

## MATHEMATICAL SIMULATION OF INFOCOMMUNICATION NETWORKS APPLYING CHAOS THEORY

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**Abstract.** The advent and wide usage of computer networks as well as an increasing number of various network services have resulted in the fact that the network traffic has become more complex and unpredictable. These properties have become especially apparent with the appearance of high-speed data transmission technology. It is connected with the fact that one of the main quality indices (QoS) of operation of packet transmission networks is the number of lost packets. The loss of packets results to additional network traffic and, finally, “congestions”. At high speeds of data transmission packet losses, expressed in portions of a per-cent, lead to considerable information losses. It has been shown in numerous papers devoted to the research of network traffic that the abovementioned phenomena are related to the properties of the traffic self-similarity which is mainly caused by the TCP protocol behaviour. However to date models have not been offered which adequately describe the behaviour of the communication networks of information systems and which allow to apply the whole arsenal of classic methods of analyzing nonlinear dynamic systems.

**Key words:** chaos, dynamic systems, self-similarity, TCP/IP.

### INTRODUCTION

Presently TCP protocol is the basic Internet transport protocol, which ensures reliable delivery of a byte flow with connection establishment and it is applied in those cases when guaranteed messages delivery to be required. Also it includes the flow control mechanism which ensures that a sender does not overflow the recipient buffer and congestion control mechanism which tries to prevent the oversized data volume injection in a network, which leads to the loss of packets.

The numerous studies of processes in the Internet network have shown that statistical characteristics traffic have the temporal scalable invariance property [13,18,25] (self-similarity) and TCP protocol being the basic data transmission protocol is the principal reason of traffic self-similarity. Applying the concept of self-similarity to telecommunication systems has been for the first time suggested by B.B. Mandelbrot [2].

Besides the Internet architecture of protocols places certain limitations. For example, competitive TCP flows [28,29] and all plurality of transport protocols used in computer networks “don’t know anything about each other”, i.e. data flow to be managed with various schemes (or it doesn’t occur at all) [11].

In addition different models of the congestion control in TCP and other protocols should coexist in harmony. If one of the TCP implementations turns out to be more aggressive than the others, then it will use the major part of the bandwidth, which will hinder the data transmission in “adjacent” connections. At the same time the unnecessary conservatism of an algorithm will have a negative impact on the general protocol performance [1,9].

In recent years several new modifications of TCP protocol have been proposed. They include Binary Increase Control TCP (BIC-TCP), CUBIC TCP, Westwood TCP (TCPW), Parallel TCP Reno (P-TCP), Scalable TCP (S-TCP), Fast TCP, HighSpeed TCP(HS-TCP), HighSpeed TCP Low

Priority (HSTCP-LP), Hamilton TCP (H-TCP), Yet Another Highspeed TCP (YeAH-TCP), Africa TCP, Compound TCP etc.

For instance TCP Vegas [12] introduces a new congestion control mechanism which tries to prevent them and it doesn't react to the appeared congestions. Almost all of them are based on the well-known old TCP versions and they differ from each other by various congestion avoidance means. To be more precise, they are based on different ways of detecting the packets loss occurrence fact, which means the appearance of congestion. Various formulas for  $cwnd$  (congestion window) calculation are used in different modifications.

A new approach to analyze the communication network behaviour of information systems with TCP protocol has been suggested in this paper – to consider these systems as nonlinear dynamic systems [6,21] manifesting chaotic properties at certain values of parameters.

### TEST BED

One of disadvantages in research of infocommunication systems is a high cost of equipment. Not nearly every laboratory (and even an Internet provider) can afford buying several routers, hardware protocol analyzers, wireless access point suites and other equipment for testing new protocols, architecture optimization solutions or certain topology selection. This is exactly why program products were created which allow infocommunication systems to be simulated [20]. Developing such software make it possible to perform required research and experiments much less expensive and to obtain practically the same results as using real hardware (OPNET [4], NS [22]). Apart from apparent saving the approach of simulator using allow to conduct experiments without a real network designing.

The congestion state occurring in a computer network has been modelled by means of a discrete-time simulator with an open source code ns-3 (Network Simulator 3) in which numerous real protocols and various types of environment data transmission to be implemented. NS-3 is able to simulate a wide range of protocols and processes in real time and to integrate the simulator with a real network as well. The NS-3 is used as a tool by a large amount of researchers during quite long time and it includes numerous tests, which guarantees validity of the obtained results. The NS-3 simulator includes a great number of tests for all the components, which ensures the reliability of the results obtained.

