NUMERICAL MODEL FOR ANALYSIS OF SPATIAL CHARACTERISTICS OF ULTRASOUND HEADS

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The paper presents the physical base of the numerical system for analysis and modelling acoustic fields generated by an acoustic head of arbitrary construction. The elaborated system allows to analyse the sending-receiving multielements ultrasound heads. In the paper modeling of the acoustic system as the linear one was proposed. Numerical model, its implementation and preliminary results was presented.

INTRODUCTION

The main aim of this work is presentation of the theoretical base and numerical implementation of the system for the analyse of acoustical heads of arbitrary construction. The elaborated system allows to describe the excitation of the array of the transducers and the analyse of spatial and time variations of the acoustical field [1–3] generated by them.

The numerical implementation of the system is presented. Its main purpose is calculating the spatial characteristics of the radiated acoustical field. More background information on the project can be found in previous publications [4–6].

1. THEORETICAL BACKGROUND OF THE PROJECTING SYSTEM

The theoretical part of the projecting system is based on the linear representation of the acoustics systems [7, 8]. Such approach assumes that the physical parameteres of the acoustic field can be represented using the convolution between the spatial impulse response of the system and the acceleration of the transducer surface under consideration. The impulse response depends on the relative position of the considered point in the field and the transducer. In most cases, this kind of dependency makes impossible to analytically solve the problem and makes necessary to use models for numerically finding of the solution.

In general, such approach can be written using the equation [4]:

$$y(t) = h(t) \star x(t) = \int_{-\infty}^{+\infty} h(t')x(t-t')dt' \tag{1}$$

where, the h(t) is the transition function of a linear system (sc. a spatial impulse response of acoustic linear system).

As a result of the analyse, the determination of the spatial impulse response for an arbitrary shaped aperture of the transducer can be achieved. The pressure field for such aperture S mounted in an infinite rigid baffle can be found by the Rayleighs' integral [7, 8]:

$$p(\vec{r_1}, t) = \frac{\rho_0}{2\pi} \int_S \frac{\frac{\partial v_n(\vec{r_2}, t - \frac{|\vec{r_1} - \vec{r_2}|}{c})}{\partial t}}{|\vec{r_1} - \vec{r_2}|} dS \tag{2}$$

where: v_n is the normal velocity of the elementary parts of a transducer surface. The above equation assumes, that according to the Huyghens' principle the radiated field can be found by summing the contributions from all small areas on the surface of the transducer. This approach assumes also that the propagation occures in a homogenous medium and that no attenuation takes place. After converting this equation and introducing the velocity potential Ψ in a form of [7, 8]:

$$\vec{v}(\vec{r}, t) = -\nabla\Psi(\vec{r}, t) \tag{3}$$

and,

$$\vec{p_0}(\vec{r}, t) = -\rho \frac{\partial \Psi(\vec{r}, t)}{\partial t} \tag{4}$$

as well as assuming that the surface normal velocity is uniform on an entire transducer one can write down the velocity potential as:

$$\Psi(\vec{r_1}, t) = v_n(t) \star \int_S \frac{\delta(t - \frac{|\vec{r_1} - \vec{r_2}|}{c})}{2\pi|\vec{r_1} - \vec{r_2}|} dS \tag{5}$$

The integral is the spatial impulse response of the system:

$$h(\vec{r_1}, t) = \int_S \frac{\delta(t - \frac{|\vec{r_1} - \vec{r_2}|}{c})}{2\pi|\vec{r_1} - \vec{r_2}|} dS \tag{6}$$

This approach was proposed by P. Stepanishen [7] and later used by J.A. Jensen in the papers [2, 8, 9]. The authors of this work propose the expansion of this approach.

According to signal theory and assuming the linearity of physical parameters of the acoustical structure, the spatial impulse response from the complex system of transducers can be presented as a sum of responses from any number of subelements which represents the system. This approach can be written as:

$$h_{all} = \sum_{i=1}^{N} h_i \tag{7}$$

where h_{all} is the complete response from transducer (or array of transducers) and h_i is the response from single element. In our approach the single element is in the form of the triangle.

In such approach we can simplify the calculation of the transducer of any kind to a number of calculation of spatial impulse response from a triangle. J.A. Jensen in his paper [2] has been proposed the additional simplification that allows to treat any triangle as a set of three basic triangles.

