

# Rational Searching Procedure in Warehouse Design

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*Designing a warehouse layout is an important problem, which plays a role in a life cycle of a company. Generally, two main types of layout decision problems are mentioned in literature. The first problem is usually called facility layout problem while the second one is internal layout designing problem or aisle configuration problem. Merging them both, the procedure for rational searching of optimal warehouse layout is prepared. It was decided to prepare the procedure as an optimisation procedure of functional and spatial areas. The procedure is a part of the logistics facilities designing method because a sketch layout of logistics facility is an important part of the method of logistic facilities designing. The paper briefly describes the location of the procedure in the method. Regarding functional and spatial layouts designing, model was expanded with mathematical formulas of certain geometrical parameters. Particular emphasis on warehouse geometry is important due to the fact that many other issues depend on these parameters in warehouse designing process. The paper consists of a problem definition, mathematical model formulation and precise descriptions adequate to mathematic notations. What is more, the example of procedure application implemented into the software is given. The software is the implementation of the procedure and the whole method itself.*

**Keywords:** facilities planning, facilities design, warehouse design, logistics, warehouse layout, searching procedure, rational searching.

## 1. INTRODUCTION

Designing a warehouse layout is an important problem, which plays a role in a life cycle of a company. Let me consider the complexity of this problem from general to specific. As research given in Anon. (2007) have informed the logistics market, the logistics costs represent almost 10% of sales in Western countries (in Anon. (2010) is a little bit less is mentioned: 8.28%). It is still a fact. Generally, three major initiating factors of total logistics cost are stressed and specified. The first one is warehousing, second one is transportation and the last one is inventory carrying. As the mentioned research informs, these initiating factors are responsible for circa 21% of total logistics costs in United States of America and circa 37% in European Union countries (treated as a one group). For further consideration let me also bear in mind the fact that a warehouse is well known as a key logistics facility in a company's supply chain and distribution strategy. Warehouses designing is the

general subject matter of this paper, Therefore, let me consider the first initiating factor, which is warehousing costs, since this type of costs is to a large extent determined already during a warehouse designing phase. Let me also keep in mind that warehouse design is a highly complex task with many trade-offs between conflicting objectives and a large number of feasible designs. This complex task is realized with consideration for different levels of decision-making process. Rouwenhorst *et al.* (2000) defines the warehouse design as an approach, which consists of three levels of decision-making. These levels are: strategic, tactical and operational. Decisions about layout designing, which is mainly considered in this paper, are made on the tactical level. Furthermore, a selection of equipment is to be made on the tactical level; nevertheless it is not a part of this paper's scope. A warehouse layout designing is extremely important since integrating all other decisions

connected to warehousing (and other operations in a warehouse) depend on infrastructure, and thus on a warehouse layout. Decisions regarding warehouse layouts are integral part of a warehouse designing process, which is considered in papers: Kostrzewski (2012a, 2012b).

According to de Koster *et al.* (2007), two main types of layout decision problems are mentioned in literature. The first problem is usually called the facility layout problem. It concerns the decision where to locate various warehouse areas such as: an entry area, an exit area, other kinds of buffer areas, an order-picking area, a storage area *etc.* “Koopmans and Beckmann (1957) were among the first to consider this class of problems, and they defined the facility layout problem as a common industrial problem in, which the objective is to configure facilities, so as to minimize the cost of transporting materials between them”, after Drira *et al.* (2007).

The second layout decision problem on the whole is called internal layout designing problem or sometimes aisle configuration problem. It is concerned with the placement of equipment, infrastructure location and aisles configuration *etc.* Most papers define the warehouse layout problem as finding an optimal, or rather sub-optimal, layout of storage or order-picking area where travel distance/time is the criterion function. As Smith (2007: 1530) says layout design is frequently subjected to accomplish following objectives: maximizing the use of space, maximizing the use of equipment, maximizing the use of labour, maximizing the accessibility to all items, maximizing protection of all items *etc.* Besides, the majority of papers are focused on the analysis of an isolated problem rather than on the synthesis. Let me try to consider the both mentioned layout decision problems. Here, in the paper, I consider some strictly chosen elements of a facility designing problem and an internal layout designing problem. Notwithstanding, let me introduce a slightly different approach. The goal is to find a warehouse layout that minimizes investment expenditures and infrastructure costs within a warehouse.

Designing of spatial and functional areas (as a sketch layout of logistics facility) is an important part of the method of logistic facilities designing. A logistics facility is understood here as a building where some logistics processes are realized. It is *e.g.* a warehouse. The method of logistic facilities designing is presented in Kostrzewski (2012a, 2012b). As one of the first steps of designing

method – step number 6 in the method – it determines designing variants of a logistics facility, and at least a character of a whole design chosen to realisation. Spatial and functional areas are connected to allocation and fulfilment of facility areas. It is supported by the fact that spatial arrangement in a warehouse is determined by some parameters. These parameters include: selection of a type of storage in the storage area, a type of storage in the order-picking area, a number of levels in the pallet rack storage (or in stack) in the storage area, a number of aisles in the storage area, a number of cross-aisles in storage area, a number of levels in pallet rack storage in the order-picking area, a number of aisles in the order-picking area, a number of cross-aisles in the order-picking area (and in case of aisles and cross-aisles – their sizes: the length and width, orientation: along or across a warehouse, location of I/O points and finally flows of materials within a warehouse). In the literature there are not many papers regarding the framework for developing sketches of functional and spatial layouts.

Due to the fact that the spatial-functional layout designing is an important aspect of warehouse designing, it was decided to amplify the designing procedure of optimisation of functional and spatial areas. The assumption of optimisation is to contribute to the further development of the functional and spatial areas designing frameworks. In practice, the optimal sizes of building blocks and areas can only be defined for one, pre-defined designing facility. Too many parameters are involved for a “global” optimisation to be realised. Therefore, it was decided that a problem of sub-optimisation of functional and spatial areas in logistics facilities should be considered. As a way of optimisation let us propose rational searching of solutions in warehouse layouts designing.

Regarding functional and spatial layouts designing, it seeks to, *inter alia*, expand models with mathematical formulas of certain geometrical parameters of a warehouse. Their notations are determined by some important assumptions and coefficients, which are precised in one of the first steps of designing method *e.g.* a type of storage or other criteria and parameters affecting the problem. Thus, for each type of storage mathematical notification of formulas takes a different form. Character of formulas is described in this paper.

Particular emphasis on warehouse geometry is important due to the fact that in warehouse designing process many other issues depend on the geometrical parameters.

In general, warehouse geometry affects investment expenditure and operational costs. Among others this was reason for decision to merge the criterion function with the investment expenditure on building and infrastructure. Thus, the main aim of the optimisation procedure is to find solution or solutions, which will generate investment expenditures as low as possible for the construction of a warehouse. The important fact is that the rational approach to warehouse space planning was also taken into account. This means that it is not only the cheapest solution that is implemented, but also the most reasonable one.

Let us consider a traditional warehouse layout. This is a kind of layout found today in the vast majority of warehouses. Its basic form is built in on a rectangular plan, with parallel straight aisles crossing main aisles, called cross aisles. Three possibilities for aisles changing by means of internal transport are defined. One of them is at the front, the second at the rear of a warehouse and the last one is without a doubt in cross aisles. These aisles are also straight and meet the cross aisles at right angles. Layouts of conventional warehouses with other types of storage systems, such as carousels, a flow rack system, drive-in or drive-through pallet racks, mobile racks *etc.* are not taken into consideration here.

In Drira *et al.* (2007) a tree representation of layout problems is given. In this paper we consider some of its branches, such as: fixed dimensions of rectangular regular shape of facility, its static layout, crisp data for layout formulation, the objectives that are minimum space costs and minimal shape irregularities, the layout configuration that is multi-rows *etc.* Taking into account the tree we can prepare strong base for future research including dynamic changes. This paper describes an approach to determine a layout for all areas in warehouses, so that the average travel distance for entities is minimised while minimising investment expenditure. According to Vrysagotis *et al.* (2011) mathematical based solutions include different types of algorithms such as: heuristics, algorithms based on geometry, "cut trees" algorithms, genetic algorithms, neighbourhood search algorithms, dynamic programming, linear and non-linear programming, mixed integer programming, stochastic programming, simulated annealing algorithms, particle swarm optimisation, general mathematical models, other algorithms. The technique presented here is heuristic-based optimisation technique.

The paper is organised as follows. In section 2 the logistics facilities layout-designing problem review is derived. In section 3 the logistic facilities designing method is described very briefly. Section 4 consists of a problem definition, mathematical model formulation and precise descriptions of adequate to mathematic notations. Finally in section 5 summary of the paper, conclusive remarks and future research proposal are provided. In addition, the summary gives precisely possible directions of further researches on the problem.

## 2. LITERATURE REVIEW

Shayan and Chittilappilly (2004) define the logistics facility layout problem. They define it as an optimisation problem that tries to make layouts more efficient by taking into account various interactions between facilities and material handling systems while layouts designing.

Better planning of logistics facility layout leads in general to reducing storage costs. A given range of products can be stored in reduced storage space adapting the idea of optimal usage of storage space. A concept of optimal size of storage space is understood as follow. It is a space whose usage coefficient's value is highest in certain conditions. At the same time it should be noted that there is no unique optimal size, nor one optimal layout of logistics facility areas. Only in individual, strictly defined logistics facility designing cases values can be specified. Therefore, the issue should be considered under the idea of sub-optimisation (not optimisation) of functional and spatial areas in logistics facilities at the stage of their design as it was mentioned before. According to Fijałkowski (1983) in '80s of previous century it was still in the theoretical stage.

The literature on warehouse designing and operations is highly extensive. Much of the research are motivated by the need to improve the efficiency of order-picking operations and to estimate travel distance or time. For these reasons order-picking process, storage strategy, batching method, routing method and many more are described in the literature on warehousing aspects. However warehouse layout design, especially in multiple-level, has received less attention. It should be also mentioned that some research papers discuss dynamic changes in facility layout, for example the paper by Chen and Lo (2014) that treats about a problem in multi-objective dynamic facility layout is achieving distance- and adjacency-based objectives for arranging facility

layouts across multiple time periods. This is an important problem without any doubts, but at the same time “static” layout of warehouse or any other logistics facilities should be designed first as it could be treated as an assumption for other kind of research e.g. dynamic research on logistics facilities layout.

