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# Dynamics simulation studies on the electric city car with an electromechanical differential and the rear wheels drive

M. KOZŁOWSKI and W. CHOROMAŃSKI*

Faculty of Transport, Warsaw University of Technology, 75 Koszykowa St., 00-662 Warszawa, Poland

**Abstract.** Here we present one of the more complex models for studying the stability of driving an electric car with electromechanical differential systems. The purpose of simulation is to choose a structure of the control system for a velocity control on driven wheels (an algorithm of a differential) most appropriate for the driver. This type of goal is particularly important in the case of a disabled driver sitting in a wheelchair. The modeling takes into account both the mechanical and electric structure of the vehicle, and finally the human element – a simple model of human impact on the steer by a wire system. Modeling and simulation have used MBS package (SimMechanics). The results of the simulation have showed the best algorithms of an electromechanical differential for the velocity control of rear drive wheels: with setting a velocity difference or with an average velocity controller in the point A of the centre of a car front axle.

**Key words:** electric city car, electromechanical differential, human driver, steer by wire system, MBS modeling.

## 1. Introduction

This paper studies an electromechanical differential of an electric city car with electric motors in the rear wheels. The presented car structure is a specific structure designed for the traffic in the separated urban areas (with banned traffic of internal combustion vehicles). The vehicle is intended to be used by healthy as well as by disabled people (in the active wheelchairs as a driver and as a passenger). It determines a number of design solutions which are not under consideration in this paper, but are listed here. First of all, we mean: steer-by-wire system, brake-by-wire system, an active suspension, systems for fixing wheelchairs and people in the wheelchairs. The electric car is also equipped with hybrid energy storage (lithium ionic batteries and supercapacitor). Some of the mentioned elements are included in the simulation model (such as steer-by-wire). As aforementioned, the work concerns mainly the research of the electro-mechanical differential mechanism. The differential system is necessary in motor vehicles to enable the vehicle movement on a curved track, where there is a difference between the outer and the inner circle. There occurs a special design of this mechanism in the electric vehicles, where driving wheels are powered by separate motors (Fig. 1). In this case we have two motors placed in two rear wheels. Acting as a differential – an electronic controller steers the torque and the velocities of motors. There is a rich literature of designing and studies on the electro-mechanical differential in electric vehicles.

It seems that this work presents one of more complex models. The target of simulation studies is to choose the structure of the velocity control system for driven wheels (an algorithm of a differential) which is most appropriate for the driver. This kind of target is particularly important in the case of a disabled driver sitting in the wheelchair, in order not to make the

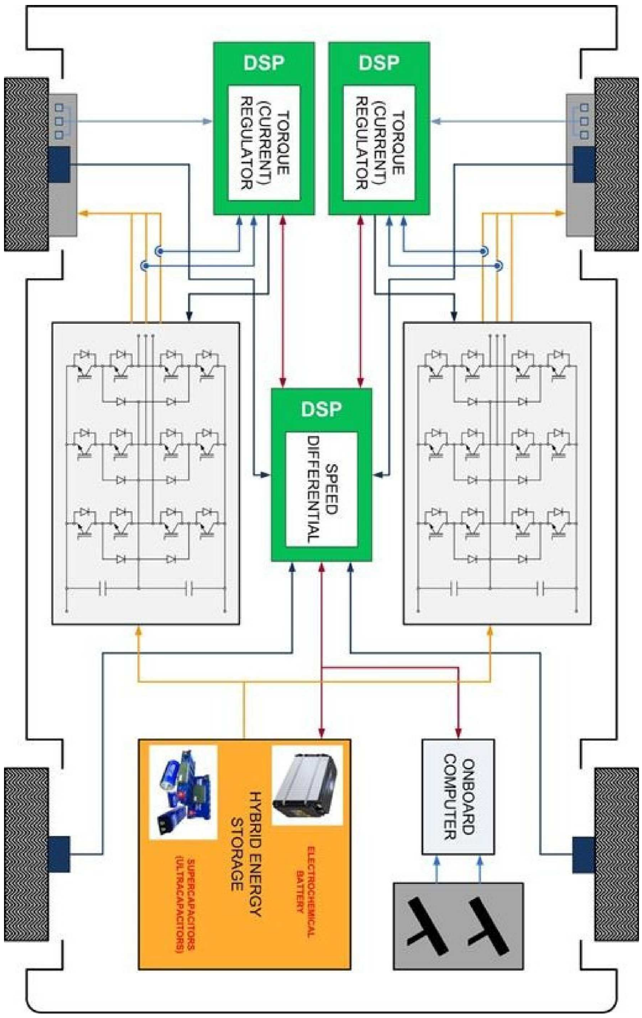


Fig. 1. The electrical structure of the vehicle

*e-mail: prof.wch@gmail.com



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function of driving a vehicle more difficult for him. Modelling and simulations used MBS [1] package (Simmechanics). The modelling takes into account both the mechanical and electrical structure of the vehicle, and finally the human element – a simple model of human impact on the steer-by-wire system. The examined vehicle, due to its functionality features, is called ECO-vehicle or ECO car in the further part of the work.

## 2. The basic relations

Under driving conditions along a curve, the electro-mechanical differential mechanism should eliminate a number of adverse effects such as vehicle lateral drift, wear of mechanical parts, etc. etc. The kinematic model of a car is shown in Fig. 2. The basic geometric data are shown in Table 1. The wheel drive is implemented through two motors placed in the rear wheels. The rotating wheels at the front are not driven.

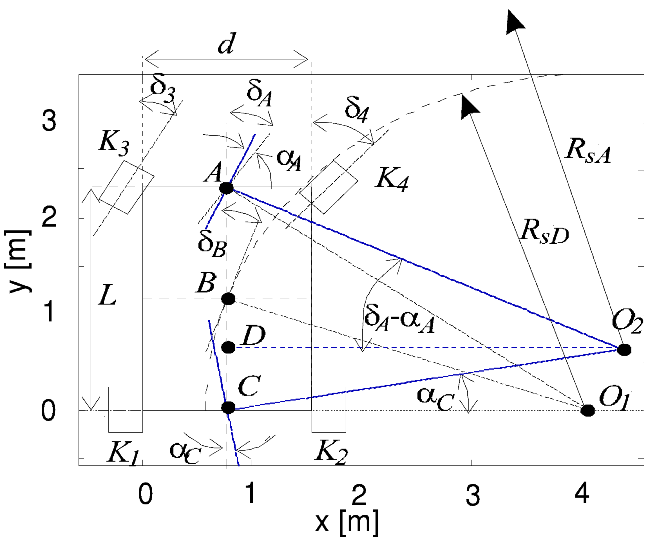


Fig. 2. An instantaneous centre of the motion curvature in the driving conditions along the arc with the turning angle 35° of a vehicle front. Marked: $K_1$, $K_2$ – driven rear wheels, $K_3$, $K_4$ – front rotating wheels, $A$, $B$, $C$ – points of bodywork projection: the centre point of the vehicle's front axle ($A$), of the car centre ($B$), the centre point of the vehicle's rear axle ($C$), $O_1$, $O_2$ – the points of instantaneous centres of curvature in the conditions of no drift driving ($O_1$) and with a side drift ($O_2$), $R_1$, $R_2$ – instantaneous turn radius, $\delta_1$, $\delta_2$ – angles of setting the odometric model of front wheels (with no drifting), $\delta_A$, $\delta_B$ – turning angles of the vehicle front and center, $\alpha_A$ – a drift angle of a vehicle front, $\alpha_C$ – a drift angle of a vehicle rear, $d$ – a vehicle wheelbase, $L$ – a distance between axles of the vehicle wheels, $r_d$ – a dynamic tyre radius

Table 1
The basic geometric data for ECO car

| Symbol | Description | Value |
|---|---|---|
| $L$ | a distance between axles of vehicle wheels | 2.330 m |
| $D$ | a vehicle wheelbase | 1.542 m |
| $r_d$ | a dynamic tyre radius | 0.3135 m |

Each of wheels moves along a different curve. At the point $D$ of bodywork, the direction of motion lies on the longitudinal symmetry axis of the vehicle:

$$v_{sD\_l} = v_{sD}, \tag{1}$$

$$v_{sD\_d} = 0, \tag{2}$$

where $v_{sD}$ – a car velocity of point $D$, $v_{sD\_l}$ – a velocity of point $D$ in the tangential direction to the motion curvature, $v_{sD\_d}$ – a velocity of point $D$ in the lateral direction to the motion curvature.

In the point $A$ of the vehicle front, one could determine tangential and lateral velocieties:

$$v_{sA\_\delta} = v_{sA} \cos \alpha_A, \tag{3}$$

$$v_{sA\_r} = v_{sA} \sin \alpha_A, \tag{4}$$

where $v_{sA}$ – a car velocity of point $A$, $v_{sA\_\delta}$ – a velocity of point $A$ in the tangential direction to the motion curvature, $v_{sA\_r}$ – a velocity of point $A$ in the direction transverse to the motion curvature.