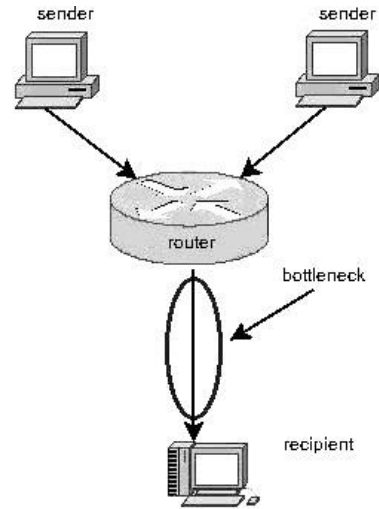


Fig. 1. Topology of a model network

The TCP/IP network model has been created by means of the NS-3 simulator where all the hosts are connected with a router by the point-to-point connection type (Fig.1). The operation of the applications has been simulated in the sender-hosts sending data with a permanent bitrate to the recipient host where the application operated and received data from the both types of hosts [8,26]. The numerical experiments have been conducted in which the sender-hosts transmitted data with different bitrate exceeding the capacity of the recipient channel, which resulted in overfilling the router buffer and unnecessary packets were rejected. The data generation rate senders ( $C_f$ ), the delay ( $d_b$ ) and capacity ( $C_b$ ) of the channels in a bottleneck as well as the delay and capacity of the channels in the sending hosts could be varied specifying particular parameters in the given model. The receiving host window was deliberately made very large to make the value of the congestion window ( $cwnd$ ) the only limiting factor. The value of congestion window ( $cwnd$ ) has been chosen as the observed parameter and the most informative one as it directly affects the transmitted data size. The TCP-Tahoe mechanism of congestion control has been used for the network operation simulation [24].

The  $cwnd$  value has been monitored for each TCP-connection throughout modeling of the congestion state in the network. The paper [27] has suggested using the values of the time sequence  $[x_t, x_{t-\sigma}, x_{t-2\sigma}, \dots]$  averaged over  $N$  as an easily measured characteristic of complex systems, and it has been shown that it is applicable to the recovery of the hidden multidimensional trajectories. The given method applied to the  $cwnd$  values leads to the relations [28]:

$$\begin{aligned} x[i] &= \frac{1}{N} \sum_{j=1}^N cwnd_x[i-j] \\ y[i] &= \frac{1}{N} \sum_{j=1}^N cwnd_y[i-j] \end{aligned}, \quad (1)$$

Here  $x$  and  $y$  denote two TCP flows. The  $N$  quantity is responsible for an averaging scale and the more  $N$  is the more hidden dimensions of the system can be recovered.

In this case  $cwnd(t)$  functions are different for each of the hosts and only moments of changing the values of this function have been fixed. Therefore applying the abovementioned method and plotting a phase portrait require taking  $cwnd$  values at the same moment of time.

For the further analysis of the data obtained one should plot a phase portrait – dependence  $cwnd1(cwnd2)$ . In order to get that done one should take  $cwnd$  values at the same moment of time.

## NUMERICAL RESULTS

Under certain parameters of the test bed such a system that seems to be rather simple, manifests quite a complicated behaviour. In particular, below are given the graphs of  $cwnd(t)$  dependence at

$C_f=5\text{Mbps}$ ,  $d_b=10\text{ms}$ ,  $C_b=5\text{Mbps}$ ,  $Q_s=20$  packets (1 packet = 536 bytes, for all the numerical experiments).

The phase portraits corresponding to Fig. 2 and obtained by data processing, according to the algorithm described in the previous section, at  $N=2000$  and  $dt=10$  ms, are shown in Fig. 3.

As it can be seen the phase trajectories shape a limit cycle which has quite a delicate structure. At that this trajectory is rather steady an image point after a little roaming begins describing the same closed trajectory at changing the start time of TCP flows relative to each other. Such trajectories are called attractors. More technical definition is proposed in [16].

For comparison, below are given data and the phase portrait for the case when the host-senders don't overfill the recipient bandwidth i.e. there is no congestion (Fig. 4 and Fig.5). In this case, as one would expect, the  $cwnd$  value of the both hosts indefinitely increases and there are no anomalies in the phase portrait.