Numerous papers consider the layout-designing problem from the order-picking point of view. Liu (2004) develops dynamic stock layout systems for storage layout and order-picking operations problems in view of the dynamic nature of customer order demand. Similar consideration can be found in Roodbergen *et al.* (2008). Ballou (1967) formulates a linear programming model for a problem involving in between order-picking areas.

Francis *et al.* (1992) present mathematical models to determine the size of the storage system and assign items to storage locations. Jarvis and McDowell (1991) develop a stochastic model to locate products in an order-picking warehouse. Liu (1999) presents a clustering model and develops a closed-form solution for improving stock location and order-picking operations for a distribution centre. Several papers consider the facility shape and pick-up and drop-off locations. In the paper of Guenov and Raeside (1992) the area shapes in a class-based storage are optimally evaluated by a simulation model.

Notwithstanding, in the literature connected to the problem, only few studies are concerned to optimisation in designing of functional and spatial areas in a general way. Therefore, the stronger the argument is for the necessity of examining the problem. One of the studies – the paper by Ashayeri and Gelders (1985) – gives the general requirements for the problem. The conclusion leads Authors to claim that the method of optimising the storage areas should be based both on analytical approach and simulation solutions. Similar conclusions, as a result of analytical research were included in Kostrzewski (2007). Further studies, with emphasis on optimising a surface of storage area for storage in blocks, are presented in Piechota-Rybarczyk (1978). In this case, the examined procedure has been developed for one type of storage only. Therefore, a certain need to prepare a procedure dealing with optimisation of other storage types arises. A useful theoretical basis in case of a warehouse layout can be found in Francis *et al.* (1992). It concerns both non-rectangular and rectangular warehouse designs. Problems of warehouse layout and

location are treated together because of the similarity between arranging the space in a single facility and arranging systems of facilities. An introduction to the field's issues and literature is included there, along with the basic tools and methodologies.

There seems to be no static base structure of a warehouse layout designing, which would minimise investment expenditures and further operating costs, which includes storage area plan, aisles and cross aisles plan, rack types and sizes, dock plan etc. It must be mentioned here that Malette and Francis (1972) applied a generalized assignment model to optimal facility layout considering the material-handling cost. Malmberg and Deutch (1988) constructed a stock location model, in which the inventory level and cost were considered. Malette and Francis (1972) and Malmberg and Deutch (1988) consider Cube per Order Index-COI, which is ratio between the volume occupied by the specific product and its popularity. It is described in Heskeet (1963). As shown in literature, this parameter guarantees a picking costs minimisation, but it requires a high computational effort. Therefore, other, lower computational effort and less time-consuming methods are needed.

### 3. A LOGISTICS FACILITIES DESIGNING METHOD

Since rational search in warehouse layouts designing understood as a procedure of sub-optimisation of functional and spatial areas is a part of a logistic facilities designing method, the method should be mentioned briefly.

In the logistics facilities designing method following parts are included:

- system functions modelling,
- the input data (assumptions),
- the procedure for logistics facilities designing, which includes three phases: *task defining* phase, *solution design* phase, *solution variant evaluation* phase,
- the procedure for optimisation of functional and spatial areas (rational search in warehouse layouts designing considered in the paper),
- output data (for each of  $n$  variants).

It must be particularly emphasized that the method consists of 2 main procedures, as it is briefly mentioned above.

Method of logistics facilities designing has been implemented in a form of a software with acronym *OL09*. Within the *OL09* software a module was distinguished, which implements the rational search in warehouse layouts designing.

#### 4. MATHEMATICAL NOTATION OF RATIONAL SEARCH IN WAREHOUSE LAYOUTS DESIGNING

Optimisation is a method of determining the best (optimal) solution (search for extreme function) from the viewpoint of a specific criterion (index) of quality (*e.g.* cost, route, efficiency).

Formulating of optimisation task (in this case: the mathematical model of rational search in warehouse layouts designing) is reduced to:

- adopting data (systematically choosing input values),
- determining the decision variables,
- identifying constraints (an allowed set),
- defining task objective function (computing the value of the function).

More generally, optimisation includes finding the “best available” values of some objective function given as a defined domain, including a variety of different types of objective functions and different types of domains.

Let me define a majority of parameters, which are used in rational search in warehouse layouts designing.

##### 4.1. ENTRY DATA

In the mathematical model of rational searching for warehouse layouts designing input data is split into technological data and economic (financial) data.

Among the technology data there are:

- $P^R$  – quantity of standardised pallet entities load being operated in the warehouse per year, [ple/year], (where: ple/year – pallet load entities per year),
- $N$  – a norm coefficient of storage,
- $d_r$  – quantity of working days per year, [days],

- $Z$  – a stock in a warehouse, quantity of pallet-places in a storage area [ple],
- $A$  – an assortment of products in a warehouse,
- $\varphi_{in}$  – an accumulation coefficient on entrance to a warehouse,
- $\varphi_{out}$  – an accumulation coefficient on exit from a warehouse,
- $\rho$  – an average filling percentage of a pallet, indicates the extent to which a pallet load entity is filled with products after an order-picking process in relation to a pallet load entity before an order-picking process,
- $\tau$  – an order-picking coefficient, indicates how many pallet entities load from the storage area will participate in an order-picking process,
- $w \times p$  – a structure of a pallet load entity after an order-picking process,  $w$  is the number of lines on the order-picking list and  $p$  is the number of goods per one line,
- $\varphi_t$  – an utilisation of work-time coefficient,
- $F_Z$  – a warehouse surface, [m<sup>2</sup>],
- $H_Z$  – a warehouse height, [m],
- $\mu$  – quantity of logistics modules (1 logistics module in a layout designing is equal to 6 [m] long).

Among the economic data there are:

- $c_g$  – land price, [currency/m<sup>2</sup>],
- $c_n^s$  – building construction price per 1 m<sup>3</sup> in a storage area (superscript:  $s$ ) in case of storage type number  $n$  ( $n = 1, \dots, 6$ ), [currency/m<sup>3</sup>], where storage types can be, as follow: a storage in blocks (stacks) on a warehouse floor, a storage in rows on a warehouse floor, a storage in rows in a warehouse rack using front fork lift trucks, a storage in rows in a warehouse rack using front-side fork lift trucks, a storage in rows in a warehouse rack using low and average height rack stacker cranes, a storage in rows in a warehouse rack using high rack stacker cranes between racks forming a support structure of a roof and walls in a warehouse,
- $c_{baz}^s$  – base (subscript: *baz*) price of a building construction per 1 m<sup>3</sup> in a storage area ( $s$ ), [currency/m<sup>3</sup>],

- $c_m^k$  – price of a building construction per  $1 \text{ m}^3$  in an order-picking area (part of superscript:  $k$ ) in case of storage type number  $m$  ( $m = 1, \dots, 6$ ), [currency/ $\text{m}^3$ ],
- $c_p$  – price of a building construction per  $1 \text{ m}^3$  in case of other areas (superscript:  $p$ ), [currency/ $\text{m}^3$ ],
- $c_{baz}^k$  – base (*baz*) price of a building construction per  $1 \text{ m}^3$  in an order-picking area ( $k$ ), [currency/ $\text{m}^3$ ],
- $c_n^{mps}$  – price of one pallet place construction (part of superscript:  $mp$ ) in a storage area (part of superscript:  $s$ ) in case of storage type number  $n$  ( $n = 1, \dots, 6$ ), [currency/ $\text{m}^3$ ],
- $c_{baz}^{mps}$  – base (*baz*) price of one pallet place construction ( $mp$ ) in a the storage area ( $s$ ), [currency/ $\text{m}^3$ ],
- $c_m^{mpk}$  – price of one pallet place construction ( $mp$ ) in an order-picking area ( $k$ ) in case of storage type number  $m$  ( $m = 1, \dots, 6$ ), [currency/ $\text{m}^3$ ],
- $c_{baz}^{mpk}$  – base (*baz*) price of one pallet place construction ( $mp$ ) in an order-picking area ( $k$ ), [currency/ $\text{m}^3$ ].

The purpose of *base*-type parameters introduction will be described in rest part of the paper (subchapter 4.5.).

#### 4.2. DECISION VARIABLE

Herein, decision variables are parameters whose values belong to the set of natural numbers and they are introduced to differentiate variants of a warehouse layout. These are the parameters related to the geometry of a warehouse.

Among the decision variables, the following parameters can be distinguished:

- $L_l^{sn}$ ;  $n = 1, \dots, 6$  – quantity of levels (subscript:  $l$ ) in a pallet stack on a floor or in a pallet rack storage in a storage area ( $s$ ) for  $n$ -type of storage,
- $L_a^{sn}$ ;  $n = 1, \dots, 6$  – quantity of aisles (subscript:  $a$ ) in a storage area ( $s$ ) for  $n$ -type of storage,
- $L_{ca}^{sn}$ ;  $n = 1, \dots, 6$  – quantity of cross aisles (subscript:  $ca$ ) in a storage area ( $s$ ) for  $n$ -type of storage,
- $g^{sn}$ ;  $n = 1, \dots, 6$  – quantity of pallet entities load on a rack shelf (or a way of putting

a pallet into a block/row: along the shorter or longer side of a pallet) in a storage area ( $s$ ) for  $n$ -type of storage,

- $L_l^{km}$ ;  $m = 1, \dots, 6$  – quantity of levels ( $l$ ) in a pallet stack on a floor or in a pallet rack storage in an order-picking area ( $k$ ) for  $m$ -type of storage,
- $L_a^{km}$ ;  $m = 1, \dots, 6$  – quantity of aisles ( $a$ ) in an order-picking area ( $k$ ) for  $m$ -type of storage,
- $L_{ca}^{km}$ ;  $m = 1, \dots, 6$  – quantity of cross aisles ( $ca$ ) in an order-picking area ( $k$ ) for  $m$ -type of storage,
- $g^{km}$ ;  $m = 1, \dots, 6$  – quantity of pallet entities load on a rack shelf (or a way of putting a pallet into block/row: along the shorter or the longer side of a pallet) in an order-picking area ( $k$ ) for  $m$ -type of storage.