A formula describing the curvature radius of car motion at point $D$ (in Fig. 2 indicated by symbol $R_{sD}$) can be determined from the analysis of the geometrical relations for right-angled triangles $AO_2D$ and $CO_2D$ [2]; and a formula defining the radius length of curvature at point A (in Fig. 2 indicated by symbol $R_{sA}$) – using the Pythagorean theorem for triangle $ADO_2$:

$$R_{sD} = \frac{L}{tg(\delta_A - \alpha_A) - tg(\alpha_C)}, \tag{5}$$

$$R_{sA} = \frac{L\sqrt{1 + tg^2(\delta_A - \alpha_A)}}{tg(\delta_A - \alpha_A) - tg(\alpha_C)}. \tag{6}$$

The rotational angular velocity of a car rigid body is given by the formula:

$$\Omega_{sD} = \frac{v_{sD}}{L}\left[tg(\delta_A - \alpha_A) - tg(\alpha_C)\right], \tag{7}$$

where $v_{sD}$ – a car velocity of point $D$, $\Omega_{sD}$ – the angular velocity of the vehicle point $D$ motion around the rotation centre $O_2$, $R_{sD}$ – a curvature radius of point $D$ of car motion.

The rotational angular velocities of all the points of rigid body are identical. Comparing the formulas for velocity of the point $D$ and point $A$ we get the following:

$$v_{sD} = \frac{v_{sA}}{\sqrt{1 + tg^2(\delta_A - \alpha_A)}}. \tag{8}$$

The angular velocities of wheels are associated with a tangent line component (3). The angular velocities of front wheels are expressed by the following formulas:

$$\omega_3 = \frac{v_{sD}\cos\alpha_A}{r_d}\sqrt{\left(1 + \frac{d}{2R_{sD}}\right)^2 + tg^2\left(\delta_A - \alpha_A\right)}, \tag{9}$$

$$\omega_4 = \frac{v_{sD}\cos\alpha_A}{r_d}\sqrt{\left(1 - \frac{d}{2R_{sD}}\right)^2 + tg^2\left(\delta_A - \alpha_A\right)}, \tag{10}$$

where $d$ – a vehicle wheelbase, $L$ – a distance between axles of the vehicle wheels, $r_d$ – a dynamic tyre radius, $R_3$, $R_4$ – the curvature radii of motion at contact points 3 and 4 of wheel tyres, $v_{sD}$ – a car velocity of point $D$.



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For the rear wheels:

$$\omega_1 = \frac{v_{sD}\cos\alpha_C}{r_d}\sqrt{\left(1+\frac{d}{2R_{sD}}\right)^2 + tg^2\left(\alpha_C\right)}, \qquad (11)$$

$$\omega_2 = \frac{v_{sD}\cos\alpha_C}{r_d}\sqrt{\left(1-\frac{d}{2R_{sD}}\right)^2 + tg^2\left(\alpha_C\right)}, \qquad (12)$$

where $R_1$, $R_2$ – the curvature radii of motion at contact points 1 and 2 wheel tyres.

Taking into account parameters included in Table 1, we can determine the angular velocity of car wheels $\omega_1$, $\omega_2$, $\omega_3$, $\omega_4$ as a function of the turning angle $\delta_A$ at the constant driving speed and at different values of a drift angle $\alpha_A$ and $\alpha_C$.

The function graphs at the constant driving speed of point $Dv_{sD} = 5$ m/s are described by formulas (9–12) as shown in Fig. 3: a) for the front wheels, b) for the rear wheels. The graphs 1–3 (in Fig. 3a, for the front wheels) and 7–9 (in Fig. 3b, for the rear wheels) describe functions of velocity without drift, and the graphs 4–6 (in Fig. 3a, for the front wheels) and 10–12 (in Fig. 3b, for the rear wheels) – functions of velocity with drift described by parameters $\alpha_A = 5°$, $\alpha_C = 5°$. The graphs 1, 4, 7, 10 refer to outside wheels, the graphs 2, 5, 8, 11 – to the inside ones. The graphs marked by the broken lines represent the values of average wheel velocities of the front or rear axis. Figure 4 shows the graphs of the angular velocities of front and rear wheels of the vehicle moving at the constant velocity of a vehicle front $v_{sA} = 5$ m/s in a varying radius curve: a) for the front wheels, b) for the rear wheels. The graphs are marked the same as in Fig. 3.

The graphs in Figs. 3 and 4 differ from each other due to the non-linear scale factor $R_{sD}/R_{sA}$ described by the formula (8). The graphs in Figs. 3 and 4 show the influence of the location point of a car body, which velocity is treated as a reference velocity of the car (this is an estimate of a car velocity) for calculating the difference in wheel velocities for driving along an arc. For a car with a rear wheel drive, you can choose two basic possibilities to determine the estimate of a car body velocity: based on the average velocity of front wheels or based on the average velocity of rear wheels. Assuming that ECO car velocity is determined on the basis of an average velocity of front wheels, the estimate of car velocity is described by the formula:

$$\widehat{\omega}_{s1} = \frac{\dfrac{v_{sA}}{r_d}\cos\alpha_A}{2\sqrt{1+tg^2(\delta_A-\alpha_A)}}\left(\sqrt{\left(1+\frac{d}{2R_{sD}}\right)^2 + tg^2\left(\delta_A-\alpha_A\right)} + \sqrt{\left(1-\frac{d}{2R_{sD}}\right)^2 + tg^2\left(\delta_A-\alpha_A\right)}\right). \qquad (13)$$

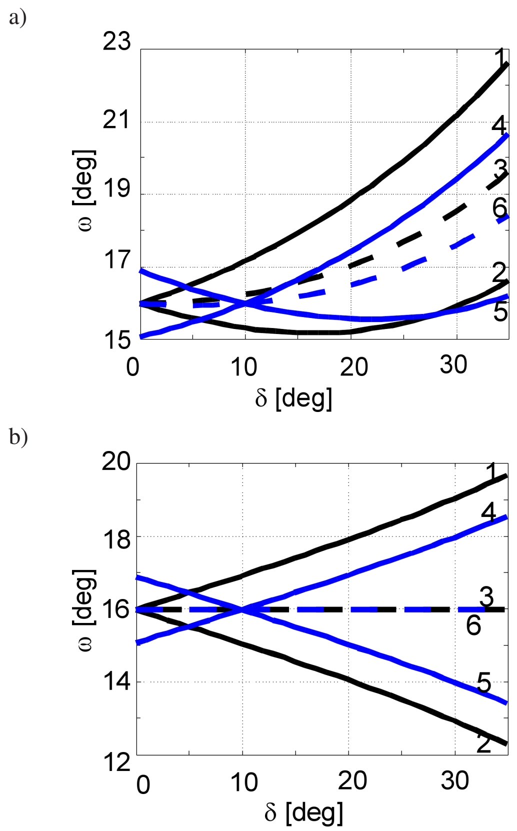


Fig. 3. Function graphs of the angular wheel velocities depending on the turning angle of the vehicle front, moving at a constant linear velocity of point $D$ (no slip): a) front wheels, b) rear wheels

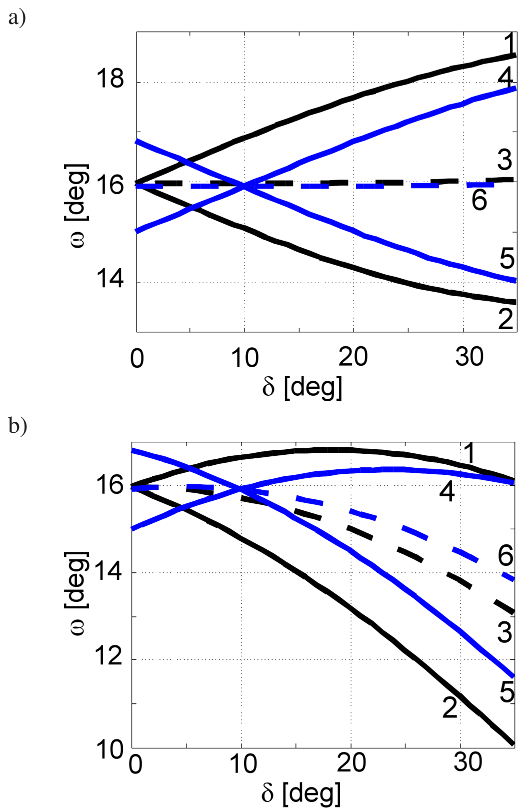


Fig. 4. Function graphs of the angular wheel velocities depending on the turning angle of the vehicle front moving at a constant linear velocity of point $A$ (no slip): a) front wheels, b) rear wheels



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Therefore, a relative error of the angular velocity estimate of the wheel replacement located in front of the vehicle at point $A$ given by the formula:

$$\frac{\omega_{sA} - \widehat{\omega}_s}{\omega_{sA}} = 1 - k_{sA}, \tag{14}$$

$$k_{sA} \approx \cos \alpha_A. \tag{15}$$

Because of the lack of knowledge of a drift angle of rear wheels, the estimates of rear wheel velocities are as following:

$$\widehat{\omega}_1 = \widehat{\omega}_s \frac{\sqrt{\left(1 + \dfrac{d}{2R_{sD}}\right)^2}}{\sqrt{1 + tg^2(\delta_A)}}, \tag{16}$$

$$\widehat{\omega}_2 = \widehat{\omega}_s \frac{\sqrt{\left(1 - \dfrac{d}{2R_{sD}}\right)^2}}{\sqrt{1 + tg^2(\delta_A)}}. \tag{17}$$

