the member strength utilisation (U_k Eqs (2, 5, 12)) values determined for the characteristic strength of the cross section are marked on the vertical axis.

The graphs shown in Figs 1–6 were drawn for $\gamma_{M1}=1.0$. In the case of other values of coefficient γ_{M1} read off the graph, one should multiply the value of U_b Eqs (3, 6, 14, 15) by γ_{M1} and check if it is lower than 1.

Fig. 1 shows the structure resistance utilisation curves U_b, respectively, for the buckling curves a_0, a, b, c, d, which in a given group (U_b=1.0;0.8;0.6, etc.) show the same utilisation level (each point on the curve in a given group represents the same utilisation).

The cases in which coordinates α_{cr} and U_k indicate the points below curve U_b=1.0 are regarded as safe states of the member/structure (Figs 1–6). The points situated above this curve represent the states in which the member loses its stability.

Moreover, it follows from the graphs (Figs 1–6) that when $\alpha_{cr} \leq 1.0$, the real member buckles, regardless of the level of cross-section strength utilisation U_k.

A great advantage of this method is that the value by which the load of a member/structure can be increased so that the member/structure reaches the ULS can be easily determined from the general stability condition. The load increasing multiplier can be calculated from the formula:

$$\alpha_{lim} = \frac{1}{U_b} \qquad (16)$$

Moreover, the analyses showed that the buckling reduction factor χ or the lateral torsional reduction factor χ_{LT} reaches
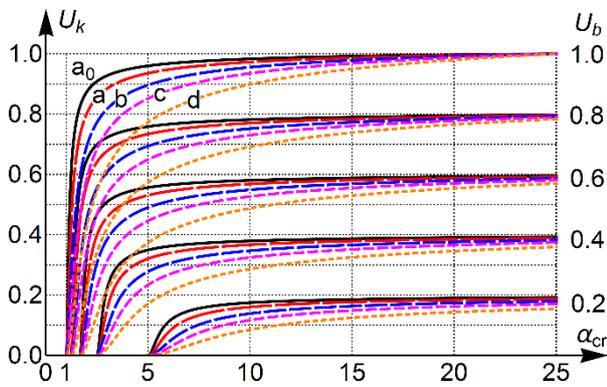
Figure 1: Member resistance utilisation U_b Eqs (3, 6, 14) with regard to global buckling for the buckling curves a_0, a, b, c, d at α_{cr} = 1–25 and γ_{M1}=1.0.
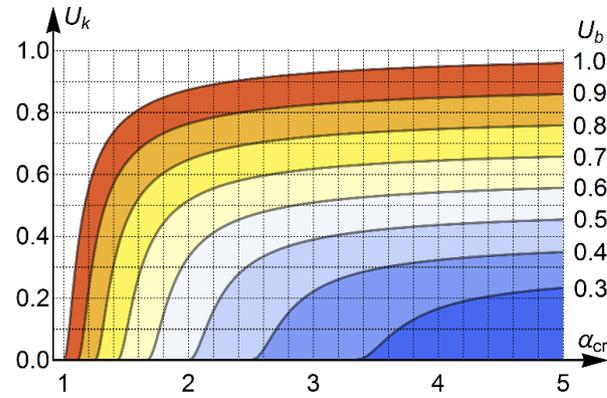
Figure 2: Member resistance utilisation U_b Eqs (3, 6, 14) with regard to global buckling for the buckling curve a_0 at α_{cr} = 1–5 and γ_{M1}=1.0.

the value of 1.0 (no reduction in member resistance with regard to buckling) when

$$\alpha_{cr} \geq \frac{25}{U_k} \qquad (17)$$

A graphical interpretation of Eq. (17) is shown in Fig. 1 where the buckling curves a_0, a, b, c, d of the ULS (U_b=1.0) converge in one point α_{cr}=25, U_k=1.0. In this case, the member's resistance with buckling taken into account reaches the cross section's resistance and further increasing α_{cr} no longer results in increased resistance of the member or the structure.

This finding explains the standard's statement (EN 1993-1-1:2005) in section 6.3.1.2: 'For slenderness $\overline{\lambda}$≤0.2 or for N_{Ed}/N_{cr} ≤0.04, the buckling effects may be ignored and only cross sectional checks apply', since it was shown above that for U_k=1.0, $1/\alpha_{cr}$ =N_{Ed}/N_{cr} =1/(25=0.04).

In the case of lower cross-section strength utilisation levels (U_k≤1.0), the reduction factors χ and $_{LT}$ reach the value of 1.0 at higher values of α_{cr}, for example, for U_k=0.2, α_{cr}=25/0.2=125.