#### 4.3. AN ALLOWED DATA SET

The mathematical formulas connected to an allowed data set must include the decision variables, as follow. The maximum value must be at least 1 time greater than the minimum value for the number of the input data in case of set point ranges such as: quantity of levels in pallet stack on a floor or in pallet rack storage in a storage area for  $n$ -type of storage  $L_{l \min}^{sn} < L_{l \max}^{sn}$ , quantity of aisles in a storage area  $L_{a \min}^{sn} < L_{a \max}^{sn}$ , quantity of cross aisles in a storage area  $L_{ca \min}^{sn} < L_{ca \max}^{sn}$ ;  $n = 1, \dots, 6$ , quantity of levels in pallet stacks on a floor or in pallet rack storage in an order-picking area for  $m$ -type of storage  $L_{l \min}^{km} < L_{l \max}^{km}$ , quantity of aisles in an order-picking area  $L_{a \min}^{km} < L_{a \max}^{km}$ , quantity of cross aisles in an order-picking area  $L_{ca \min}^{km} < L_{ca \max}^{km}$ ;  $m = 1, \dots, 6$ . After Fijałkowski (1995, 2002), value ranges for quantities of levels in a storage area or an order-picking area (for the following type of storage) are: a storage in stacks on a warehouse floor:  $1 \leq L_l^{s1} \leq 5$ ,  $1 \leq L_l^{k1} \leq 5$ , a storage in rows on a warehouse floor:  $1 \leq L_l^{s2} \leq 4$ ,  $1 \leq L_l^{k2} \leq 4$ , a storage in rows in a warehouse rack using front fork lift truck:  $1 \leq L_l^{s3} \leq 6$ ,  $1 \leq L_l^{k3} \leq 6$ , a storage in rows in a warehouse rack using front-side fork lift trucks:  $1 \leq L_l^{s4} \leq 13$ ,  $1 \leq L_l^{k4} \leq 13$ , a storage in rows in a warehouse rack using low and average height rack stacker cranes:  $1 \leq L_l^{s5} \leq 15$ ,

$1 \leq L_l^{k5} \leq 15$ , a storage in rows in a warehouse rack using high rack stacker cranes between racks forming a support structure of a roof and walls in a warehouse:  $1 \leq L_l^{s6} \leq 31$ ,  $1 \leq L_l^{k6} \leq 31$ . Minimum quantity of aisles in a storage area or an order-picking area is 1, while the maximum quantity of aisles in a storage area or an order-picking area is 10:

$$1 \leq L_a^{sn} \leq 10; n = 1, \dots, 6,$$

$1 \leq L_a^{km} \leq 10; m = 1, \dots, 6$ . Minimum quantity of cross aisles in a storage area or an order-picking area is 1, while the maximum quantity of cross aisles in a storage area or an order-picking area is 5:  $1 \leq L_{ca}^{sn} \leq 5; n = 1, \dots, 6$ ,  $1 \leq L_{ca}^{km} \leq 5; m = 1, \dots, 6$ .

Quantities of pallet entities load on rack shelf (or a way of putting a pallet into stack/row: with the shorter or the longer side of pallet) in storage area for  $n$ -type of storage or in order-picking area for  $m$ -type of storage is 2 or 3  $g^{sn} = \{2, 3\}; n = 1, \dots, 6$ ,  $g^{km} = \{2, 3\}; m = 1, \dots, 6$ .

Beside the decision of variables' limitations, other kinds of limitations are identified. A permissible surface of land for a building is  $F_{Zdop} \geq F_Z$  and a warehouse surface must be greater than zero:  $F_Z > 0$ . The surface area adopted for warehouse surface is rectangular. The permissible height of a building is  $H_{Zdop} \geq H_Z$  and a warehouse height must be greater than zero:  $H_Z > 0$ . In case of value sets in quantity of storage levels, the only variants that will be considered are those whose maximum quantity of storage levels is higher than the set point value. This means that if a reference quantity of storage levels is 3 then types of storage of quantities: 4, 5, 6 will be considered. And other limitations are as follow:

- quantity of pallet load entity being operated in warehouse per year:  $P^R > 0$ ,
- a norm coefficient of storage:  $N > 0$ ,
- quantity of working days per year:  $d_r > 0$ ,
- quantity of pallet-places in stock:  $Z_p > 0$ ,
- an assortment:  $A > 0$ ,
- an accumulation coefficient on entrance:  $\varphi_{in} > 0$ ,
- an accumulation coefficient on exit:  $\varphi_{out} > 0$ ,

- a medium filling percentage of a pallet:  $0 < \rho < 1$ ,
- an order-picking coefficient:  $0 < \tau < 1$ ,
- a medium structure of pallet load entity after order-picking process:  $w > 0, p > 0$ ,
- an utilisation of work-time coefficient:  $0 < \varphi_t < 1$ .

#### 4.4. MATHEMATICAL APPARATUS

There are some formulas that are constructed for criterion function. Their notations need discussion.

Formulas connected to investment expenditure on the building, investment expenditure on pallet places are discussed in Fijałkowski (1995, 2002).

Other necessary formulas (distinguishing types of storage) are explained below.

To calculate parameters related to the geometry of a warehouse in a storage area in case of storage in blocks (stacks) on a the warehouse floor ( $n = 1$ ) the following equations can be used.

Quantity of columns (blocks) in a storage area  $L_{col}^{sn}$  can be calculated by formula (1). It is dimensionless parameter. It is integer type of number, Therefore, ceil-function is used. This formula is needed to define some of other formulas.

$$L_{col}^{sn} = \left\lceil \frac{Z}{25 \cdot L_l^{sn}} \right\rceil \quad (1)$$

Number 25 in formula (1) is a constant number of pallet load entities in block per 1 level. It means that 25 [ple] are located at one level of a storage block.

The total width of a storage area in a warehouse  $S^{sn}$  can be calculated by formula (2). Values of parameter are calculated in meters.

$$S^{sn} = (L_a^{sn} + 2) \cdot sz_a + (L_a^{sn} + 1) \cdot sz_r \quad (2)$$

In formula (2)  $sz_a$  is the width of an aisle in a storage area in a warehouse and its values are calculated in meters. In the same formula  $sz_r$  is width of a column (block) in a storage area in a warehouse and its values are also calculated in meters. The meaning of part  $(L_a^{sn} + 2)$  of the

formula (2) is that two additional aisles are situated at extreme locations of a storage area (at its left and right sides). Part  $(L_a^{sn} + 1)$  of the formula (2) means quantity of columns (blocks) in a storage area.

To calculate quantity of blocks per one aisle in a storage area  $L_{cola}^{sn}$  formula (3) can be used. It is integer type of number Therefore, ceil-function is used. A value of this dimensionless parameter is needed in purpose of calculating a storage area length, whose notation is presented as formula (4). Values of a storage area length  $D^{sn}$  are calculated in meters.

$$L_{cola}^{sn} = \left\lceil \frac{L_{col}^{sn}}{L_a^{sn}} \right\rceil \quad (3)$$

In formula (4) the parameter  $d_{pal}$  is a pallet length and its values are calculated in meters. Quantity of cross aisles is increased by 2 because of two additional cross aisles: one at the front and the second at the rear of a warehouse. Number 6 in formula (4) is width of a cross aisle given after Fijałkowski (2002). It is a logistics module equals to 6 meters. From the point of view of optimal warehouse layout, it is obvious that the rear aisle in layout is not needed. Furthermore, adding one or more cross-aisles is not beneficial as this leads only to an increased total space required but on the other hand it is facilitating the use of operation policies, makes easier to travel between different places in a warehouse *etc.*

$$D^{sn} = L_{cola}^{sn} \cdot d_{pal} + 6 \cdot (L_{ca}^{sn} + 2), \quad (4)$$

Similar notations can be used in an order-picking area in case of storage in blocks on a warehouse floor ( $m = 1$ ). However, it is theoretical since an order-picking process is not realised in storage in blocks on a warehouse floor. Formulas (5) – (8) are given to maintain the order of considerations.

To calculate the parameters related to geometry of a warehouse in an order-picking area in case of storage in blocks on a warehouse floor ( $m = 1$ ) the following equations can be used.

Quantity of columns (stacks) in an order-picking area  $L_{col}^{km}$  is given as formula (5).

$$L_{col}^{km} = \left\lceil \frac{A}{25 \cdot L_l^{km}} \right\rceil, \quad (5)$$

Total width of an order-picking area in a warehouse  $S^{km}$  is given as formula (6). Its values are calculated in meters.

$$S^{km} = (L_a^{km} + 2) \cdot sz_a + (L_a^{km} + 1) \cdot sz_r, \quad (6)$$

To calculate quantity of blocks per 1 aisle in an order-picking area  $L_{cola}^{km}$  (dimensionless parameter) formula (7) can be used.

$$L_{cola}^{km} = \left\lceil \frac{L_{col}^{km}}{L_a^{km}} \right\rceil, \quad (7)$$

A length of an order-picking area in a warehouse  $D^{km}$  is given as formula (8). Its values are calculated in meters.

$$D^{km} = L_{cola}^{km} \cdot d_{pal} + 6 \cdot (L_{ca}^{km} + 2), \quad (8)$$

To calculate the parameters related to warehouse geometry in a storage area in case of storage in rows on a warehouse floor ( $n = 2$ ) following equations can be used.