On this basis, the relative errors of the estimate of rear wheel velocities are given by the following formulas:

$$\frac{\overset{\wedge}{\omega}_2}{\omega_2} = \frac{\overset{\wedge}{\omega}_s}{\omega_{sA}} \frac{\sqrt{1 + tg^2(\delta_A - \alpha_A)}}{\sqrt{1 + tg^2(\delta_A)}} \cdot \frac{\left(1 - \dfrac{d}{2R_{sD}}\right) \cos \alpha_C}{\sqrt{\left(1 - \dfrac{d}{2R_{sD}}\right)^2 + tg^2(\alpha_C)}}, \tag{18}$$

$$\frac{\overset{\wedge}{\omega}_1}{\omega_1} = \frac{\overset{\wedge}{\omega}_s}{\omega_{sA}} \frac{\sqrt{1 + tg^2(\delta_A - \alpha_A)}}{\sqrt{1 + tg^2(\delta_A)}} \cdot \frac{\left(1 + \dfrac{d}{2R_{sD}}\right) \cos \alpha_C}{\sqrt{\left(1 + \dfrac{d}{2R_{sD}}\right)^2 + tg^2(\alpha_C)}}. \tag{19}$$

Wherein the velocity estimate error of a car front is determined by the formula (14).

Figure 5 shows the function graphs of relative errors of the estimate of rear wheel velocities (no slips) for driving curves of different radius, calculated with the formulas (17) and (18), taking into consideration the relation (14). The charts are marked: 1 and 2 – velocity estimate errors for driving without drift, 3 and 4 - velocity estimate errors for driving with drift ($\alpha_A = 10°$, $\alpha_C = 5°$), 1 and 3 – outside wheels, 2 and 4 – inside ones. Figure 5 shows that velocity estimate errors of rear wheels when drifting, have significant values.

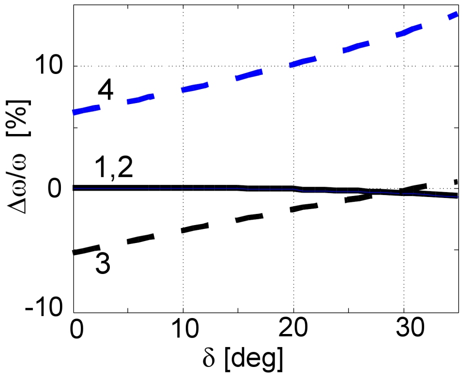


Fig. 5. Function graphs of relative errors of estimate of rear wheel velocities (no slips) set on the basis of average front wheels velocity for driving curves with different radii calculated on the base of the formulas (17) and (18), taking into account the relation (14) (slipping of wheels is omitted)

## 3. Research methodology of electromechanical car differential based on the car motion simulation model

In the virtual pre-prototyping phase, the studies on the object trajectory is carried out on models. To perform the tests we need: a model of a car, a model of the control system, a control model, a set route and evaluation criteria for the ride. The concept study is shown in Fig. 6. It is assumed that a vehicle model should perform a driving along a curve with a specified constant average velocity determined at point A of the centre of car front axles.

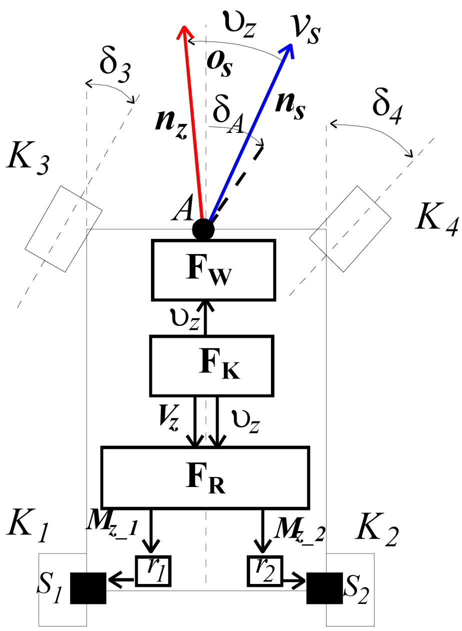


Fig. 6. A structure of a car motion control model: $F_K$ – a control block, $F_R$ – a block of the propulsion control system, $F_W$ – a block of the steer-by-wire system

Marked: $F_K$ – a vehicle control block, $F_R$ – a block of the propulsion control system, $F_W$ – a block of the steer-by-wire system, $S_1$ – a motor in the wheel 1, $S_2$ – a motor in the



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wheel 2, $r_1$ – a motor controller 1, $r_2$ – a motor controller 2, $o_s$ – a car axle, $n_s$ – a driving direction, $n_z$ – a driving direction setpoint, $\upsilon_s$ – a driving velocity, $V_z$ – a driving velocity setpoint, $\delta_A$ – a turning angle of a steering wheel, $\upsilon_z$ – a turning angle of a steering wheel setpoint, $M_{z\_1}$ – a torque of motor 1 setpoint, $M_{z\_2}$ – a torque of motor 2 setpoint. A steering wheel (a steering joystick) generates the input signal for the control system. In the real car, the adjuster of input signal) is operated by the driver. In the virtual tests, an input signal is determined by a control block replacing the functional interaction of human – environment. The relations implemented in the block are expressed in Fig. 6 by the function marked $F_K$ describing psychological reactions to the changes of driving direction and velocity. The vehicle control block has an influence on the models of control systems, which are performing a direct control of the motion process. One can distinguish two basic vehicle control systems: a propulsion system and a steer-by-wire system. It is assumed that their models can be expressed with functions marked accordingly: $F_R$ and $F_W$ (a diagram – Fig. 6).

a)

b)

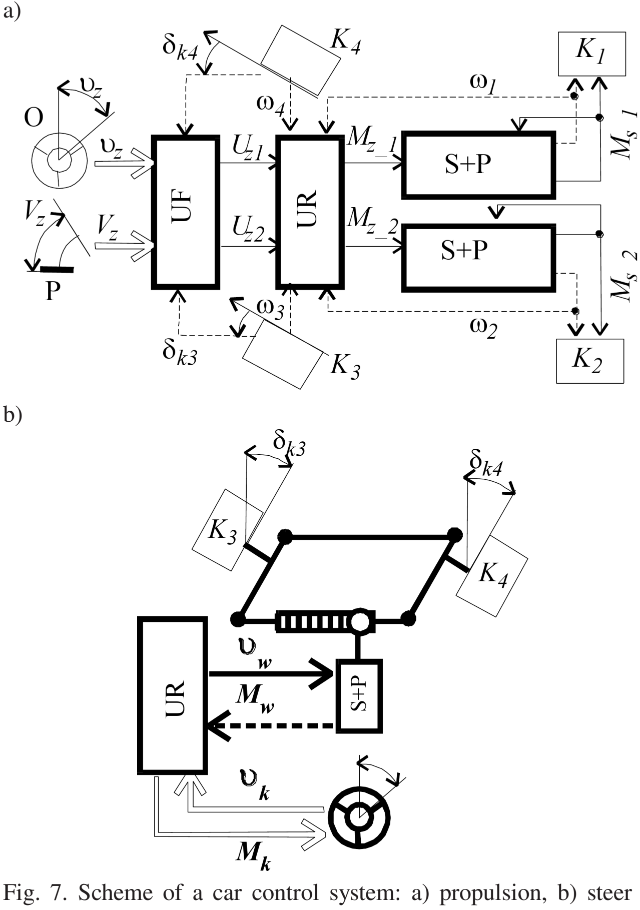


Fig. 7. Scheme of a car control system: a) propulsion, b) steer by wire. Marked: $K_1$, $K_2$ – driven rear wheels, $K_3$, $K_4$ – rotating front wheels, $P$ – a button or a joystick, $O$ – a steering wheel or a joystick, $M_{s\_1}$, $M_{s\_2}$ – torques of motors, $M_{z\_1}$, $M_{z\_1}$ – engine torques setpoints, $M_z$ – a car torque setpoint, $V_z$ – a linear car velocity setpoint, U$_{z1}$, $U_{z2}$ – formed input values of a drive adjustment system, $\omega 1$, $\omega 2$ – angular velocities of motors, $\omega 3$, $\omega 4$ – angular velocities of front wheels, $\upsilon_k$, $\upsilon_z$ – a turning angle of a steering wheel, actual and setpoint

In the simulation studies carried out on models, a trajectory must be evaluated on the basis of the unequivocal criteria. A distance of the car body (solid model) from the setpoint trajectory is most often taken as the evaluation criteria for the accuracy of the ride. An objective function expressing this criterion is indicated with $\Delta D$. Such an approach is often used in the studies of technical-stochastic stability (the concept of car stability means a definition specified by ISO standard 8855:1991 [3, 4].) A general schema of a car motion control system is shown in Fig. 7: a) a propulsion system implementing the function $F_R$, b) steer by wire – $F_W$ function. Using an adjuster, a driver sets two values: $\upsilon_z$ – a turning angle of a steering wheel setpoint, and $M_z$ – a car torque setpoint or $V_z$ – a linear car velocity setpoint (one of these values, a torque or a velocity).