In general the impulse response from triangle in two dimensions can be calculated from Eq. 6) by rewriting it to polar coordinative system:

$$h(\vec{r}_1, t) = \int_0^r \int_0^{2\pi} \frac{r\delta(t - \frac{|R|}{c})}{2\pi|R|} d\theta dr \tag{8}$$

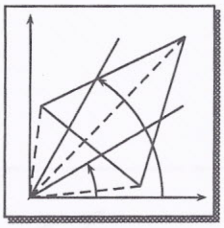

Figure 1: Schema for the calculation of spatial impulse response from triangle.

After switching to effective angles, which are shown in Fig.1 the equation for tringle under consideration can be rewriten as:

$$h(\vec{r}_1, t) = \int_0^r \int_{\theta_a}^{\theta_b} \frac{r\delta(t - \frac{|R|}{c})}{2\pi|R|} d\theta dr \tag{9}$$

where θ_a and θ_b are the boundary angles for triangle in time t of the simulation. After some substitutions the impulse response from the triangle at given time point t can be presented as:

$$h(\vec{r}_1, t) = c\frac{\theta_b(t) - \theta_a(t)}{2\pi} \tag{10}$$

This form is very usefull for numeric computations and gives the possibility to greatly enchance the modeling process of acoustic transducers. Exemplary response from uniform triangle aperture is shown on Fig. 2. On figure on its left side the aperture geomentry tranformation for specific calculation point is presented. On the right one the spatial impulse response as a function of discrete simulation time is presented. The total singnal is presented as well as the responses from elementary subtriangles.

2. NUMERICAL SIMULATION

As stated before the process of analyse of the acoustic transmiting and/or reciving head of any kind can be treat as a set of calculations of simple, basic elements. Such approach makes the automatic analysis of any array of whatever kind transducers quite possible. Numerical implementation of this method envolves:

- division of the transducers to a set of basic apertures,
- application of the weighting function to the transducers,
- transformation of the apertures to the local basis of the calculation points,
- detection of aperture type and calculation of sign vector,
- calculation of impulse response from single aperture for given time vector,
- summarisation of the resonses and creation of the directivity characteristics.

In presented simulation model as it was admited earlier, the basic element for computation is in the form of triangle. This is for reason that in general any polygon can be easily represented as a set of