Quantity of rack columns in a storage area  $L_{col}^{sn}$ , which is a dimensionless parameter, can be calculated using formula (9). It is integer type of number Therefore, ceil-function is used. The meaning of part  $(2 \cdot L_a^{sn})$  of the formula (9) is quantity of rows in a storage area (two rows fall on one aisle).

$$L_{col}^{sn} = \left\lceil \frac{Z}{L_l^{sn} \cdot (2 \cdot L_a^{sn})} \right\rceil, \quad (9)$$

Total width of a storage area in a warehouse  $S^{sn}$  can be calculated by formula (10). Its values are calculated in meters.

$$S^{sn} = (L_a^{sn} + 2) \cdot sz_a + L_a^{sn} \cdot 2 \cdot sz_r, \quad (10)$$

In formula (10) parameter  $sz_a$  is the width of an aisle in a storage area in a warehouse. At the



same,  $sz_r$  is width of a row in a storage area in a warehouse. Values of two previous parameters are calculated in meters. The meaning of part  $(L_a^{sn} + 2)$  of the formula (10) is that two addition aisles are situated at extreme locations of a storage area (at its left and right sides). Part  $2 \cdot L_a^{sn}$  of the formula (10) means quantity of rows in a storage area.

A length of a storage area in a warehouse  $D^{sn}$  is given as formula (11). It is similar to formula (4), however quantity of blocks per one aisle in a storage area  $L_{cola}^{sn}$  is replaced by quantity of pallets per one aisle in a storage area in one level  $L_{col}^{sn}$ .

$$D^{sn} = L_{col}^{sn} \cdot d_{pal} + 6 \cdot (L_{ca}^{sn} + 2), \quad (11)$$

Again similar notations can be used in case of an order-picking area where the storage is in rows on the logistics facility floor ( $m = 2$ ).

To calculate the parameters related to geometry of a warehouse in an order-picking area in case of storage in rows on a logistics facility's floor ( $m = 2$ ) following equations can be used.

Quantity of rows in a storage area  $L_{col}^{km}$  (dimensionless parameter) can be calculated by formula (12).

$$L_{col}^{km} = \left\lceil \frac{A}{L_l^{km} \cdot (2 \cdot L_a^{km})} \right\rceil, \quad (12)$$

The total width of an order-picking area in a warehouse  $S^{km}$  (a parameter in meters) can be calculated by formula (13).

$$S^{km} = (L_a^{km} + 2) \cdot sz_a + L_a^{km} \cdot 2 \cdot sz_r, \quad (13)$$

A total length of an order-picking area in a warehouse  $D^{km}$  (in meters) is given as formula (14).

$$D^{km} = L_{col}^{km} \cdot d_{pal} + 6 \cdot (L_{ca}^{km} + 2), \quad (14)$$

Let me, in case of three following types of storage, introduce more universal formulas. The mentioned storage types are: storage in rows in a warehouse rack using front fork-lift truck ( $n = 3$ ), storage in rows in a warehouse rack using front-

side fork-lift trucks ( $n = 4$ ), storage in rows in a warehouse rack using low and average height rack stacker cranes ( $n = 5$ ).

To calculate the parameters related to geometry of a warehouse in a storage area in case of the mentioned types of storage, the following notations can be used.

Quantity of rack columns in a storage area  $L_{col}^{sn}$  is dimensionless parameter. It can be calculated by formula (15). It is integer type of number, Therefore, ceil-function is used. The meaning of part  $(L_a^{sn} \cdot 2)$  of the formula (15) is quantity of rows in a storage area (two rows fall per one aisle).

$$L_{col}^{sn} = \left\lceil \frac{Z}{g^{sn} \cdot L_l^{sn} \cdot (L_a^{sn} \cdot 2)} \right\rceil, \quad (15)$$

The total width of a storage area in a warehouse  $S^{sn}$  (expressed in meters) can be calculated by formula (16).

$$S^{sn} = (L_a^{sn} + 2) \cdot sz_a + L_a^{sn} \cdot 2 \cdot sz_r, \quad (16)$$

The total length of a storage area in a warehouse  $D^{sn}$  is given as formula (17). Its values are calculated in meters.

$$D^{sn} = L_{col}^{sn} \cdot 3 + 6 \cdot (L_{ca}^{sn} + 2), \quad (17)$$

The meaning of part  $(L_{ca}^{sn} + 2)$  of the formula (17) is that two addition cross aisles are situated at extreme locations of a storage area (at its left and right sides). Part  $L_{col}^{sn} \cdot 3$  of the formula (17) means a constant length of rack column, which is 3 meters long.

Again, similar notations can be used in case of an order-picking area where three mentioned types of storage are discussed.

To calculate the parameters related to geometry of a warehouse in an order-picking area following equations can be used.

Quantity of rows in a storage area  $L_{col}^{km}$  (dimensionless parameter) can be calculated by formula (18).

$$L_{col}^{km} = \left\lceil \frac{A}{g^{km} \cdot L_l^{km} \cdot (L_a^{km} \cdot 2)} \right\rceil, \quad (18)$$

The total width of an order-picking area in a warehouse  $S^{km}$  can be calculated in meters by formula (19).

$$S^{km} = (L_a^{km} + 2) \cdot sz_a + L_a^{km} \cdot 2 \cdot sz_r, \quad (19)$$

The total length of an order-picking area in a warehouse  $D^{km}$  is given as formula (20). Its values are calculated in meters.

$$D^{km} = L_{col}^{km} \cdot 3 + 6 \cdot (L_{ca}^{km} + 2), \quad (20)$$

The mathematical apparatus is not used for an order-picking area in case of a storage in rows in a warehouse rack using a high rack stacker cranes between racks forming a support structure of a roof and walls of a warehouse. That is because it is assumed that an order-picking process takes place in a storage area. Therefore, to calculate the parameters related to geometry of a warehouse in a storage area in case of a storage in rows in a warehouse rack using a high rack stacker cranes between racks (racks form a support structure of the roof and walls of a warehouse;  $n = 6, m = 6$ ), following equations can be used.

Quantity of rack columns in a warehouse, as a dimensionless parameter, can be calculated by formula (21).

$$L_{col}^{sknm} = \left\lceil \frac{Z + A}{g^{sknm} \cdot L_l^{sknm} \cdot (L_a^{sknm} \cdot 2)} \right\rceil, \quad (21)$$

The total width of a storage area in a warehouse  $S^{sknm}$  can be calculated in meters by formula (22).

$$S^{sknm} = L_a^{sknm} \cdot sz_a + L_a^{sknm} \cdot 2 \cdot sz_r, \quad (22)$$

The total length of a storage area in a warehouse  $D^{sknm}$  is given as formula (23).

$$D^{sknm} = L_{col}^{sknm} \cdot 3 + 6 \cdot (L_{ca}^{sknm} + 2), \quad (23)$$

Some other formulas used in the research (e.g.: a minimal investment expenditure coefficient, a warehouse sizes balance coefficient) are described below.

The parameter called “sustainable warehouse building investment expenditure” was developed due to two facts. The first is the diversity of prices per 1 m<sup>3</sup> of the building construction in a storage area for each subsequent type of storage, whereas the second is 1 m<sup>3</sup> building construction price differentials of storage types in an order-picking area. The differences are levelled by parameters such as: a base price of building construction per 1 m<sup>3</sup> in a storage area and a base price of building construction per 1 m<sup>3</sup> in an order-picking area.

A sustainable warehouse building investment expenditure should be evaluated especially while diversifying the ways of storage types take place (in case of different variants of warehouse layout).

A sustainable warehouse building investment expenditure can be calculated using formula (24).

$$\bar{N}_w^M = N_w^g + N_w^s \cdot \frac{c^{sn}}{c_{baz}^s} + N_w^k \cdot \frac{c^{km}}{c_{baz}^k} + N_w^e + N_w^x + N_w^b, \quad (24)$$

Notations are:

- $\bar{N}_w^M$  – a sustainable warehouse building investment expenditure in case of  $w$ -variant,  $w = 1, \dots, w, \dots, W$ , [currency],
- $N_w^g$  – an investment expenditure on ground (superscript:  $g$ ) under a warehouse building in case of  $w$ -variant,  $w = 1, \dots, w, \dots, W$ , [currency],
- $N_w^s$  – an investment expenditure in a storage area ( $s$ ) in a warehouse building in case of  $w$ -variant,  $w = 1, \dots, w, \dots, W$ , [currency],
- $c^{sn}$  – a building construction price per 1 m<sup>3</sup> in a storage area ( $s$ ) in case of storage type number  $n$  ( $n = 1, \dots, 6$ ), [currency],
- $c_{baz}^s$  – a base ( $baz$ ) price of a building construction per 1 m<sup>3</sup> in a storage area ( $s$ ), [currency/m<sup>2</sup>],
- $N_w^k$  – an investment expenditure on an order-picking area ( $k$ ) in a warehouse building in case of  $w$ -variant,  $w = 1, \dots, w, \dots, W$ , [currency],
- $c^{km}$  – a price of building construction per 1 m<sup>3</sup> in an order-picking area ( $k$ ) in case of storage type number  $m$  ( $m = 1, \dots, 6$ ), [currency/m<sup>3</sup>],

- $c_{baz}^k$  – a base (*baz*) price of building construction per 1 m<sup>3</sup> in an order-picking area (*k*), [currency/m<sup>3</sup>],
- $N_w^e$  – an investment expenditure on an entry area (superscript: *e*) in a warehouse building in case of *w*-variant,  $w = 1, \dots, w, \dots, W$ , [currency],
- $N_w^x$  – an investment expenditure on an exit area (superscript: *x*) in a warehouse building in case of *w*-variant,  $w = 1, \dots, w, \dots, W$ , [currency],
- $N_w^b$  – an investment expenditure on an order-picking buffer area (superscript: *b*) in a warehouse building in case of *w*-variant,  $w = 1, \dots, w, \dots, W$ , [currency].

The parameter called “sustainable investment expenditure on pallet places in a warehouse” was developed due to two facts. The first one is the diversity of prices per one pallet place of building construction in a storage area for each subsequent type of storage, whereas the second fact is that pallet place construction price diversity in various storage types in an order-picking area occurs. The differences are levelled by parameters such as: a base price of one pallet place construction in a storage area and a base price of one pallet place construction in an order-picking area.