If a velocity is the value which is being set, then a system implementing this control is called a "velocity system", if a torque is the value – it is called a "torque system". When driving corners, the drive wheels move at different velocities (Fig. 2). A function of steering the front wheels is performed with steer by wire system [5]. The task of the propulsion control system is to actuate the motors properly, meeting all the required limits of their working conditions [6]. The control signals of motors PB BDCM (Permanent Magnet Brushless Direct Current Motor) are usually given in the form of torque curves setpoints: $M_{z\_1}$, $M_{z\_2}$. These torques are determined in the automatic regulation system, with different internal structures.Independently of the system structure, to control its dynamics it is necessary to have two formed input values: $U_{z1}$ and $U_{z2}$. Usually, there are two of the following values: $\omega_{z1}$, $\omega_{z2}$ – an angular engine velocity setpoint, $\Delta\omega_{12}$ – a velocity difference setpoint of the rear wheels, $\omega_{zC}$, $\omega_{zA}$ – an angular velocity setpoint of the replacement wheel at point $C$ of the rear axle centre or at point A of the front axle centre, $\Delta\omega_{1A}$, $\Delta\omega_{2A}$ – a setpoint difference between the angular velocity of the replacement wheel and the first or the second wheel, $M_{z\_1}$, $M_{z\_2}$ – engine torques setpoints, $\Delta M_{12}$ – difference of torques setpoint. During determining an engine torque setpoints, an automatic control system can also use the measuring velocity signals of the vehicle wheels: $\omega_1$, $\omega_2$ – angular velocities of motors or $\omega_3$, $\omega_4$ – angular velocities of front wheels. Forming the input values of the automatic control system is performed in a special block of the forming system. The forming algorithm performs the transformation function of the values being set from the adjuster (turning angle and velocity $\upsilon_z$ and $V_z$) or (turning angle and torque $\upsilon_z$ and $M_z$) using formulas (9, 10, 11, 12).

In the described case, the direct control of the model refers to setting the three values: two torques of motors $M_{z1}$ and $M_{z2}$ and a turning angle of the steering wheel $\upsilon_z$. These values are related to the setpoint route with the use of appropriate function transformations $F_K$, $F_R$, $F_W$ which analyse a location and a velocity in relation to the route and describing operations of the control system. These assumptions clearly result in the methodology of studies, which purpose is to select a structure of the car propulsion control system. The studies are carried out through a method of rides simulations



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on the unequivocally set driving route, with a clearly defined function of the control block $F_K$ and with a model function controlling a turning angle of the front wheels $F_W$ under conditions of controlling with various automatic regulation systems – described with various functions $F_R$. Each simulation ride of a setpoint route section is evaluated using an objective function (described later). In this way, the values of functions already implemented $\Delta D$ have been assigned to the individual control function $F_R$.

## 4. The basic structures of the control systems

A review of the selected topologies of control systems for twin-engine drives has been made for example in the work by [6]. The selected applications of electronic differentials are presented in the works by [7–9].

In the systems with a "velocity" control, we distinguish the following basic structures of automatic control systems (in the brackets, there are adopted marks of functions of the control system, containing the specified controller type): a) with setting two velocities ($F_{VA}$), b) with setting a velocity difference ($F_{VB}$), c) with a leading engine ($F_{VC}$), d) with setting an average velocity ($F_{VD}$). The schematic diagrams of the systems are shown in Fig. 8. In the systems with a "torque" control, we distinguish the following basic structures of automatic control systems: a) a structure of a direct system ($F_{TA}$), b) a structure of a crossing system with a slave controller of velocity difference ($F_{TB}$), c) with a leading engine ($F_{TC}$). The schematic diagrams of the systems are presented in Fig. 9. With the adopted symbols, a set of available control functions is given by (20):

$$\mathbf{F_R} = \{F_{VA}, F_{VB}F_{VC}, F_{VD}, F_{TA}, F_{TB}, F_{TC}\}. \qquad (20)$$

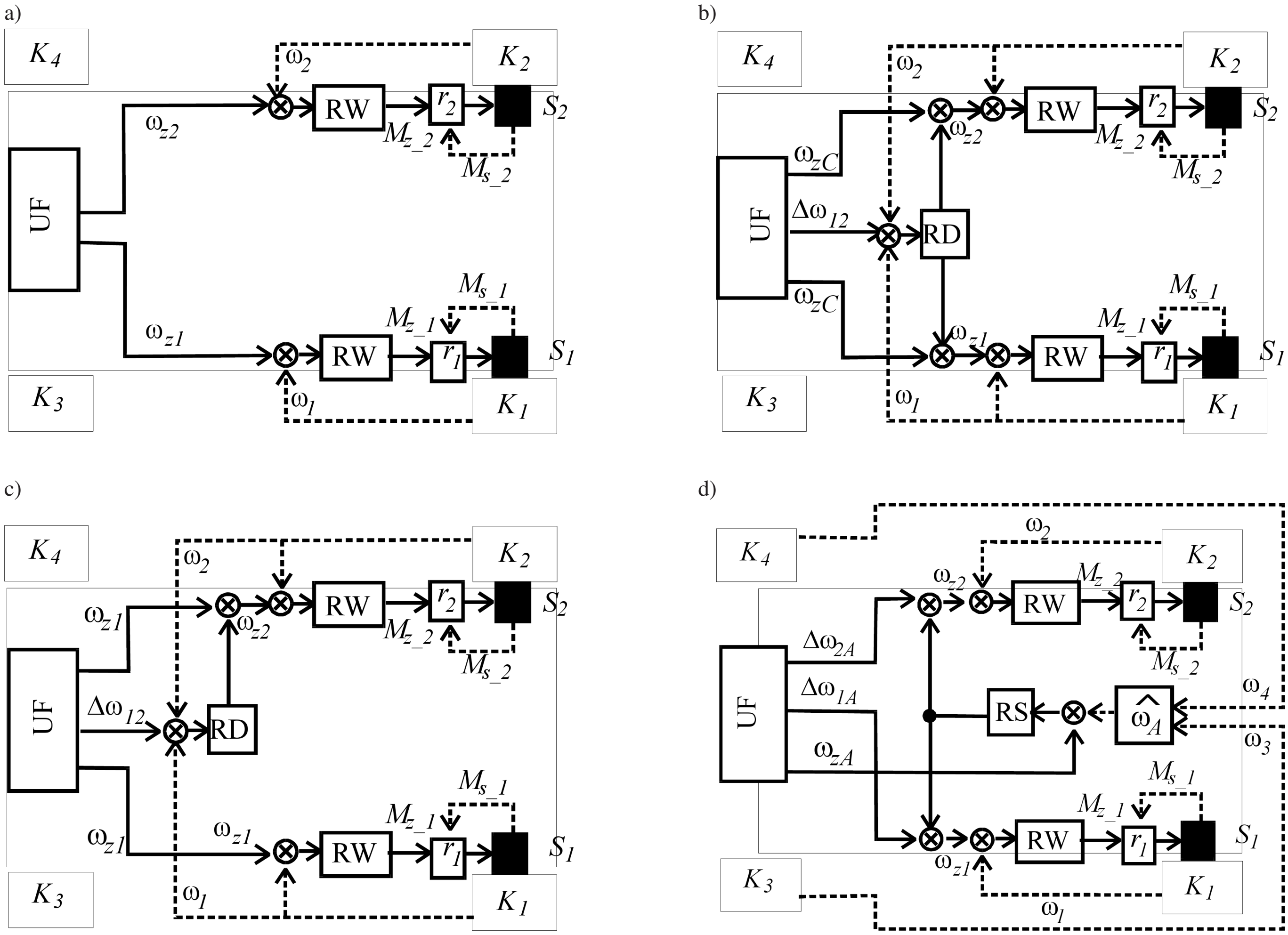


Fig. 8. Block diagrams of control systems with velocity setting considered for applications in the designed ECO car: a) with setting of two velocities, b) with setting of velocity difference, c) with the leading engine $S_1$, d) with a controller of an average velocity at point $C$ of the centre of the car rear axle. Marked: RW – a velocity controller, RD – a controller of velocity difference, RS – a controller of an average velocity, $S_1$, $S_2$ – electric traction motors, $r_1$, $r_2$ – controllers of traction motors, $\Delta M_{12}$ – a setpoint torque difference, $\omega_{zC}$, $\omega_{zA}$ – a setpoint angular velocity of the replacement wheel at point $C$ of the rear axle centre or at point A of the front axle centre, $\Delta\omega_{1A}$, $\Delta\omega_{2A}$ – a setpoint difference of angular velocities between the replacement wheel and the first wheel, - and the second wheel, $\omega_{z1}$, $\omega_{z2}$ – angular engine velocities setpoints, $\Delta\omega_{12}$ – a velocity difference setpoint