By analysing the properties of the obtained boundary surfaces (Figs 1–6), it can be concluded that for any utilisation level of a structure element Ub,i<1 resulting from the parameters $U_{k,i}$, $\alpha_{cr,i}$, it is easy to determine the parameters for the limit state $U_{b,lim}$=1 and they are equal to $U_{k,lim}$=$U_{k,i}/U_{b,i}$, $\alpha_{cr,lim}$=$\alpha_{cr,i} U_{b,i}$ (applies to sections 1, 2, 3).

6 Workout Example

Determine the utilisation of load-bearing capacity of frame columns in Fig. 7a. The frame is restrained against out-of-plane buckling and lateral torsional buckling. Verification is carried out by the classical method and by the general method using the nomographs proposed in this paper.

Data:
Steel: S235, f_y=235 MPa, E=210 GPa
Height of the frame: h_c=4.0 m
Span of the frame: L_b=4.0 m
Compression force in the column: N_{Ed}=1700 kN
Column and girder cross section: HEA 300, h=290 mm, b_f=300 mm, A=112.5 cm², I_y=18.260 cm⁴
Static calculations and stability analysis were performed in the software Consteel v.14.

Verification by classical method sect. 6.3.1-6.3.3 (EN 1993-1-1:2005)
Buckling length of a column was determined according to Boissonnade et al. (2006).
Distributor factor in the bottom node of the column
$K_c = \frac{I_y}{h_c} = 45.65 \text{ cm}^3$
$K_1 = \infty -$ rigid support
$\eta_1 = \frac{K_c}{K_c + K_1} = 0$