As for a sustainable warehouse building investment expenditure, a sustainable investment expenditure on pallet places in a warehouse should be evaluated especially while diversifying the types of storage in case of different variants of warehouse layout.

A sustainable investment expenditure on pallet places in a warehouse is calculated by formula (25).

$$\bar{N}_w^{mp} = N_w^{mps} \cdot \frac{c_n^{mps}}{c_{baz}^{mps}} + N_w^{mpk} \cdot \frac{c_m^{mpk}}{c_{baz}^{mpk}}, \quad (25)$$

Notations are:

- $\bar{N}_w^{mp}$  – a sustainable investment expenditure on pallet places (*mp*) in a warehouse in case of *w*-variant,  $w = 1, \dots, w, \dots, W$ , [currency],
- $N_w^{mps}$  – an investment expenditure on pallet places (*mp*) in a warehouse storage area (*s*) in case of *w*-variant,  $w = 1, \dots, w, \dots, W$ , [currency],

- $N_w^{mpk}$  – an investment expenditure on pallet places (*mp*) in a warehouse order-picking area (*k*) in case of *w*-variant,  $w = 1, \dots, w, \dots, W$ , [currency],
- $c_n^{mps}$  – a price of one pallet place construction (*mp*) in a storage area (*s*) in case of storage type number *n* ( $n = 1, \dots, 6$ ), [currency/m<sup>3</sup>],
- $c_{baz}^{mps}$  – a base (*baz*) price of one pallet place construction (*mp*) in a storage area (*s*) [currency/m<sup>3</sup>],
- $c_m^{mpk}$  – a price of one pallet place construction (*mp*) in an order-picking area (*k*) in case of storage type number *m* ( $m = 1, \dots, 6$ ), [currency/m<sup>3</sup>],
- $c_{baz}^{mpk}$  – a base (*baz*) price of one pallet place construction (*mp*) in an order-picking area (*k*) [currency/m<sup>3</sup>].

The parameter called sustainable warehouse investment expenditure  $\bar{N}_w$  is a sum of a sustainable warehouse building investment expenditure and a sustainable investment expenditure on pallet places in a warehouse, as it can be seen in formula’s (26) notation.

$$\bar{N}_w = \bar{N}_w^M + \bar{N}_w^{mp}, \quad (26)$$

“Since for a given capacity of storage area (number of storage locations) one could design various layouts (altering e.g. the number of aisles and length of aisles), the problem is, which layout is optimal regarding the design objective. For example, a design objective could be costs (investment expenditure costs and operational costs). For a given capacity, different layouts have slightly different costs of required area due to the length (and Therefore, space) of front aisle. Please note that differences are in most cases negligible and it could be assumed that a given capacity defines the required total storage space. They also have different perimeter costs if walls should be built around the area. Operational costs are the costs of cycles in a considered layout. Most models in the literature optimize the layout minimizing the expected travel distance to store/retrieve an item”, Dukic et al. 2012. Let me suggest slightly different point of view.

A minimal investment expenditure coefficient is a parameter, which is used for evaluation of warehouse designs variants and to find the best

sub-optimal warehouse design variant (or possibly the best variants). The most preferred one, from the viewpoint of expenses associated with the construction of a warehouse, is a variant whose value of a coefficient is minimum.

Calculating a value of a minimal investment expenditure coefficient  $W^{N_w}$  is based on formula (27).

$$W^{N_w} = \frac{\bar{N}_w}{\max\{\bar{N}_1, \dots, \bar{N}_w, \dots, \bar{N}_W\}}, \quad (27)$$

The proposal to introduce a warehouse investment expenditure coefficient could be recognized as unnecessary. Simply, the minimum investment expenditure from a group for all designing variants could be considered. Such a solution is apparently true, however it would create a risk of significant under evaluation. When comparing design variants, it would not recognize the interdependence between variants, knowledge of which is necessary to make constructive criticism (evaluation) of design variants. Another of conditions for the application of a warehouse investment expenditure coefficient is that it works very well as the criterion function for the sub-optimisation of the functional and spatial areas (which means: for this research), Kostrzewski (2007, 2011a, 2012a, 2012b).

While simulation research of warehouse geometry was done, a problem associated to oversized values of transverse warehouse sizes occurred, Kostrzewski (2007, 2012a, 2012b). This is connected to a significant difference between the length and width of a warehouse building. It was attempted to eliminate this problem by developing a warehouse size sustainability coefficient.

A warehouse sizes sustainability coefficient aims to assist in decision-making choice of variants, which is characterised by no significant difference between length and width of a warehouse. A value of a warehouse sizes sustainability coefficient in case of  $w$ -variant will be assessed in relation to the appraised value determined on the basis of specific calculations presented in table 1. The calculations were based on data contained in the catalogue of warehouses, attached to Fijałkowski (1995). The length and width of warehouses are taken from tables I - XV, in Fijałkowski (1995: 242-271). The minimum value of a warehouse sizes sustainability coefficient for the data in table 1. is calculated for

the third variant and is equal to 0.166667. Therefore, it is concluded that the default value of a warehouse sizes sustainability coefficient will be  $W_{emp}^z = 0.17$ . This value is used to be compared with a warehouse sizes sustainability coefficient in case of any  $w$ -variant.

Table 1. Warehouse catalogue data.

Nr	$S_w$ [m]	$D_w$ [m]	$W_w^z$
I	2	3	4
I	42	78	0.538462
II	24	84	0.285714
III	15	90	0.166667
IV	63	90	0.700000
V	33	102	0.323529
VI	21	102	0.205882
VII	75	108	0.694444
VIII	42	102	0.411765
IX	24	108	0.222222
X	81	108	0.750000
XI	57	102	0.558824
XII	30	114	0.263158
XIII	87	114	0.763158
XIV	69	102	0.676471
XV	33	120	0.275000

Source: based on Fijałkowski (1995: 242-271).

Theoretically, according to Dukic *et al.* (2012), the shape of the resulting optimal layout is rectangular with the proportion of  $D_w : S_w = 1 : 2$ , while in practice it should usually be slightly modified under some circumstances. That is why it is good to check a warehouse sizes sustainability coefficient, while computing different variants in case of checking whether the proportion is reasonable.

A warehouse size sustainability coefficient is calculated using formula (28).

$$W_w^z = \frac{S_w}{D_w}, \quad \text{for } D_w > S_w; \quad W_w^z = \frac{D_w}{S_w}, \quad \text{for } S_w \geq D_w, \quad (28)$$

where:

- $W_w^z$  – warehouse sizes sustainability coefficient in case of  $w$ -variant,  $w = 1, \dots, w, \dots, W$ , dimensionless,
- $S_w$  – width of a warehouse in case of  $w$ -variant,  $w = 1, \dots, w, \dots, W$ , [m],

- $D_w$  – length of a warehouse in case of  $w$ -variant,  $w = 1, \dots, w, \dots, W$ , [m].

Sustainable parameters differ from their "not sustainable" counterparts in the fact that they can be safely compared with various designing variants. This is because the size is somehow balanced, which is "reduced to a common denominator." This is particularly necessary when looking for an optimal design solution. For example, let me assume that in case of variant  $a$  a type of storage is different than in case of variant  $b$ . Therefore, investment expenditure of one pallet place in case of both variants may differ (e.g. option  $a$ :  $N_a^{mps} = 120$  [PLN/pallet place], option  $b$ :  $N_b^{mps} = 140$  [PLN/pallet place]. As in case of a sustainable investment expenditure on pallet places in a warehouse, these differences are offset by applying a common denominator, which is a base price of one pallet place construction. The comparison does not pose a risk of inaccuracy that would occur when compared to "not sustainable" investment expenditure.

#### 4.5. OUTPUT DATA

Due to the relatively long-time "hand-written" calculating process the software was prepared (*OL09*). While preparing an implementation of rational searching procedure for functional and spatial areas designing (so-called sub-optimisation) into the software a simulation idea was used. A computer simulation is used in this case as a mean to generate the input data, which are generated from a range defined by the software user. It is also used to receive output data after the application of mathematical apparatus implemented in the software.

In the software data is generated in two steps: after the simulation and after sub-optimisation.

As a result of the simulation idea implemented into *OL09* software, the following output parameters are received:

- $F_{Zw}$  – a warehouse surface in case of variant No.  $w$  ( $w = 1, \dots, w, \dots, W$ ), [m<sup>2</sup>],
- $H_{Zw}$  – a warehouse height in case of variant No.  $w$  ( $w = 1, \dots, w, \dots, W$ ), [m],
- $N_w^M$  – an investment expenditure on a warehouse building in case of variant No.  $w$  ( $w = 1, \dots, w, \dots, W$ ), [currency],

- $N_w^{mp}$  – an investment expenditure on a pallet place in a warehouse in case of variant No.  $w$  ( $w = 1, \dots, w, \dots, W$ ), [currency],
- $\bar{N}_w^M$  – a sustainable investment expenditure on a warehouse in case of variant No.  $w$  ( $w = 1, \dots, w, \dots, W$ ), [currency],
- $\bar{N}_w^{mp}$  – a sustainable investment expenditure on pallet places in case of variant No.  $w$  ( $w = 1, \dots, w, \dots, W$ ), [currency],
- $\bar{N}_w$  – a sustainable investment expenditure on a warehouse in case of variant No.  $w$  ( $w = 1, \dots, w, \dots, W$ ), [currency],
- $W_w^{N_w}$  – a minimal investment expenditure coefficient in case of variant No.  $w$  ( $w = 1, \dots, w, \dots, W$ ),
- $W_w^z$  – a warehouse size sustainability coefficient in case of variant No.  $w$  ( $w = 1, \dots, w, \dots, W$ ).