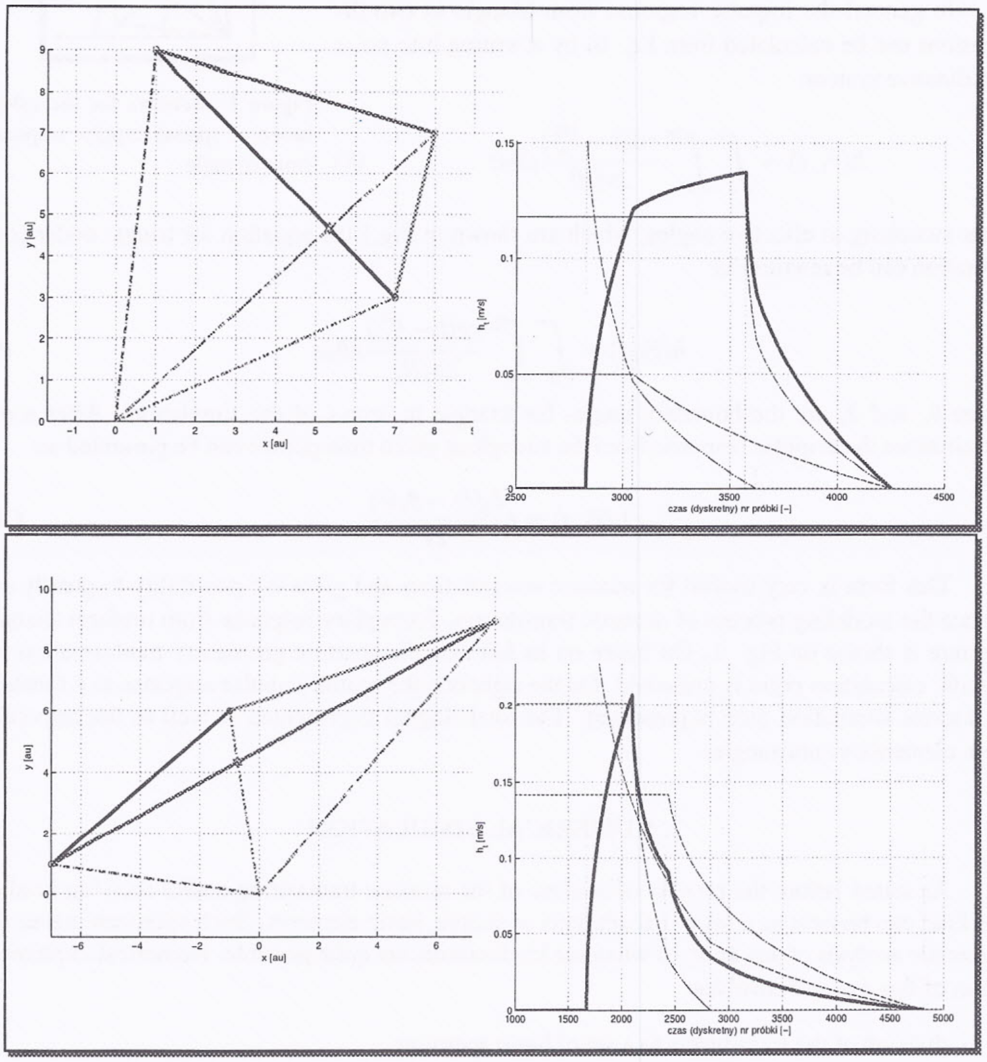

Figure 2: The exemplary spatial impulse response from triangle aperture for two different calculation examples.

triangles. This is additionaly true, because the basic shapes for transducers are rectangle and circle and trangularization methods for such geometries are well known. Exemplary triangularization effects for typical transducers are presented on Fig. 3.

The first part of the simulation involves the triangularization of the transducers. As a result we recieve a set of triangles which future computation is completly independent from each other. To such set of triangles the weighting function can be applied.

The weighting function is a solution for modeling the real world transducers in a simple manner. In general the ultrasound transducers do not vibrate in piston mode, and the transducer velocity is nonuniform through the surface. In simulation the weighting can be of any type, e.g. gaussian and can be even independly applied to each aperture.

Transformation of the three-dimensional representation of the set of aperture-calculation point to two-dimensional one can be greatly simplified and speed-up the calculation procedure. This method allows us to convert any triangle which is defined in three-dimensional certhesian world to a simple, two-dimensional form with an additional elevation factor. The conversion method which has been proposed by authors of this paper envolves some basis conversion of global and local frames. Such approach makes the conversion more general. Entire conversion procedure was performed in four-dimensional affine space using Barycentric coordination system [10]. This part of the procedure is one of the most power consuming ones because it involves some transformations of 4x4 matrixes (such as finding the inverse matix or solving the equations for determinants). As an effect the aperture form in well definied two-dimensional environment can be found.

Further procedure involves detection of the location and orientation of the aperture in new basis and its clasification. To simplify the process, the procedure of "sign" vector was proposed. As a result of procedure we obtain the simple vector which allows the interchangable calculation of the aperture's spatial impulse response.

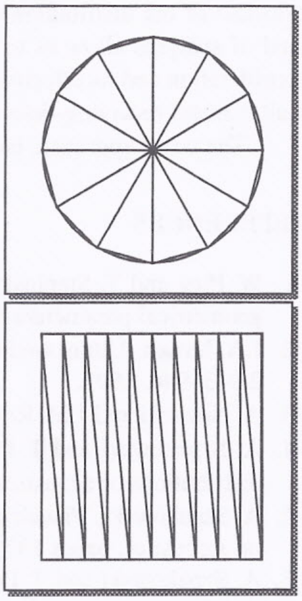
Figure 3: Effect of the triangularization procedure fo typical transducers geometry.

The calculation of the aperture spatial impulse response involves the modified procedure proposed earlier by Jensen [9]. This modified procedure bases on the division of the simulation time vector to a set of subvectors which represents the singiulartity points in spatial impulse response function. This modified approach greatly simplifies the calculation to a set of basic matrix operations.

The final part of the calculations involves the sumarization of the responses from all of the apertures, its normalization and conversion to a directivity angles form.

3. IMPLEMENTATION OF THE SIMULATION

Proposed numerical model was implemented in GNU Octave/Matlab Environments. Just numerical environment was chosen because it's high level language and its the robust implementatnion of numerical procedures.

In the future the model will be implemented in Python using SciPython clases. Such approach is in our opinion quite intresting because of its free status (OpenSource license), great numeric support (ScientyficPython) and its MPI and BSP support (for parallel computation).

4. CONCLUSION

The main aim of this work was the creation of the system which makes able to theoretically calculate of the distribution of acoustic pressure caused by ultrasound multielement head. This kind of analyses allow us to estimate the directivity characteristics of the ultrasound head under consideration and its effectiveness. The elaborated system will be applied for the projecting of the multielement receiving-detecting heads for hydroacoustic as well as medical applications.

The novel approach, based on the P. Stepanishen and J.A. Jensen works has been presented.

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