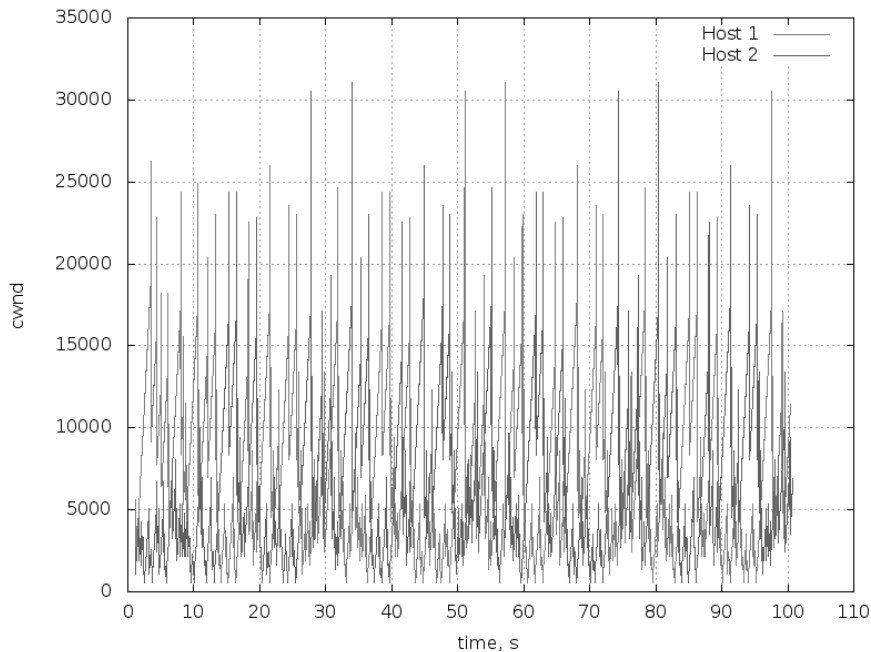


Fig. 2. Congestion window dependence on time under  $C_f=5\text{Mbps}$ ,  $d_b=10\text{ms}$ ,  $C_b=5\text{Mbps}$ ,  $Q_s=20$

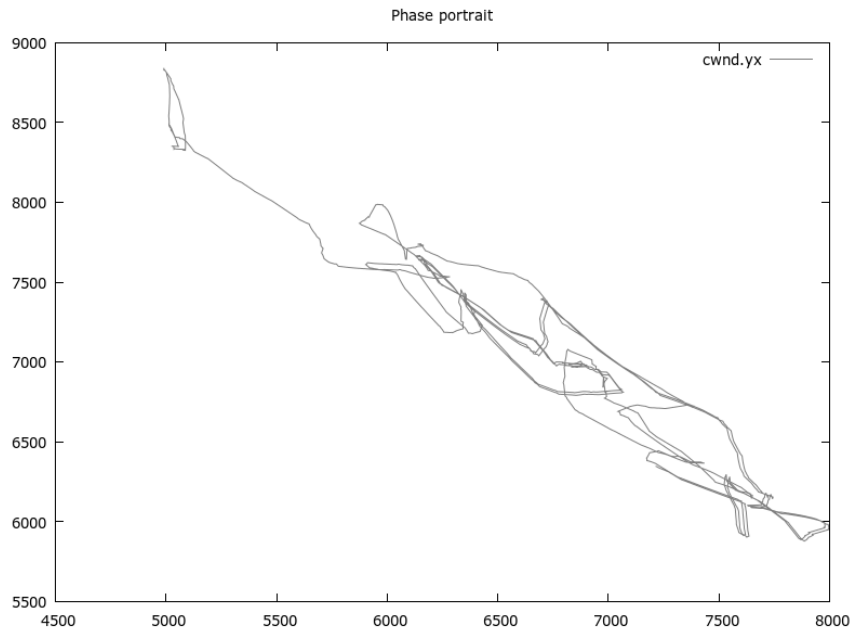


Fig. 3. Phase portrait under  $C_f=5\text{Mbps}$ ,  $d_b=10\text{ms}$ ,  $C_b=5\text{Mbps}$ ,  $Q_s=2$