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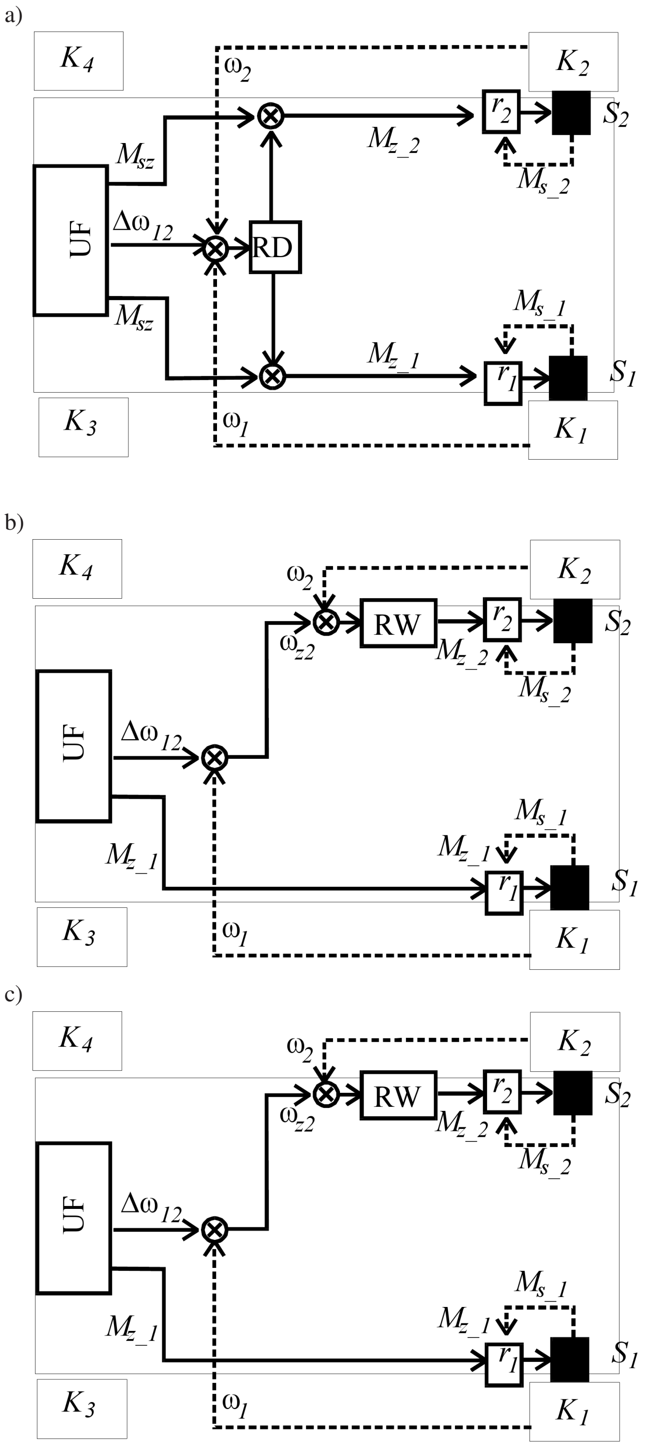


Fig. 9. Block diagrams of control systems with setting a torque: a) a structure of a direct system, b) a structure of a crossing system with a slave controller of velocity difference, c) with a leading engine $S_1$. Marked: $M_z$ – a car torque setpoint

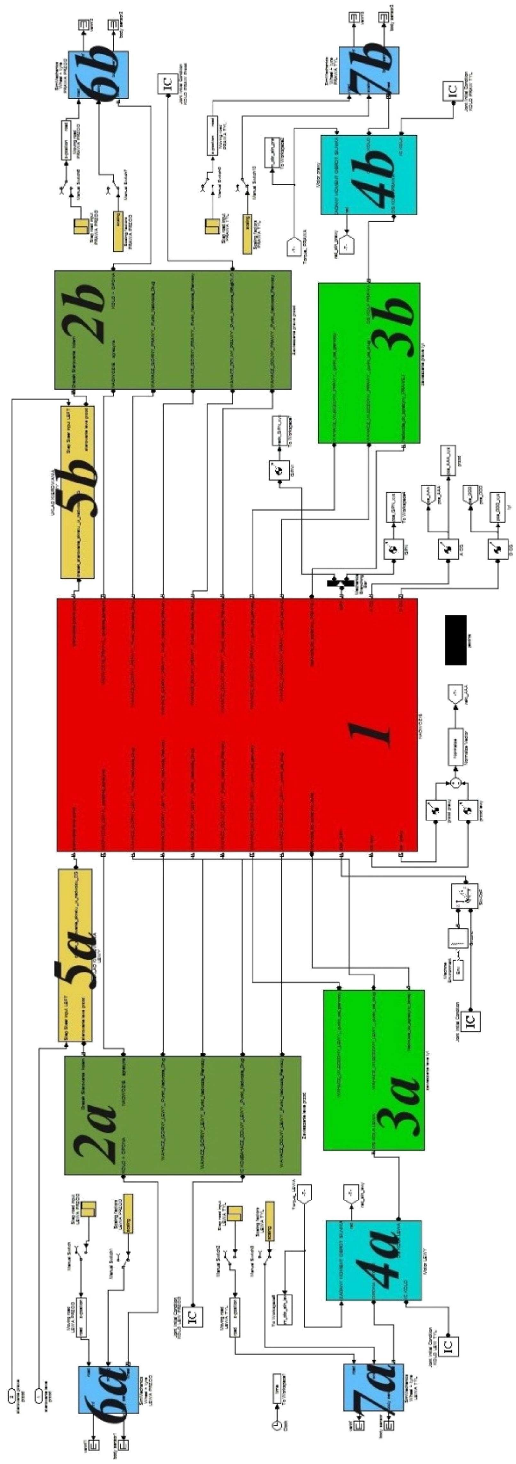


Fig. 10. A structure of the car simulation model

## 5. Car model

Modelling and simulation used MBS package (Simmechanics). A structure of adopted simulation model of ECO car is shown in Fig. 10. It is assumed that the car simulation model includes the following subsystems (representing its partial models): 1 – a bodywork, 2a and 2b – a double arm suspension front, left and right, 3a and 3b – trailing-link rear suspension, left and right, 4a and 4b – BLDC electric propulsion motors, placed in the rear wheels, left and right, 5a and 5b – steering systems for front wheels, linked with a shared power steering rack, 6a and 6b – front wheels with tyres, 7a and 7b – rear wheels with tyres. To introduce the dynamic properties of tyres, it is used a package TNO Delft



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Tyre [10] with the tyre parameters defined by the library file 'TNO_car205_60R15.tir'.

## 6. Car model parameters identification

Car parameters (given in Table 2) have been identified on the basis of the design made in Catia [11] and transferred into SimMechanics program, in which the main parts of the computer simulations have been made. Figure 11 shows a view of a dimensioned rigid body moddeling a car bodywork. Table 2 summarizes basic parameters of this body, considering that there are 4 people travelling in this vehicle, including a driver in a wheelchair. Figure 11 additionally shows the elements of vehicle suspension systems. A low location of the bodywork gravity center has been obtained by placing accumulator's batteries and electric control systems (with a total mass of about 400 kg) in the floor (maximum floor plate height 0.12 m). A partial model of a double arm suspension is shown in Fig. 12. It contains: rigid bodies: upper arm, lower arm, wheel carrier and a steering rod, couplings – 3 revolving (rev), 4 spherical (sph), 2 universal (uni) and a spring damping element (tha). The dimensions of suspension elements are shown in Table 3. We used spring damping elements (tha) with the following parameters: spring constant $c_{THA} = 507$ N/mm, damper constant $k_{THA} = 10$ N/(mm/s), spring natural length $l_{THA} = 198$ mm. Figure 13 presents a trailing-link rear suspension model. The dimensions of suspension elements are shown in Table 4. It has been applied a spring damping element (tha) with the following parameters: spring constant $c_{THA} = 880$ N/mm, damper constant $k_{THA} = 10$ N/(mm/s), spring natural length $l_{THA} = 198$ mm.