Distributor factor in the top node of the column
$K_c = \frac{I_y}{h_c} = 45.65 \text{ cm}^3$
$K_2 = 1.5 \frac{I_y}{L_b} = 68.475 \text{ cm}^3$
$\eta_1 = \frac{K_c}{K_c + K_2} = 0.400$
Buckling length
$l_{cr,y} = h_c * \sqrt{\frac{1 - 0.2(\eta_1 + \eta_2) - 0.12\eta_1\eta_2}{1 - 0.8(\eta_1 + \eta_2) - 0.60\eta_1\eta_2}} = 4.0 \text{ m} * 1.163 = 4.653 \text{ m}$
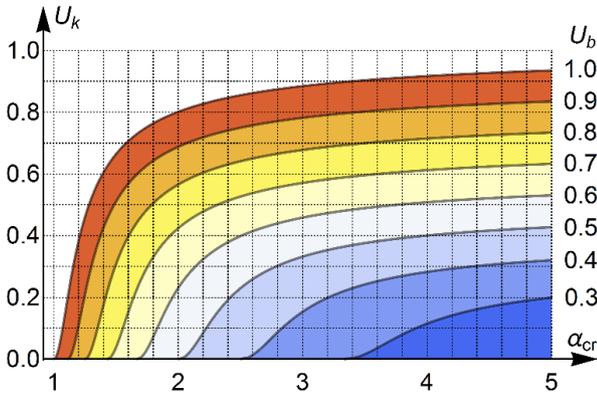
Figure 3: Member resistance utilisation U_b Eqs (3, 6, 14) with regard to global buckling for the bucking curve *a* at α_{cr} = 1–5 and γ_{M1}=1.0.
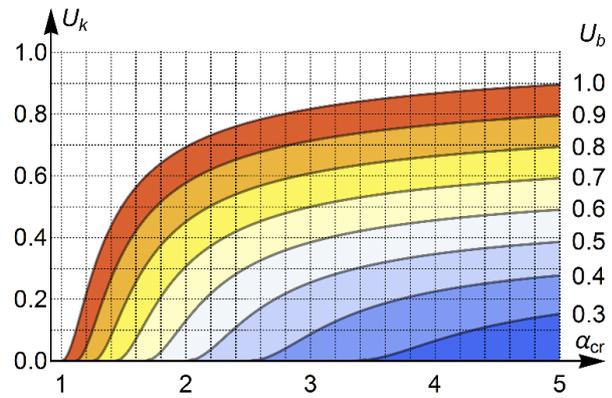
Figure 4: Member resistance utilisation U_b Eqs (3, 6, 14) with regard to global buckling for the buckling curve *b* at α_{cr} = 1–5 and γ_{M1}=1.0.
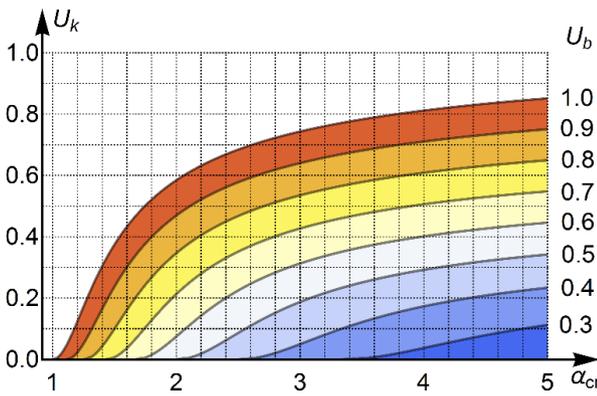
Figure 5: Member resistance utilisation U_b Eqs (3, 6, 14) with regard to global buckling for the buckling curve *c* at α_{cr} = 1–5 and γ_{M1}=1.0.
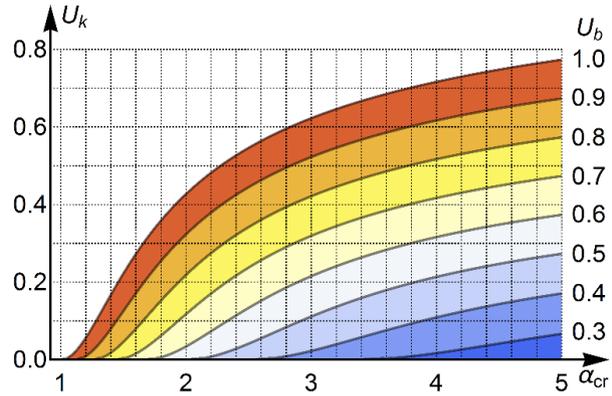
Figure 6: Member resistance utilisation U_b Eqs (3, 6, 14) with regard to global buckling for the buckling curve *d* at α_{cr} = 1–5 and γ_{M1}=1.0.
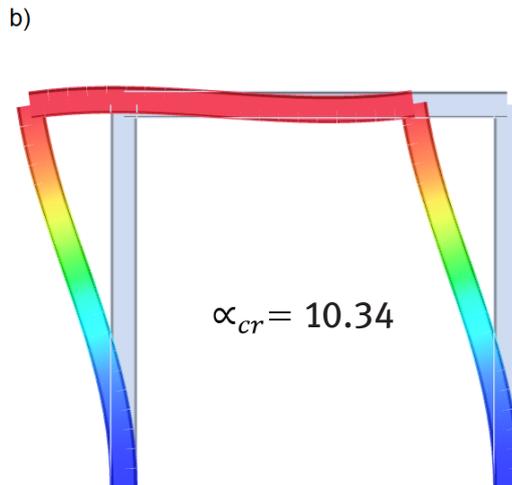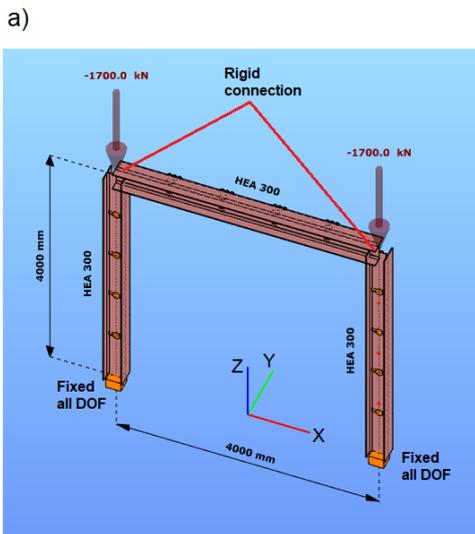
Figure 7: Analysed steel frame: a) static scheme, b) first (in-plane) buckling mode and critical load amplifier.

Critical force of buckling

$$N_{cr,y} = \frac{\pi^2 E I_y}{(l_{cr,y})^2} = 17483 \text{ kN}$$

Buckling reduction factor

$$\frac{h}{b_f} = \frac{290 \text{ mm}}{300 \text{ mm}} = 0.967 \Rightarrow \text{imperfection buckling curve 'b', } \alpha=0.34$$

$$\lambda = \sqrt{\frac{A f_y}{N_{cr,y}}} = 0.3889$$

$$\phi = 0.5(1+\alpha(\lambda-0.2)+\lambda^2) = 0.6077$$

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \lambda^2}} = 0.9305$$

$$N_{b,Rd} = \frac{\chi A f_y}{\gamma_{M1}} = 2460 \text{ kN}$$

Buckling resistance of compression member

$$\frac{N_{Ed}}{N_{b,Rd}} = \frac{1700 \text{ kN}}{2460 \text{ kN}} = 0.691$$

Verification by general method and nomographs

$$\frac{h}{b_f} = \frac{290 \text{ mm}}{300 \text{ mm}} = 0.967 \Rightarrow \text{imperfection buckling curve 'b', Fig. 1}$$
or Fig. 4

$$U_{k,N} = \frac{N_{Ed}}{A f_y} = 0.634$$

$\alpha_{cr,N}$=10.34 – calculation in Consteel using 7DOF beam element (Fig. 7b)