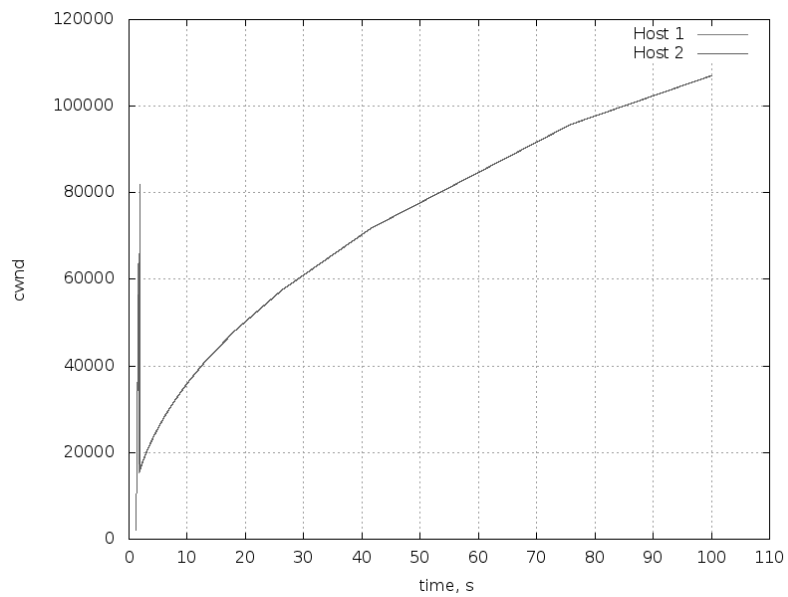


Fig. 4. Congestion window dependence on time under  $C_f=2\text{Mbps}$ ,  $d_b=10\text{ms}$ ,  $C_b=5\text{Mbps}$ ,  $Q_s=100$

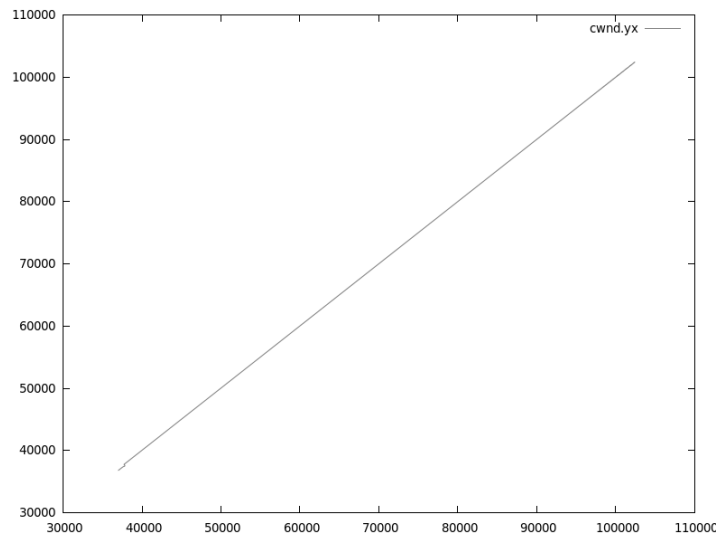


Fig. 5. Phase portrait under  $C_f=2\text{Mbps}$ ,  $d_b=10\text{ms}$ ,  $C_b=5\text{Mbps}$ ,  $Q_s=100$

### MAXIMUM LYAPUNOV EXPONENT

Phase portraits are suitable as they are able to visualize the state of a dynamic system. Having a phase portrait one can calculate a maximum Lyapunov exponent (MLE), for example, using Benettin's algorithm [3], – the quantity which is characteristic of the recession rate of close trajectories whose positive value is considered to be an indication of the system chaotic behaviour [3, 17]. The utility package TISEAN has been chosen as an instrument which would allow the obtained data to be analyzed independently of the number of available TCP-sessions and it is intended for analysis of time series and which is based on the theory of nonlinear deterministic systems or chaos theory [7].