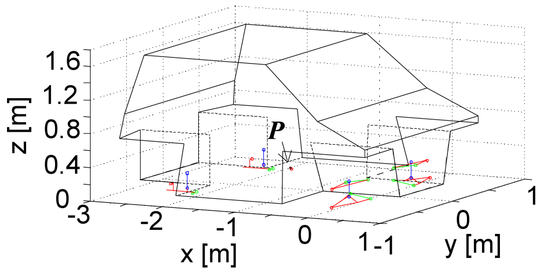


Fig. 11. A rigid body modelling ECO car bodywork. With applied suspension system elements

Table 2
Basic parameters of a rigid bodywork together with a driver model + biomechanical models of 3 passengers

| Symbol | Definition | Value |
|---|---|---|
| G(x,y,z) | Center of gravity | Gx −1701 mm<br>Gy 6 mm<br>Gz 522 mm<br>(in coordinate system given in Fig. 11) |
| MG | Mass | 1402 kg |
| JG | Interia Matrix | JoxG 603 kg/m$^2$<br>JoyG 1281 kg/m$^2$<br>JozG 1187 kg/m$^2$<br>JxyG 3.0 kg/m$^2$<br>JxzG 82.4 kg/m$^2$<br>JyzG 3.6 kg/m$^2$ |

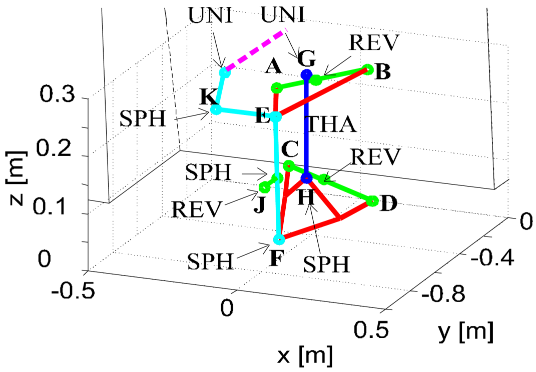


Fig. 12. A view of the front suspension model

Table 3
Coordinates of the points of the front right suspension model

| Point | $x$ [mm] | $y$ [mm] | $z$ [mm] |
|---|---|---|---|
| A | 162.6 | −330 | 288 |
| B | −142.8 | −330 | 235.4 |
| C | −112.5 | −300 | 96.4 |
| D | 167.8 | −300 | 54 |
| E | −10 | −660 | 247 |
| F | 10 | −682 | 38 |
| G | 0 | −430 | 283.8 |
| H | 0 | −430 | 103.8 |
| J | 0 | −671 | 142.5 |
| K | −210 | −660 | 247 |
| L | 0 | −771 | 142.5 |

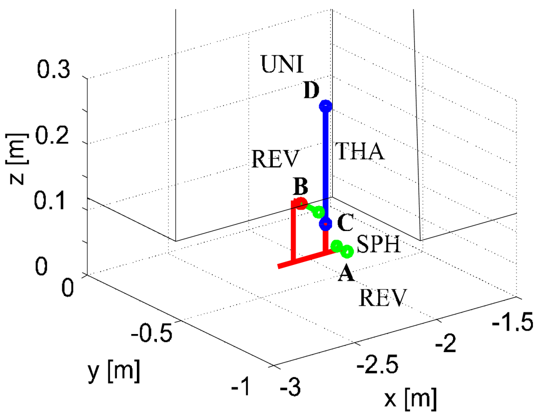


Fig. 13. A view of the rear suspension model

Table 4
Coordinates of points of the rear suspension model

| Point | $x$ [mm] | $y$ [mm] | $z$ [mm] |
|---|---|---|---|
| A | −2035 | −530 | 45 |
| B | −2330 | −663 | 140 |
| C | −2135 | −530 | 90 |
| D | −2135 | −530 | 270 |

## 7. The model of a vehicle control block

To describe the behavior of the driver during cornering, it is used a so-called anticipatory control block [12–14]. A method to determine a drive direction, described in the literature, has been modified so that the car can be driven on any setpoint



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route. For this purpose, a direction vector is designated for a leading straight line (marked with $n_z$ in Fig. 6) connecting the center point of the front wheel axis (point $A$ in Fig. 2) with a point located on the route at the certain fixed distance ahead of the vehicle $R_D$. The angle between this straight line and the vector of current driving direction (indicated in Fig. 6 as the symbol $n_s$) is a setpoint turning angle of wheels marked $\upsilon z$. The Angle $\delta_A$ is the replacement angle of setting the front wheels relative to the longitudinal car axis os. The more extended models of the vector control of a car are shown for example in work by [15].

As a measure of the objective functions $\Delta D$ it is assumed an average distance of car point A from the given road (setpoint). This distance can be determined with the yaw angle of the driving direction to the set direction of road, as follows:

$$\Delta D = R_D \sin\left(\frac{1}{T_F}\int\limits_0^{T_F} |\sigma_z|\, dt\right), \tag{21}$$

where: $R_D$ – a leading radius length, $\Delta D$ –an average distance of point $A$ of the front axle to the setpoint route, $\sigma_z$ – a yaw angle of the driving direction from the direction of the road. The principle of steering a car model shown in Fig. 14: a) determining the driving direction in the traffic straight ahead, b) implementation of the driving bend. In Fig. 14a for example, a front of the vehicle is located at points $A$, $B$, $C$, $D$, $E$. These positions correspond to the subsequent points (appearing) on the road, in front of the vehicle

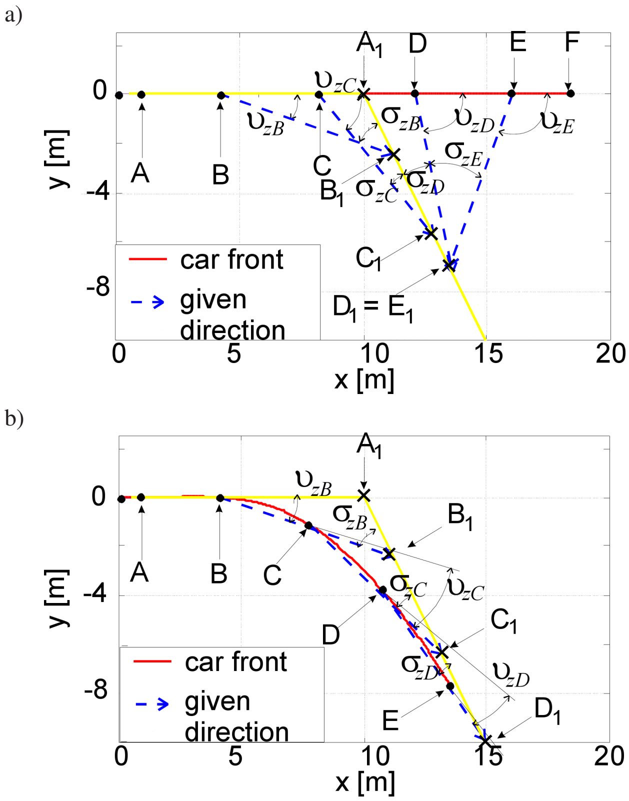


Fig. 14. A principle of anticipatory determination of driving direction: a) driving straight ahead, b) driving around a corner

within a distance $R_D$: $A_1$, $B_1$, $C_1$, $D_1$ and $E_1$ and the next setpoint angles of a driving direction: $\upsilon_{zA}$, $\upsilon_{zB}$, $\upsilon_{zC}$, $\upsilon_{zD}$, $\upsilon_{zE}$, and yaw angles of a driving direction from a road direction: $\sigma_{zA}$, $\sigma_{zB}$, $\sigma_{zC}$, $\sigma_{zD}$, $\sigma_{zE}$. If the "virtual driver" does not take a turn, then in point $F$, a position of point $F_1$ on the setpoint trajectory cannot be determined any more (for a particular leading radius setpoint ($R_D$). Then the algorithm is interrupted and an error report is sent. Driving corner simulation results, in case if the "driver" is turning the steering wheel and the vehicle is turning, are shown in Fig. 14b. The objective function is calculated with the formula (20) and its amount is $\Delta D = 1.59$ m for $R_D = 7.5$ m.

## 8. The simulation model

Model consists of the following elements: 1 – a control block, 2 – a car model in SimMechanics, 3a – block of a set speed, 3b - block of a set trajectory, 4 – a model of the propulsion control system, 5 – a block of analysis of driving direction, 6 – block of setting the turn of the steering wheel, 7 – Block of the initial and final conditions, 8 – Block of the reports on the reasons for detention before the planned end of the ride, 9 – presentation block of the objective and reporting functions. Figure 15 shows the structure of the control block, executing a process to steer the driving direction. The input value is a setpoint angle of the motion direction, understood as an angle between a bodywork directional vector and a set direction vector (e.g. $\upsilon_{zC}$ angle at point $C$ in Fig. 14). PID controller describes the driver reaction while turning the steering wheel. The controller reflects the psychological factors of the driver, but it is not a part of the automatic control system. A "limit" segment (Rate Limiter) describes the maximum speed of turning the wheels, which a driver can perform, and a "Saturation" segment – maximum values of the turning angle. At the output of the system there are obtained turning angles of front wheels of the values which follow the setpoint values, considering typical driver's reactions and technical limitations of the steering system operation (the output "a turning angle of the steering wheel") or the turning angle of the wheels without taking into account the psychological factor (the output "limited angle of the direction of motion").