As a result of the sub-optimisation implemented into *OL09* software, the following output parameters are received:

- $w^*$  – sub-optimal variant(s),
- $W_w^{N_w^{m^*}}$  – a minimal investment expenditure coefficient in case of sub-optimal variant No.  $w^*$ ,
- $n^*$  – a number of a type of storage in a storage area in case of sub-optimal variant No.  $w^*$ ,
- $L_{lw}^{sn^*}$  – quantity of levels in a pallet stack on a floor or in pallet rack storage in a storage area in case of sub-optimal variant No.  $w^*$ ,
- $L_{aw}^{sn^*}$  – quantity of aisles in a storage area in case of sub-optimal variant No.  $w^*$ ,
- $L_{caw}^{sn^*}$  – quantity of cross aisles in a storage area in case of sub-optimal variant No.  $w^*$ ,
- $g_{lw}^{sn^*}$  – quantity of pallet load entity on a rack shelf (or a way of putting a pallet into stack/row: with shorter or longer side of pallet) in a storage area in case of sub-optimal variant No.  $w^*$ ,
- $m^*$  – a number of a type of a storage in an order-picking area in case of sub-optimal variant No.  $w^*$ ,
- $L_{lw}^{km^*}$  – quantity of levels in a pallet stack on a floor or in a pallet rack storage in an

order-picking area in case of sub-optimal variant No.  $w^*$ ,

- $L_{lw}^{km*}$  – quantity of aisles in an order-picking area in case of sub-optimal variant No.  $w^*$ ,
- $L_{caw}^{km*}$  – quantity of cross aisles in an order-picking area in case of sub-optimal variant No.  $w^*$ ,
- $g_{w^*}^{km*}$  – quantity of pallet load entity on a rack shelf (or a way of putting a pallet into stack/row: with shorter or longer side of pallet) in a storage area in case of sub-optimal variant No.  $w^*$ .

4.6. MATHEMATICAL MODEL FORMULATION

Having in mind the notations that are mentioned above, the mathematical model of rational searching procedure can be formulated. It is formulated below.

**Input data:**

- $c_g, c_g \in R^+$ , -  $\varphi_{in/out}, \varphi_{in/out} \in R^+$ ,
- $c^{sn}, c_n^s \in R^+$ , -  $\rho, \rho \in R^+$ ,
- $c_{baz}^s, c_{baz}^s \in R^+$ , -  $\tau, \tau \in R^+$ ,
- $c^{km}, c_m^k \in R^+$ , -  $w \times p; w \in N^+, p \in N^+$ ,
- $c_p, c_p \in R^+$ , -  $\varphi_l, \varphi_l \in R^+$ ,
- $c_{baz}^k, c_{baz}^k \in R^+$ , -  $F_Z, F_Z \in R^+$ ,
- $c_n^{mps}, c_n^{mps} \in R^+$ , -  $H_Z, H_Z \in R^+$ ,
- $c_{baz}^{mps}, c_{baz}^{mps} \in R^+$ , -  $L_{l\min}^{sn}; L_{l\min}^{sn} \in N^+$ ,
- $c_m^{mpk}, c_m^{mpk} \in R^+$ , -  $L_{a\min}^{sn}; L_{a\min}^{sn} \in N^+$ ,
- $c_{baz}^{mpk}, c_{baz}^{mpk} \in R^+$ , -  $L_{ca\min}^{sn}; L_{ca\min}^{sn} \in N^+ + \{0\}$ ,
- $P^R, P^R \in N^+$ , -  $L_{l\max}^{sn}; L_{l\max}^{sn} \in N^+$ ,
- $N, N \in N^+$ , -  $L_{a\max}^{sn}; L_{a\max}^{sn} \in N^+$ ,
- $d_r, d_r \in N^+$ , -  $g^{sn}; g^{sn} \in N^+$ ,
- $Z_p, Z_p \in N^+$ , -  $L_{ca\max}^{sn}; L_{ca\max}^{sn} \in N^+ + \{0\}$ ,
- $A, A \in N^+$ ,

**An allowed set:**

- $L_{l\min}^{km}; L_{l\min}^{km} \in N^+$ , -  $1 \leq L_l^1 \leq 5$ ,
- $L_{a\min}^{km}; L_{a\min}^{km} \in N^+$ , -  $1 \leq L_l^{k1} \leq 5$ ,
- $L_{ca\min}^{km}; L_{ca\min}^{km} \in N^+ + \{0\}$ , -  $1 \leq L_l^{s2} \leq 4$ ,
- $L_{l\max}^{km}; L_{l\max}^{km} \in N^+$ , -  $1 \leq L_l^{k2} \leq 4$ ,
- $L_{a\max}^{km}; L_{a\max}^{km} \in N^+$ , -  $1 \leq L_l^{s3} \leq 6$ ,
- $L_{ca\max}^{km}; L_{ca\max}^{km} \in N^+ + \{0\}$ , -  $1 \leq L_l^{k3} \leq 6$ ,
- $g^{km}; g^{km} \in N^+$ , -  $1 \leq L_l^{s4} \leq 13$ ,
- $W_{emp}^z, W_{emp}^z \in R^+$ , -  $1 \leq L_l^{k4} \leq 13$ ,
- $1 \leq L_l^{s5} \leq 15$ ,
- $1 \leq L_l^{k5} \leq 15$ ,
- $1 \leq L_l^{s6} \leq 31$ ,
- $1 \leq L_l^{k6} \leq 31$ ,
- $1 \leq L_a^{sn} \leq 10$ ,
- $1 \leq L_a^{km} \leq 10$ ,
- $1 \leq L_{ca}^{sn} \leq 5$ ,
- $1 \leq L_{ca}^{km} \leq 5$ ,
- $1 \leq n \leq 6$ ,
- $1 \leq m \leq 6$ .

**Decision variables:**

- $n$ , -  $m$ ,
- $L_{lw}^{sn}$ , -  $L_{lw}^{km}$ ,
- $L_{aw}^{sn}$ , -  $L_{aw}^{km}$ ,
- $L_{caw}^{sn}$ , -  $L_{caw}^{km}$ ,
- $g_w^{sn}$ , -  $g_w^{km}$ .

**Task objective function (the criterion function):**

$$W^{N_w^{n^*m^*}} = \min_{w \in W} \left\{ \frac{\bar{N}_w^{nm} (n, L_l^{sn}, L_a^{sn}, L_{ca}^{sn}, g^{sn}, m, L_l^{km}, L_a^{km}, L_{ca}^{km}, g^{km})}{\max \{ \bar{N}_1^{nm}, \dots, \bar{N}_w^{nm}, \dots, \bar{N}_W^{nm} \}} \right\} =$$

$$= \min_{w \in W} \left\{ \frac{\min \{ \bar{N}_w^{11}, \dots, \bar{N}_w^{1m}, \dots, \bar{N}_w^{16} \}}{\max \{ \bar{N}_1^{nm}, \dots, \bar{N}_w^{nm}, \dots, \bar{N}_W^{nm} \}}; \frac{\min \{ \bar{N}_w^{21}, \dots, \bar{N}_w^{2m}, \dots, \bar{N}_w^{26} \}}{\max \{ \bar{N}_1^{nm}, \dots, \bar{N}_w^{nm}, \dots, \bar{N}_W^{nm} \}}; \frac{\min \{ \bar{N}_w^{31}, \dots, \bar{N}_w^{3m}, \dots, \bar{N}_w^{36} \}}{\max \{ \bar{N}_1^{nm}, \dots, \bar{N}_w^{nm}, \dots, \bar{N}_W^{nm} \}}; \right.$$

$$\left. \frac{\min \{ \bar{N}_w^{41}, \dots, \bar{N}_w^{4m}, \dots, \bar{N}_w^{46} \}}{\max \{ \bar{N}_1^{nm}, \dots, \bar{N}_w^{nm}, \dots, \bar{N}_W^{nm} \}}; \frac{\min \{ \bar{N}_w^{51}, \dots, \bar{N}_w^{5m}, \dots, \bar{N}_w^{56} \}}{\max \{ \bar{N}_1^{nm}, \dots, \bar{N}_w^{nm}, \dots, \bar{N}_W^{nm} \}}; \frac{\min \{ \bar{N}_w^{61}, \dots, \bar{N}_w^{6m}, \dots, \bar{N}_w^{66} \}}{\max \{ \bar{N}_1^{nm}, \dots, \bar{N}_w^{nm}, \dots, \bar{N}_W^{nm} \}} \right\}$$

$n = 1, \dots, 6; m = 1, \dots, 6; w = 1, \dots, w, \dots, W; \max \{ \bar{N}_1^{nm}, \dots, \bar{N}_w^{nm}, \dots, \bar{N}_W^{nm} \} \neq 0; \bar{N}_w^{nm}$  exists;

$$\bar{N}_w^{nm} = \bar{N}_w^M + \bar{N}_w^{mp} = N_w^g + N_w^s \cdot \frac{c^{sn}}{c_{baz}^s} + N_w^k \cdot \frac{c^{km}}{c_{baz}^k} + N_w^e + N_w^x + N_w^b + N_w^{mps} \cdot \frac{c_n^{mps}}{c_{baz}^{mps}} + N_w^{mpk} \cdot \frac{c_m^{mpk}}{c_{baz}^{mpk}} =$$

$$= D_w^M \cdot S_w^M \cdot c_g + D_w^s \cdot S_w^s \cdot \frac{c^{sn} \cdot c^{sn}}{c_{baz}^s} + D_w^k \cdot S_w^k \cdot \frac{c^{km} \cdot c^{km}}{c_{baz}^k} + D_w^e \cdot S_w^e \cdot c_p + D_w^x \cdot S_w^x \cdot c_p + D_w^b \cdot S_w^b \cdot c_p +$$

$$+ L_w^{mps} \cdot \frac{c_n^{mps} \cdot c_n^{mps}}{c_{baz}^{mps}} + L_w^{mpk} \cdot \frac{c_m^{mpk} \cdot c_m^{mpk}}{c_{baz}^{mpk}} = [*]$$

In case of  $n = 3/4/5, m = 3/4/5$  the model takes a form as follow below. Storage types  $n = 3/4/5, m = 3/4/5$  were chosen due to the fact that these configuration occur most frequently in warehouses.