The utility `lyap_k` from TISEAN package has been used for calculating the maximum Lyapunov exponent. The result of its work is a set of data which represents the dependence of the logarithm of trajectories recession on time  $S(\varepsilon, m, \Delta n)$  [19], which is calculated in the following way:

$$S(\varepsilon, m, \Delta n) = \frac{1}{N} \sum_{n_0=1}^N \ln \left( \frac{1}{|U(S_{n_0})|} \times \sum_{S_n \in U(S_{n_0})} |S_{n_0+\Delta n} - S_{n+\Delta n}| \right) \quad (2)$$

Where  $\varepsilon$  is the neighbourhood of point  $S_{n_0}$ ,  $m$  is dimensionality of the phase space,  $\Delta n$  is time, and  $U(S_{n_0})$  is the neighbourhood of the point  $S_{n_0}$  of diameter  $\varepsilon$ .

In the given algorithm the point  $S_{n_0}$  is chosen in the phase space and its “neighbours” are marked which are remote from  $S_{n_0}$  within distance  $\varepsilon$ .

If quantity  $S(\varepsilon, m, \Delta n)$  shows a linear increase with the same slope in the reasonable range of values  $\varepsilon$ , then the slope ratio approximating this part of the line can be considered to be roughly equal to the maximum Lyapunov exponent.

The results obtained with the help of the `lyap_k` utility after processing and visualizing  $cwnd(t)$  time series, corresponding to Fig.6, Fig.7 are represented here. The curves  $S(\varepsilon, m, \Delta n)$  for five different values  $\varepsilon$  and the straight line  $y = a + bx$  approximating the linear section of these curves are shown in the figures. Thus, value  $b$  equals the maximum Lyapunov exponent numerically.

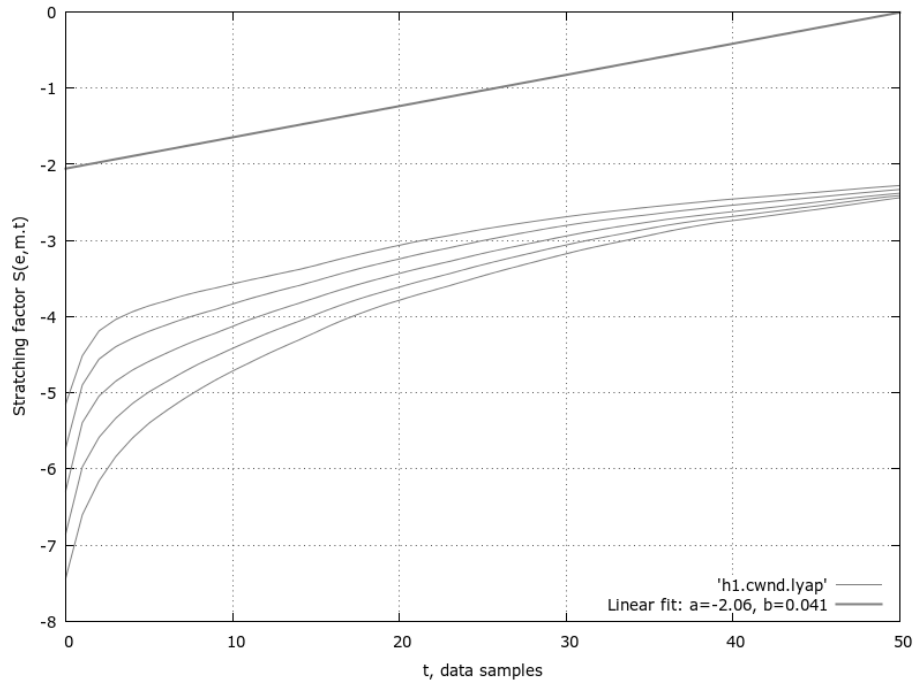


Fig. 6. The Lyapunov exponent calculation  $C_f=5\text{Mbps}$ ,  $d_b=10\text{ms}$ ,  $C_b=5\text{Mbps}$ ,  $Q_s=2$ :  $\lambda \sim 0,041$

The data obtained in the process of the network work simulation in the absence of congestion have been analyzed. As it can be seen from the graph (Fig. 7), in this case  $\lambda < 0$

and this means that such a system does not show a chaotic behaviour [5].