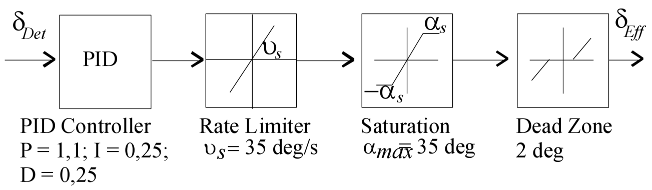


Fig. 15. The block of a control model. PID controller settings: P = 1.1, I = 0.25, D = 0.25, $\delta_{Det}$ – determinet steering angle, $\delta_{Eff}$ – effective steering angle

## 9. Simulation results of driving in corners with a speed control

It has been made a comparative study simulation for a performance of a car equipped with an electromechanical differential with a velocity setting systems (Fig. 8) for moving at a



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speed of 50 km/h on the road including two corners of 45°, driving in two different circumstances: the first – a dry road, the second – a slippery road. The resulting trajectories of the model motion are shown in Fig. 16: a) on dry roads (maximum values of longitudinal and lateral friction coefficients), b) on the slippery road (50% of the maximum values of longitudinal and transverse friction coefficients). The charts are marked: 0 – a set route, 1 – $F_{VA}$ (with system of two-velocity setting in Fig. 7a), 2 – $F_{VB}$ (with system of velocity difference setting in Fig. 7b), 4 – $F_{VD}$ (with an average velocity controller at point $C$ of the centre point of the car's rear axle, the system in Fig. 7d). Table 5 shows the values of controllers' settings. The function values of evaluating trajectory deviation from the set route are shown in Table 6. Figure 17 shows the transient courses of angles of vehicle's position for driving on dry surface road (the courses are determined in the control block marked with symbol 1 in Fig. 15, and which internal structure is shown in Fig. 15). The charts are marked: "1" – a setpoint angle of the driving direction (angle $\upsilon_{zC}$ in Fig. 14), "2" – a turning angle of the steering wheel (the angle $\sigma_z$ in Fig. 14), "3" – angle of the bodywork relative to the road. Figure 18 shows the transient courses of angular velocity of rear drive wheels and front rolling wheels for the ride on a dry road. The charts are marked: "1" – left engine, "2" – right engine, "3" – left tyre front, "4" – right tyre, front. Figure 19 shows the transient courses of torques of motors for the ride on a dry road: a) of a system with two-velocity setting, b) of a system with setting a velocity difference, c) of an average velocity controller in point C of the centre of the car rear axle. The charts are marked: "1" – an engine on the left, "2" – an engine on the right. Figure 20 shows the transient courses of the values characterizing the ride on a slippery road with the use of an average velocity control system: a) transient courses of the vehicle's position angle (the graphs determined as in Fig. 17), b) transient courses of angular velocities of the rear drive wheels and the front rolling wheels (the charts determined as in Fig. 18), c) transient courses of torques of engines for the ride (the graphs marked as shown in Fig. 19).

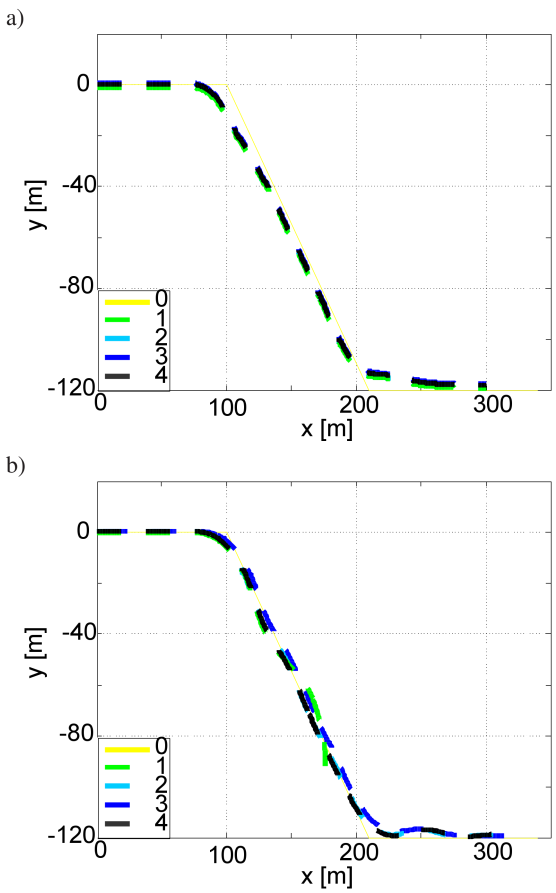


Fig. 16. Simulation trajectories of model motion with a velocity control (compare Fig. 8): a) on a dry road (maximum values of longitudinal and lateral friction coefficients), b) on a slippery road (50% of the maximum values of longitudinal and lateral friction coefficients)

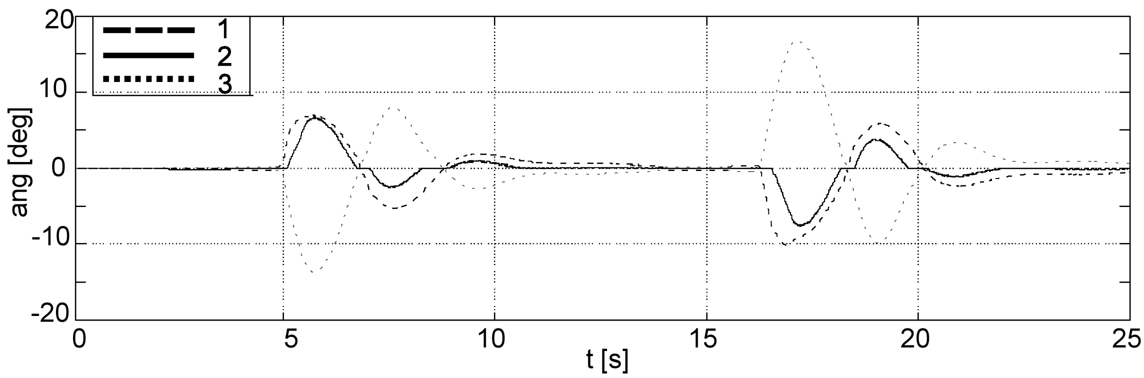


Fig. 17. The transient courses of vehicle's position angles for the ride on a dry road, the waveforms are determined in the control block in order to travel a set section of the road. The charts are marked: "1" – a set angle of a driving direction (angle $\upsilon_{zC}$ in Fig. 14), "2" – a turn angle of a steering wheel (angle $\sigma_z$ of Fig. 14), "3" – an angle of the bodywork relative to the road



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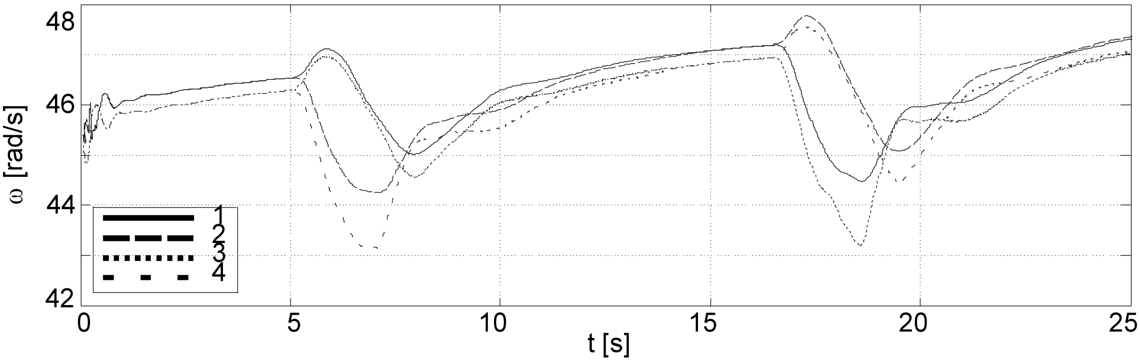


Fig. 18. Transient courses of angular velocities of the vehicle's drive wheels and rolling wheels for the ride on a dry road. The charts are marked: "1" – left motor, "2" – right motor, "3" – left tyre front, "4" – right tyre front