$$\bar{N}_w^{3/4/5/3/4/5} = [*] = \bar{N}_w^g + D_w^s \cdot S_w^s \cdot \frac{c^{s3/4/5} \cdot c^{s3/4/5}}{c_{baz}^s} + D_w^k \cdot S_w^k \cdot \frac{c^{k3/4/5} \cdot c^{k3/4/5}}{c_{baz}^k} + D_w^e \cdot S_w^e \cdot c_p +$$

$$+ D_w^x \cdot S_w^x \cdot c_p + D_w^b \cdot S_w^b \cdot c_p + L_w^{mps} \cdot \frac{c_{3/4/5}^{mps} \cdot c_{3/4/5}^{mps}}{c_{baz}^{mps}} + L_w^{mpk} \cdot \frac{c_{3/4/5}^{mpk} \cdot c_{3/4/5}^{mpk}}{c_{baz}^{mpk}} = \bar{N}_w^g +$$

$$+ \left\{ \left[ \frac{Z}{g_w^{s3/4/5} \cdot L_{lw}^{s3/4/5} \cdot (2 \cdot L_{aw}^{s3/4/5})} \right] / L_{aw}^{s3/4/5} \cdot 3 + 6 \cdot (L_{caw}^{s3/4/5} + 2) \right\} \cdot$$

$$\cdot \left\{ (L_{aw}^{s3/4/5} + 2) \cdot sz_a^s + L_{aw}^{s3/4/5} \cdot 2 \cdot sz_r^s \right\} \cdot c^{s3/4/5} \cdot c^{s3/4/5} / c_{baz}^s +$$

$$+ \left\{ \left[ \frac{A}{g_w^{k3/4/5} \cdot L_{lw}^{k3/4/5} \cdot (2 \cdot L_{aw}^{k3/4/5})} \right] / L_{lw}^{k3/4/5} \cdot 3 + 6 \cdot (L_{caw}^{k3/4/5} + 2) \right\} \cdot$$

$$\cdot \left\{ (L_{aw}^{k3/4/5} + 2) \cdot sz_a^k + L_{aw}^{k3/4/5} \cdot 2 \cdot sz_r^k \right\} \cdot c^{k3/4/5} \cdot c^{k3/4/5} / c_{baz}^k +$$

$$+ 2 \cdot \mu \cdot 6 \cdot \left[ (L_{aw}^{s3/4/5} + 2) \cdot sz_a^s + L_{aw}^{s3/4/5} \cdot 2 \cdot sz_r^s \right] / 2 \cdot c_p +$$

$$+ \left\{ \left[ \frac{Z}{g_w^{s3/4/5} \cdot L_{lw}^{s3/4/5} \cdot (2 \cdot L_{aw}^{s3/4/5})} \right] / L_{aw}^{s3/4/5} \cdot 3 + 6 \cdot (L_{caw}^{s3/4/5} + 2) + \mu \cdot 6 + \right.$$

$$\left. - \left[ \frac{A}{g_w^{k3/4/5} \cdot L_{lw}^{k3/4/5} \cdot (2 \cdot L_{aw}^{k3/4/5})} \right] / L_{aw}^{k3/4/5} \cdot 3 + \right.$$

$$\left. + 6 \cdot (L_{caw}^{k3/4/5} + 2) \right\} \cdot \left\{ (L_{aw}^{k3/4/5} + 2) \cdot sz_a^k + L_{aw}^{k3/4/5} \cdot 2 \cdot sz_r^k \right\} \cdot c_p +$$

$$\begin{aligned}
 &+ 2 \cdot L_{aw}^{s3/4/5} \cdot L_{lw}^{s3/4/5} \cdot \left[ \left[ \frac{Z}{g_w^{s3/4/5} \cdot L_{lw}^{s3/4/5} \cdot (2 \cdot L_{aw}^{s3/4/5})} \right] / L_{aw}^{s3/4/5} \right] \cdot \frac{C_{3/4/5}^{mps} \cdot C_{3/4/5}^{mps}}{C_{baz}^{mps}} + \\
 &+ 2 \cdot L_{aw}^{k3/4/5} \cdot L_{lw}^{k3/4/5} \cdot \left[ \left[ \frac{A}{g_w^{k3/4/5} \cdot L_{lw}^{k3/4/5} \cdot (2 \cdot L_{aw}^{k3/4/5})} \right] / L_{aw}^{k3/4/5} \right] \cdot g_w^{k3/4/5} \cdot \frac{C_{3/4/5}^{mpk} \cdot C_{3/4/5}^{mpk}}{C_{baz}^{mpk}}
 \end{aligned}$$

$w = 1, \dots, w, \dots, W;$

if  $g_w^{s/k3} = 2$  then  $sz_a^{s/k} = 3,35[m]$   $sz_r^{s/k} = 1,4[m]$

if  $g_w^{s/k3} = 3$  then  $sz_a^{s/k} = 3,05[m]$   $sz_r^{s/k} = 1[m]$

if  $g_w^{s/k4} = 2$  then  $sz_a^{s/k} = 1[m]$   $sz_r^{s/k} = 1[m]$

if  $g_w^{s/k4} = 3$  then  $sz_a^{s/k} = 1,4[m]$   $sz_r^{s/k} = 1,4[m]$

if  $g_w^{s/k5} = 3$  then  $sz_a^{s/k} = 1,4[m]$   $sz_r^{s/k} = 1,4[m]$

for  $D_w^s \geq D_w^k$

$$\bar{N}_w^g = (D_w^s + D_w^k) \cdot (S_w^s + S_w^k) \cdot c_g =$$

$$\begin{aligned}
 &= \left\{ \left[ \left[ \frac{Z}{g_w^{s3/4/5} \cdot L_{lw}^{s3/4/5} \cdot (2 \cdot L_{aw}^{s3/4/5})} \right] / L_{aw}^{s3/4/5} \right] \cdot 3 + 6 \cdot (L_{caw}^{s3/4/5} + 2) + \mu \cdot 6 \right\} \cdot \\
 &\cdot \left\{ (L_{aw}^{s3/4/5} + 2) \cdot sz_a^s + L_{aw}^{s3/4/5} \cdot 2 \cdot sz_r^s + (L_{aw}^{k3/4/5} + 2) \cdot sz_a^k + L_{aw}^{k3/4/5} \cdot 2 \cdot sz_r^k \right\} \cdot c_g
 \end{aligned}$$

for  $D_w^s < D_w^k$

$$\bar{N}_w^g = (D_w^s + D_w^k) \cdot (S_w^s + S_w^k) \cdot c_g =$$

$$\begin{aligned}
 &= \left\{ \left[ \left[ \frac{A}{g_w^{k3/4/5} \cdot L_{lw}^{k3/4/5} \cdot (2 \cdot L_{aw}^{k3/4/5})} \right] / L_{aw}^{k3/4/5} \right] \cdot 3 + 6 \cdot (L_{caw}^{k3/4/5} + 2) + \mu \cdot 6 \right\} \cdot \\
 &\cdot \left\{ (L_{aw}^{s3/4/5} + 2) \cdot sz_a^s + L_{aw}^{s3/4/5} \cdot 2 \cdot sz_r^s + (L_{aw}^{k3/4/5} + 2) \cdot sz_a^k + L_{aw}^{k3/4/5} \cdot 2 \cdot sz_r^k \right\} \cdot c_g
 \end{aligned}$$

As it can be seen a warehouse size sustainability coefficient do not occur in the model, however it is included in the *OL09* software.

#### 4.7. COMPUTATIONAL EXAMPLE IN THE *OL09* SOFTWARE

Rational searching procedure for functional and spatial designing has been implemented into *OL09* software. Let me discuss the computing example of using the software as a mean to find sub-optimal solution.

Since the model, formulated in subchapter 4.5., was presented for storage types:  $n = 3/4/5$  and  $m = 3/4/5$ , Therefore, also the example solution searching concerns these types of storage. The entry data for the example is given in table 2.



Table 2. Technological data (input data) for warehouse design.

Parameter	Value
Quantity of entities being operated in warehouse per year, [ple/year]	129 000
Norm factor of storage	20
Quantity of working days per year, [days]	285
Storage rotation	15
Number of pallet-places in stock	8 600
Assortment	468
Accumulations coefficient on entrance	1.24
Accumulations coefficient on exit	1.54
Medium filling percentage of pallet	0.54
Order-picking coefficient	0.67
Medium structure of pallet load entity after order-picking process	10 x 4
Utilisation of work-time coefficient	0.80
Warehouse permissible surface, [m <sup>2</sup> ]	8 000
Warehouse permissible height, [m]	12
Land price, [currency/m <sup>2</sup> ]	10
Price of building construction per 1 m <sup>3</sup> in case of other areas, [currency/m <sup>3</sup> ]	10
Building construction price per 1 m <sup>3</sup> in a storage area in case of storage type number $n = 3$ , [currency/m <sup>3</sup> ]	100
Building construction price per 1 m <sup>3</sup> in a storage area in case of storage type number $n = 4$ , [currency/m <sup>3</sup> ]	110
Building construction price per 1 m <sup>3</sup> in a storage area in case of storage type number $n = 5$ , [currency/m <sup>3</sup> ]	130
Price of building construction per 1 m <sup>3</sup> in an order-picking area in case of storage type number $m = 3$ , [currency/m <sup>3</sup> ]	100
Price of building construction per 1 m <sup>3</sup> in an order-picking area in case of storage type number $m = 4$ , [currency/m <sup>3</sup> ]	110
Price of building construction per 1 m <sup>3</sup> in an order-picking area in case of storage type number $m = 5$ , [currency/m <sup>3</sup> ]	130
Price of one pallet place construction in a storage area in case of storage type number $n = 3$ , [currency/m <sup>3</sup> ]	100
Price of one pallet place construction in a storage area in case of storage type number $n = 4$ , [currency/m <sup>3</sup> ]	130
Price of one pallet place construction in a storage area in case of storage type number $n = 5$ , [currency/m <sup>3</sup> ]	125
Price of one pallet place construction in an order-picking area in case of storage type number $n = 3$ , [currency/m <sup>3</sup> ]	100
Price of one pallet place construction in an order-picking area in case of storage type number $n = 4$ , [currency/m <sup>3</sup> ]	130
Price of one pallet place construction in an order-picking area in case of storage type number $n = 5$ , [currency/m <sup>3</sup> ]	125
Base price of building construction per 1 m <sup>3</sup> in a storage area, [currency/m <sup>3</sup> ]	100
Base price of building construction per 1 m <sup>3</sup> in an order-picking area, [currency/m <sup>3</sup> ]	100
Base price of one pallet place construction in a storage area [currency/m <sup>3</sup> ]	100
Base price of one pallet place construction in an order-picking area [currency/m <sup>3</sup> ]	100