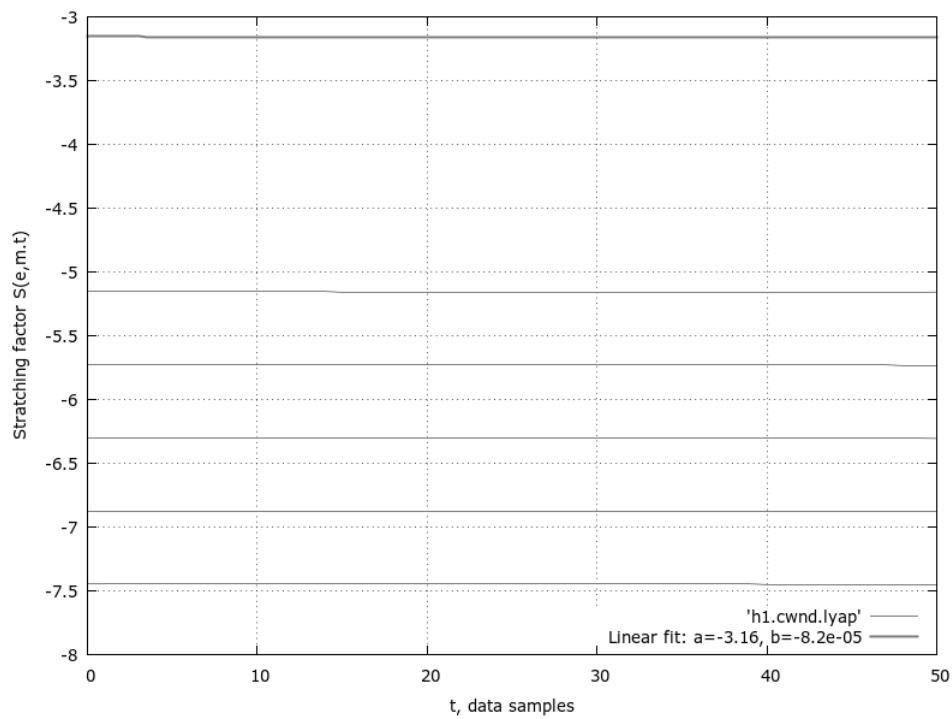


Fig. 7. The Lyapunov exponent calculation  $C_f=2\text{Mbps}$ ,  $d_b=10\text{ms}$ ,  $C_b=5\text{Mbps}$ ,  $Q_s=100$ :  $\lambda \sim -0,00008$  – absence of chaos

A number of numerical experiments have been conducted in order to analyze all the possible states of the model network under various parameters. The data obtained from NS-3 simulator have been processed by TISEAN package [23] for calculating maximum Lyapunov exponent [14]. The graph can be plotted by means of the data obtained, which demonstrates possible dynamic operation modes of the system. Thus one is able to define the parameters of the computer network affect the occurrence of chaotic phenomena and the parameters don't. The router buffer size, the data generation rate senders and maximum Lyapunov exponent as an indicator of chaotic processes in the system have been chosen as the coordinate axis. Such a choice of the axis allows to plot the surface which the peaks and the hills on it to be considered as occurrence of chaos ( $MLE > 0$ ) and the cavities and the hollows ( $MLE < 0$ ) – absence of chaos. The grid with the chosen step has been applied to the graph and therefore the values in the grid nodes provide information about the current state of the computer network. At the same time a few parameters hasn't

been changed in the process of calculation: the delay (10ms) and the recipient bandwidth (5MB/s). When one adds a new sender into the simulated network, then the analyzed phase space dimensionality will also increase by one and to analyze obtained data will be more complicated, not to mention the fact that the phase space visualization is possible, only if its dimensionality is smaller than 4. During carrying out of the calculation series the value of the data generation rate senders was changing in a range of 1 to 10 MB/s with the step 1 MB/s and the router buffer size – from 2 to 100 packets with the step 10 packets. The simulation time for each experiment was 100 seconds.

The result of the conducted calculation series is the graph of maximum Lyapunov exponent dependence on the data generation rate senders and the router buffer size (Fig.8,9) for each TCP-connection of the model network, which obtained after modeling of the operation computer network by means of NS-3 and the further processing by the lyap\_k utility from TISEAN package.