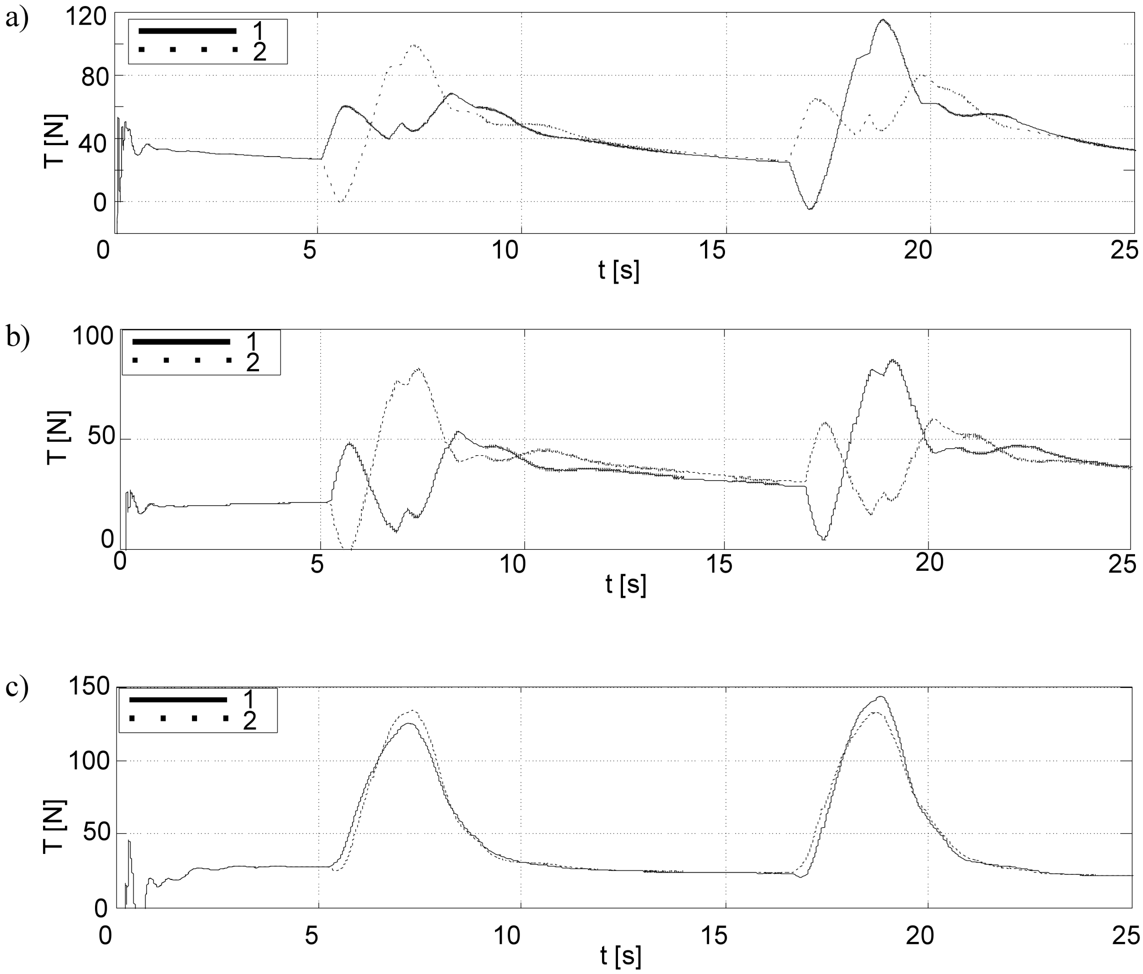


Fig. 19. Transient courses of torques of motors for the ride on a dry road: a) of the system with setting two velocities, b) of the system with setting a velocity difference, c) of the system with an average velocity controller in point $C$ of the centre of the car's rear axle. The charts are marked: "1' – an motor on the left, "2" – an motor on the right



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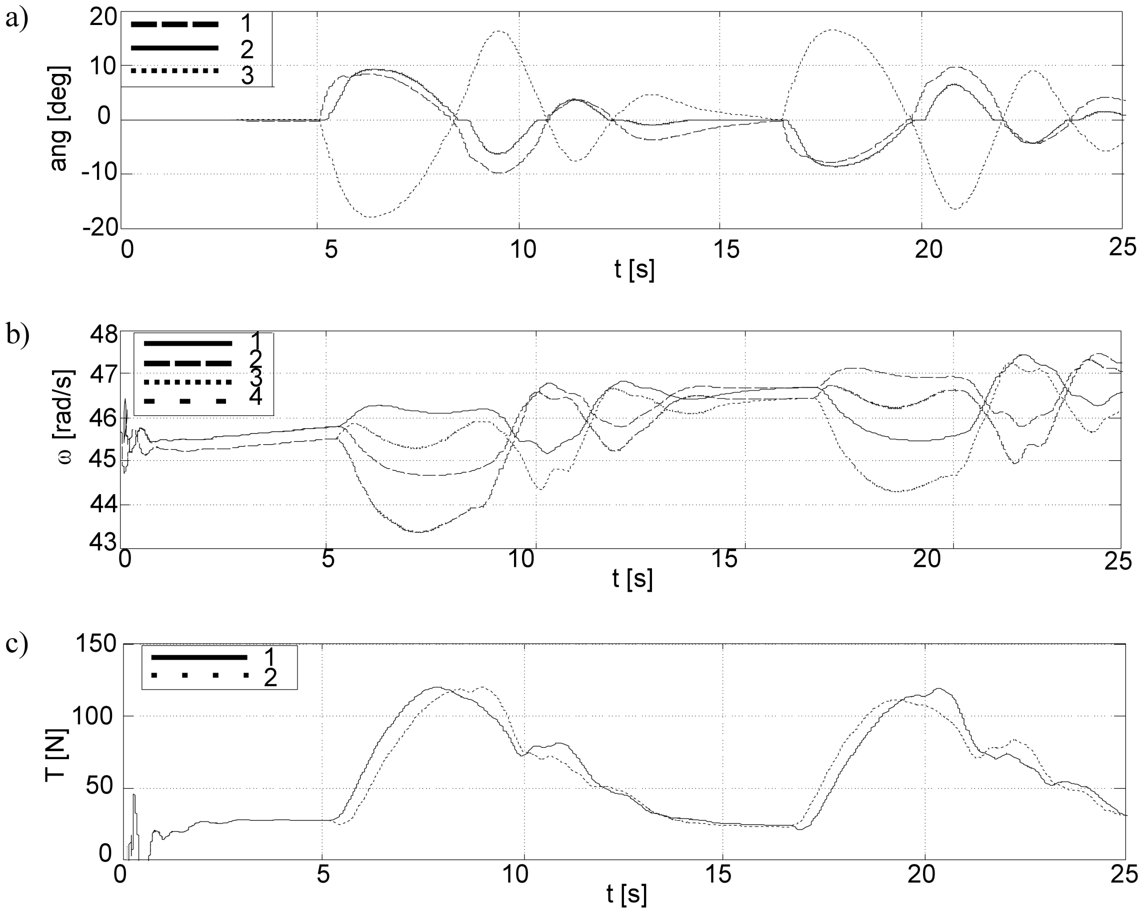


Fig. 20. Shows transient courses of the values characterizing the ride on a slippery road using the average velocity control system: a) transient courses of angles of vehicle's position (the graphs determined as in Fig. 17), b) transient courses of angular velocities of rear drive wheels and front rolling wheels (the graphs determined as in Fig. 18), c) transient courses of torques of motors for the ride (graphs in Fig. 19)

Table 5
The values of controllers' settings

| Type of system | The values of settings | |
|---|---|---|
| 1 – $F_{VA}$ (system with two-velocity setting in Fig. 8a) | RW_Prop_1 = 25<br>RW_Int_1 = 1.25<br>RW_Der_1 = 0 | |
| 2 – $F_{VB}$ (with system of velocity difference setting in Fig. 8b) | RW_Prop_2 = 12.5<br>RW_Der_2 = 0<br>RD_Int_2 = 0.1 | RW_Int_2 = 0.25<br>RD_Prop_2 = 2<br>RD_Der_2 = 0 |
| 4 – $F_{VD}$ (with an average velocity controller at point A of the centre of car's front axle, the system in Fig. 8d) | RW_Prop_4 = 5<br>RW_Der_4 = 0<br>RS_Int_4 = 1.25 | RW_Int_4 = 0.25<br>RS_Prop_4 = 25<br>RS_Der_4 = 0 |

Table 6

| Evaluation functions of simulation results as shown in Fig. 17 calculated using the formula (21) | Dry road [m] (values of objective function given by relations (21)) | Slippery road [m] |
|---|---|---|
| 1 – $F_{VA}$ | 3.2899 | not reached |
| 2 – $F_{VB}$ | 3.2953 | 2.9738 |
| 4 – $F_{VD}$ | 3.2963 | 2.9329 |



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## 10. Concluding remarks

Stable car driving, as one of the most important criteria for evaluating a design quality is extremely important for cars driven by a disabled driver sitting in a wheelchair. In fact, this type of driver is more exposed to all events that hinder driving with the use of joystick or a steering wheel. The design of vehicle is specific to allow driving by a disabled driver. The motion properties depend, among others, on the software quality of an electromechanical differential. The article presents the issue of selecting the structure of the relevant control systems. The comparative simulation studies have been carried out on performance of a car model with an electromechanical differential with velocity setting systems, moving at the speed of 50 km/hr on the road including two corners, each of 45°, the corners driven around in two different circumstances: the first – a dry road, the second – a slippery road. As a criteria function of evaluating the motion stability of the vehicle model (here in stochastic – technical sense), it is assumed an average distance of the car front point from the set path. In this way, the simulation results showed the best algorithms of an electromechanical differential for controlling velocities of rear drive wheels: with setting of velocity difference - the system in Fig. 8b, the average velocity controller in point A of the centre of the car front axle – a system in Fig. 8d.The applied model is quite complex, because it includes both mechanical and electric vehicle's structure and finally a human element - a simple model of human impact on the steer by wire system. However, the model is further extended by additional elements. The results of the simulation tests are to be reviewed after construction of a riding pre-prototype is completed. This paper has been developed within the ECO - Mobility project [16–19].

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