Quantity of solutions considered in the software, taking into account the warehouse size sustainability coefficient, was equal to 60 000 solutions. Solutions that do not meet the limit on the warehouse sizes sustainability coefficient were rejected. After computing two sub-optimal solutions of warehouse layout searching were chosen. One of them, more reasonable one, is presented below. Unreasonableness of one solution was connected with the fact that while in case of a storage area a storage in rows in a warehouse rack using front-side fork-lift truck ( $n = 4$ ) was chosen, in an order-picking area a storage in rows in a warehouse rack using low and average height rack stacker cranes ( $m = 5$ ) were proposed by the software. This is not a solution, which could be implemented in the real conditions of warehouse operating. The better solution is presented in figure 1. The chosen variants number is  $w^* = 58615$ . The values of the sustainable warehouse investment

expenditure is  $\bar{N}_{58615} = 90\,720$  [currency]. The value of the minimal investment expenditure coefficient is  $W^{N^*58615} = 0.1586$ . Results of chosen layout solution are:

- the storage area: the storage in rows in the warehouse rack using a low and an average height rack stacker cranes,
- the storage area geometry parameters:
  - the quantity of levels ( $l$ ) in pallet stack on the floor or in pallet rack storage in the storage area ( $s$ ) for 5th-type of storage:  $L_l^{s5} = 5$ ,
  - the quantity of aisles ( $a$ ) in the storage area ( $s$ ) for 5th-type of storage:  $L_a^{s5} = 5$ ,
  - the quantity of cross aisles ( $ca$ ) in the storage area ( $s$ ) for 5th-type of storage:  $L_{ca}^{s5} = 0$ ,

- the quantity of pallet load entities on rack shelf in the storage area (*s*) for 5th-type of storage,  $g^{s5} = 3$ ,
- the total quantity of slots in a rack along the aisle in one level: 58,
- the width of the storage area (*s*) in the warehouse:  $S^{s5} = 24$  [m],
- the length of the storage area (*s*) in the warehouse:  $D^{s5} = 186$  [m],
- the order-picking area: the storage in rows in the warehouse rack using a low and an average height rack stacker cranes,
- the order-picking area geometry parameters:
  - the quantity of levels (*l*) in pallet stack on a floor or in pallet rack storage in the order-picking area (*k*) for 5th-type of storage:  $L_l^{k5} = 2$ ,
  - the quantity of aisles (*a*) in the order-picking area (*k*) for 5th-type of storage:  $L_a^{k5} = 2$ ,
  - the quantity of cross aisles (*ca*) in the order-picking area (*k*) for 5th-type of storage:  $L_{ca}^{k5} = 0$ ,
  - the quantity of pallet entities load on rack shelf in the order-picking area (*k*) for 5th-type of storage,  $g^{k5} = 3$ ,
  - the total quantity of slots in a rack along the aisle on one level: 20,
  - the width of the order-picking area (*k*) in the warehouse:  $S^{k5} = 12$  [m],
  - the length of the order-picking area (*k*) in the warehouse:  $D^{k5} = 72$  [m],
- the warehouse geometry parameters:
  - the length of the storage area (*s*) in the warehouse:  $D^{s5} = 186$  [m],
  - the width of the storage area (*s*) in the warehouse:  $S^{s5} = 24$  [m],
  - the length of the order-picking area (*k*) in the warehouse:  $D^{k5} = 72$  [m],
  - the width of the order-picking area (*k*) in the warehouse:  $S^{k5} = 12$  [m],
  - the length of the entry area (*e*) in the warehouse:  $D^{e5} = 12$  [m],
  - the width of the entry area (*e*) in the warehouse:  $S^{e5} = 24$  [m],
  - the length of the exit area (*x*) in the warehouse:  $D^{x5} = 12$  [m],
  - the width of the exit area (*x*) in the warehouse:  $S^{x5} = 12$  [m],
  - the length of the after order-picking buffer area (*b*) in the warehouse:  $D^{b5} = 126$  [m],
  - the width of the after order-picking buffer area (*b*) in the warehouse:  $S^{b5} = 12$  [m],
  - the length of the warehouse:  $D^5 = 198$  [m],
  - the width of the warehouse:  $S^5 = 36$  [m],
  - the surface of the warehouse:  $F_Z^5 = 7128$  [m<sup>2</sup>],
  - the height of the warehouse:  $H_Z^5 = 8$  [m],
  - the quantity of pallet places: 8 700.

Calculations, using the software, were done on two different computers. In case of computer 1., rational searching procedure computing time of all solutions in the software was 1 minute 15 seconds. The parameters of the computer used for computing are: INTEL®Core™2 CPU T7200 @ 2.00 GHz, 2.00 GB RAM with 32-bit operating system Windows Vista™ Business. Meanwhile, in case of computer 2., rational searching procedure computing time of all solutions in the software was 40 seconds. The parameters of the computer used for computing are: INTEL®Core™ i3 CPU M370 @ 2.40 GHz, 4.00 GB RAM with 32-bit operating system Windows 7 Professional.

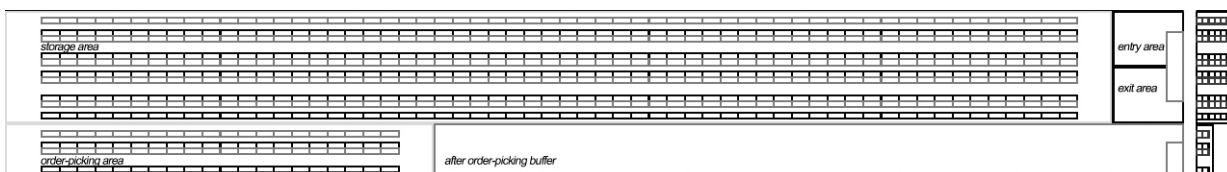


Fig. 1. The sketch of the warehouse layout in solution using OL09 software – variant No. 58615; the top view on the left side of the figure and the cross section view on the right side of the figure.

## 5. SUMMARY

In the mentioned logistics facilities designing method, particular emphasis has been put on layout sub-optimising. This problem is connected to preparing the rational searching procedure for functional and spatial areas and its implementation in the form of *OL09* software. Sub-optimisation, among others, refers to choosing the appropriate types of storage in the storage area and the order-picking area. The types are selected in a way that investment expenditure on the construction (and building itself) of the warehouse were as low as possible (a minimal investment expenditure coefficient) taking into account the rational approach to planning of space inside the building (a warehouse size sustainability coefficient). Increasing the use of a warehouse surface is one of the ways to reduce costs of warehouse investment. At the same time it has to be understood that there is not only one optimal area of the warehouse building. The optimum dimensions can only be determined in the individual, strictly defined variants of designing solution. Therefore, the research was focused particularly on warehouse geometry due to the fact that many other issues depend on geometrical parameters. Comprehensive discussion of the designing method and its implementation in the software allowed distinguishing a number of theoretical and functional aspects of the problem. It is specifically described in the summary of Kostrzewski (2012b). "Some main theoretical aspects are described below. (...) assessment of designing method based on existing methods and the fact that a decision was made to redefine existing methods with addition of some designing steps and enrichment with the procedure for optimisation of functional and spatial areas (...) And last but not the least, an effort was made to elaborate on a proposition of mathematical formulas for calculations connected with logistics facility geometry. Formulas were prepared as a function of:

- selected type of storage in a storage area,
- selected type of storage in an order-picking area,
- a number of levels in pallet rack storage in a storage area,
- a number of aisles in a storage area,
- a number of perpendicular corridors in a storage area,
- a number of levels in pallet rack storage in an order-picking area,
- a number of aisles in an order-picking area,
- a number of perpendicular corridors in an order-picking area,
- and other", Kostrzewski (2012b: 338-339).

"Among the practical aspects there are identified such as the following:

- the graphical module in computer software (2D module),
- (...)
- obtaining variant solutions using the software (freeing the designer from the painstaking work, repetitive tasks such as calculations, drawing sketches of logistics facilities layout, etc.)", Kostrzewski (2012b: 339).

In addition, in the summary of mentioned paper, possible directions of further researches on the problem of logistic facilities designing method were given. There are mentioned demands to:

- "increase the flexibility of computer software by implementing other types of spatial systems and types of storage,
- extend the software by making a module of sub-areas in storage area for different types of entities (...)", Kostrzewski (2012b: 339).

In order to perform an implementation of the problem of preparing the rational searching procedure for functional and spatial areas, an idea of a computer simulation was used. The computer simulation is used to generate the input data and receive outputs around mathematical apparatus application implemented in the software as it was mentioned in sub-chapter 4.5.

The usage of simulation in framework of the procedure implementation is to exclude time-consuming complete search of variance solutions.

The complexity of the algorithm is thus a function of the input parameters. Complete search is of course the optimal method, but gaining a result would be wasting of time. That is why rationalisation of searching was proposed. Let me consider only four input parameters. If parameters  $\varphi_{in}$ ,  $\varphi_{out}$ ,  $\rho$ ,  $\tau$  would change every 0.01 entity and maximal value of  $\varphi_{in}$ ,  $\varphi_{out}$  would be  $\varphi_{in} = \varphi_{out} = 2$  then the quantity of possible variants would be equal to  $2.0175 \cdot 10^{15}$ .

Instead of a complete search (brut force attack method), a rational search is used in the research. In future further efforts should be concerned with the investigation of adequate mathematical programming models that can integrate

coefficients related to the dynamic nature of the customer order demand, the configuration of an order-picking area and the dynamic products flows.

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