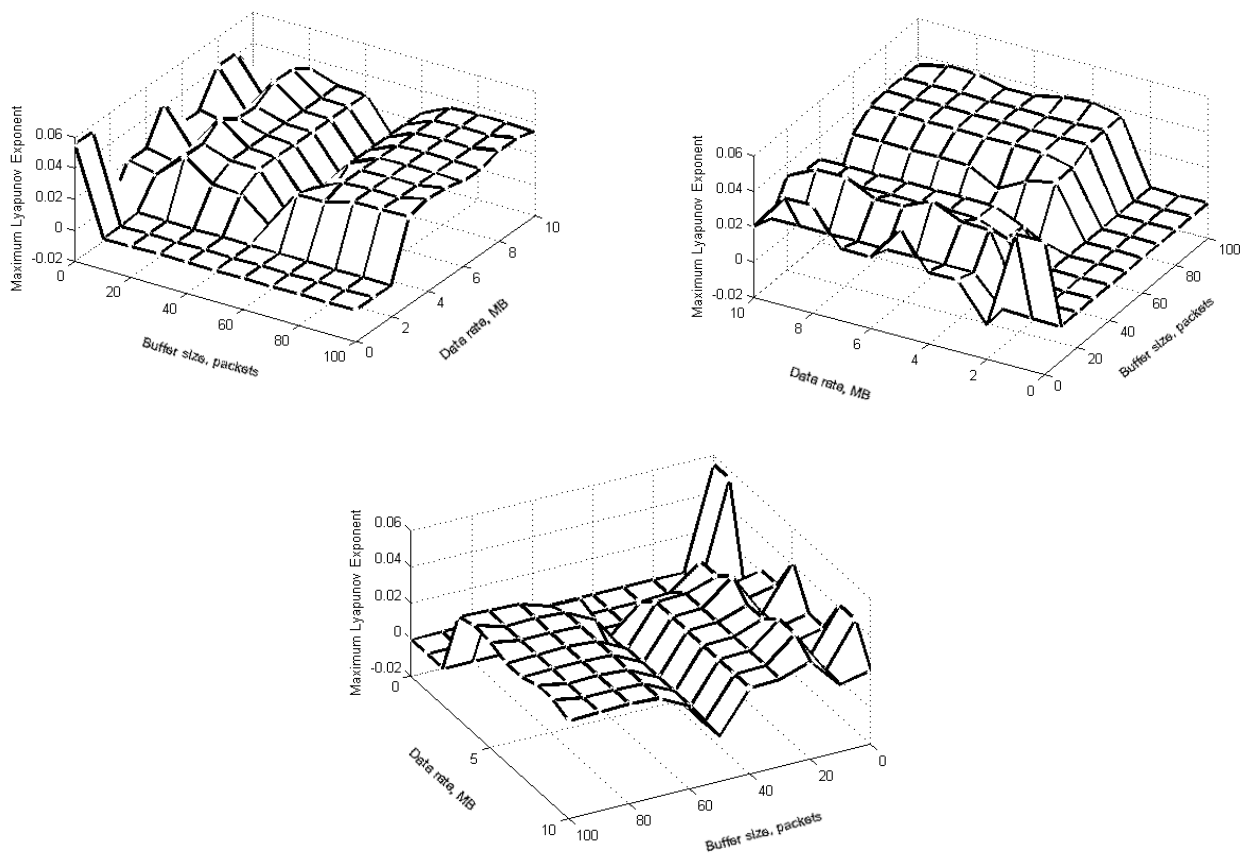


Fig. 8. Maximum Lyapunov exponent dependence on the data generation rate senders and the router buffer size for the 1<sup>st</sup> TCP connection under  $C_f = 1-10$ Mbps,  $d_b = 10$ ms,  $C_b = 5$ Mbps,  $Q_s = 2-100$