$U_{b,N}$=0.68 – utilisation of column load-bearing capacity read from Fig. 1.

There is satisfactory convergence of calculations between the two methods, which is adequate in practical design.

7 Conclusions

The analyses conducted allowed for the formulation of the following conclusions regarding the proposed modified method of assessing the ultimate load-bearing capacity of a structural member from the stability loss condition:

1) This method can be applied to arbitrarily supported members being in compression and bending relative to the axis of greater stiffness, for any load distribution. The method can also be used to calculate members with a cross section variable along their length, but the results obtained are conservative.

2) The method eliminates the necessity to determine the member buckling length coefficient and to choose between a sway system and a non-sway system. The method enables a smooth transition between the two kinds of systems.

3) The so-formulated method is easy in practical use and time-efficient (short computation time). By means of the nomographs (Figs 1–6), one can quickly check the load-bearing capacity utilisation of even complex structures. Consequently, errors due to an incorrect

interpretation of the results of numerical analyses are reduced. The method can be used as a tool for quick verification of structural design.

4) The method eliminates the need to separately check the different buckling forms (flexural, torsional, flexural–torsional and lateral torsional). Maximum member resistance utilisation U_b is determined on the basis of the minimal value of α_{cr}. Therefore, it is essential that the software used for structural computations is able to reliably determine not only the flexural modes of buckling, but also the torsional and flexural–torsional ones. In the case of beam finite elements, this means the seventh degree of freedom.

5) In the case of compression-only elements, the proposed nomographs (Figs 1–6) provide the same results of the assessment of the ultimate load-bearing capacity of the elements as the classical approach presented in sect. 6.3.1 (EN 1993-1-1:2005). This method enables a quick and easy assessment of the load-bearing capacity of columns with any support conditions, any axial force distribution and variable cross section along the length.

6) In the case of bending-only elements, the proposed nomographs (Figs 1–6) provide the same results of the assessment of the ultimate load-bearing capacity of the elements as the approach presented in sect. 6.3.2.2 (EN 1993-1-1:2005) (lateral torsional buckling curves – general case). This method enables a quick and easy assessment of utilisation of the load-bearing capacity of single-span and multi-span beams with any support conditions, any bending moment distribution and a variable cross section along the length. Of course, one can use other lateral torsional buckling curves, for example, the ones presented in sect. 6.3.2.3 (EN 1993-1-1:2005), and calculate $\overline{\lambda}_{LT}$ according to Eq. (4).

7) In the case of elements being simultaneously in bending and axial compression, the estimates of their ultimate load capacity according to the proposed modification are the same as the estimates presented in sect. 6.3.4 (4) a) and b) (EN 1993-1-1:2005). The precision of general method mainly depends on which procedure (sect. 6.3.2.2 [EN 1993-1-1:2005] or sect. 6.3.2.3 [EN 1993-1-1:2005]) was chosen to calculate lateral–torsional reduction factor. In Bijlaard et al. (2010), it was shown that the general method is conservative. In the general method, calculations are conducted for the cross section with maximum utilisation of load-bearing capacity. According to more accurate calculation with first mode of buckling shape adopted as equivalent geometrical imperfection of member, the representative cross section is less utilised.

Despite the fact that the method is conservative and overestimates utilisation of the structure load-bearing capacity, it can be useful in detecting the so-called 'thick' design errors or for quick selection of cross section of elements.

8) As opposed to geometrically and material-wise non-linear analyses covering geometric imperfections, especially arduous in the case of models with a large number of bars and load combinations, the proposed method is easily implementable in computing programs and fast in computation.

9) The application of the general method with the use of the proposed procedure and nomographs is intuitive for the user. The nomograms clearly show the degree of utilisation of the resistance of the cross section, the degree of utilisation of the element-bearing capacity, taking into account the global stability, and allow for a quick assessment of the limit load of the structure.

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