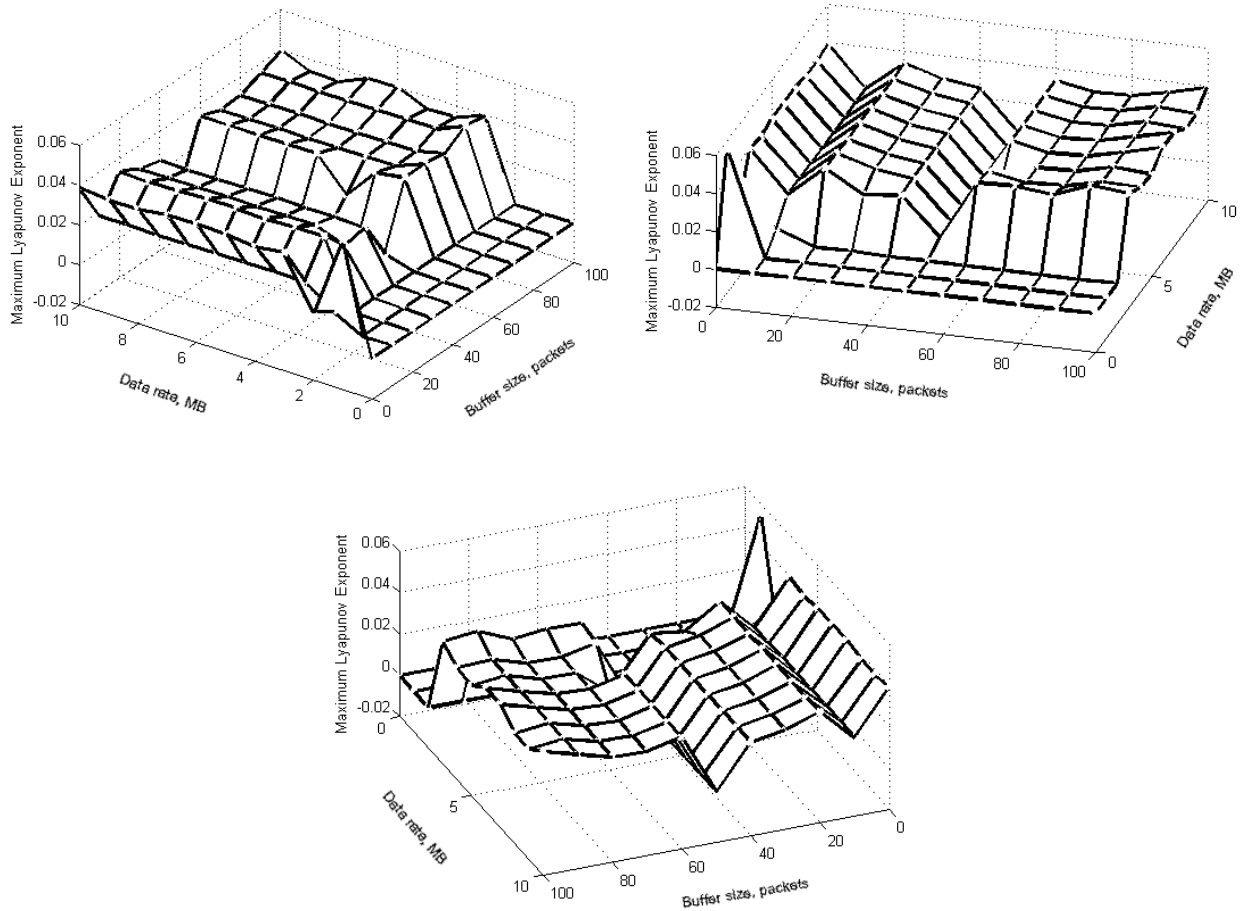


Fig. 9. Maximum Lyapunov exponent dependence on the data generation rate senders and the router buffer size for the 2<sup>nd</sup> TCP connection under  $C_f=1-10$ Mbps,  $d_b=10$ ms,  $C_b=5$ Mbps,  $Q_s=2-100$

The considered system doesn't manifest any chaotic behaviour ( $MLE < 0$ ) in case the host-senders transmits data with a rate not exceeding the recipient bandwidth and overfilling the router buffer. At the same time  $MLE > 0$  corresponds the congestion state of the network. Consequently the router buffer size and the pipe bandwidth are the key parameters affect the congestion state in the considered network. Hence the behaviour of such systems can be predicted with a percent of probability having such a set of the instruments for analysis of dynamic systems.

## CONCLUSIONS

Even such a simple system consisted of two competitive TCP flows shows very complex chaotic behaviour under certain conditions. Obviously, to solve the problem of congestions and the loss of packets is not possible on a global scale of the whole Internet network as one can't

reorganize the entire network in virtue of technical and economical reasons. However it is possible to give some recommendations on design and further operation of the networks limited in scale (even large enough) that will allow negative phenomena of chaotization to be minimized. The advent of congestion in networks especially strongly affects its working efficiency in some areas of science and technology [10,15,30].

The diagrams plotted in the Fig. 8,9 allow estimating the values of the parameters of infocommunication networks in terms of availability (or absence) undesirable chaotic phenomena. By proper choosing of these parameters it is possible to make sure stable operation of infocommunication networks in the modes without chaos, which accords with non-occurrence of congestions.



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