

WATER-EXIT PROCESS MODELING AND ADDED-MASS CALCULATION OF THE SUBMARINE-LAUNCHED MISSILE

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ABSTRACT

In the process that the submarine-launched missile exits the water, there is the complex fluid solid coupling phenomenon. Therefore, it is difficult to establish the accurate water-exit dynamic model. In the paper, according to the characteristics of the water-exit motion, based on the traditional method of added mass, considering the added mass changing rate, the water-exit dynamic model is established. And with help of the CFX fluid simulation software, a new calculation method of the added mass that is suit for submarine-launched missile is proposed, which can effectively solve the problem of fluid solid coupling in modeling process. Then by the new calculation method, the change law of the added mass in water-exit process of the missile is obtained. In simulated analysis, for the water-exit process of the missile, by comparing the results of the numerical simulation and the calculation of theoretical model, the effectiveness of the new added mass calculation method and the accuracy of the water-exit dynamic model that considers the added mass changing rate are verified.

Keywords: Submarine-launched missile; Fluid-structure interaction; Water-exit dynamic model; Time-varying added-mass; Numerical simulation

INTRODUCTION

With the development of modern military equipment, depending on the features such as good concealment and strong survivability, at present the submarine-launched missile has become an indispensable part of the national strategy and tactics forces [1]. Different from other types of missiles, the whole course of the submarine-launched missile is divided into three parts: underwater stages, water-exit stages and aerial stages. The water-exit stage is the special stage of the missile, in which the missile depends on the inertia or makes use of the thrust of the underwater engine to rise until its body is completely out of water. In the stage, the attitude and trajectory of the missile is affected by the wave,

which makes the missile deviate from the designed motion state and results in the unstable flight or even unsuccessful launch after the missile exits from water. Therefore, in order to provide the necessary initial conditions for the air trajectory of the missile, it is required to accurately predict and calculate the motion law of the missile in water-exit process [2]. As the basis of the research on the motion law of missile in water-exit process, the accuracy of the dynamic model determines the effect of the research.

In the whole course of the missile, the water-exit stage is the stage that the external conditions change the most severely and the load conditions are the most complex [3]. In the stage, on the one hand, the missile causes a series of complex changes in the flow field, such as the development and collapse

of vacuole; on the other hand, the flow field evolution process adversely acts on the missile by pressure or even the impact load generated by cavity collapse, which constantly changes the strained state of the missile, thus forming a complex fluid solid coupling process [4]. In order to establish an accurate dynamic model, the fluid solid coupling problem must be studied in depth, and the change law of the external flow field in the water-exit process must be accurately mastered.

At present, there are mainly two methods to study the fluid solid coupling problem [5]. One is the strong coupling method, which is mainly to establish the motion equations for the fluid and the structure and then to use the numerical method to solve the equations. The other method is the weak coupling method, which is also the added mass method. In the method, the fluid is simplified. And the load generated by the fluid at a certain point on the structural interface is equivalent to an inertial force produced by the movement of the structure and the fluid with a certain mass that is assumed to attach on the structure, which is so-called the added mass force. The method is simple in form and less in computation. At present, it has become the most mature method to solve this kind of problems and is widely used in engineering [6-8]. Therefore, as one of the important parameters of the hydrodynamic and dynamic load characteristics in the weak coupling method, the accuracy of the calculated results of the added mass will directly influence the effectiveness of the missile water-exit dynamic model.

For the calculation of the added mass, there are three main methods. 1. Theoretical calculation method. For the object with simple shape, the corresponding added mass theoretical solutions can be obtained by using the slender-body theory, slice theory and the engineering calculation method that uses the correction coefficient to consider the three-dimensional effect. 2. The method of pool experiment [9]. Weiqi Chen [10] designed the oblique water-exit experiment for the experimental model and carried out the parameter identification for the obtained experimental data. Then the added mass and drag coefficient of the model in axial direction are obtained. Gang Li [11] used the method of model constraint experiment to measure the added mass of the sphere, the ellipsoid and a type of submersible. The obtained conclusions were instructive for the hydrodynamic force design of the submersible. 3. Numerical calculation method of ideal fluid added mass based on potential flow theory. Huiping Fu [12] used the software Fluent as the platform and then applied the technology of dynamic mesh to carry out the research of the added mass calculation method on the basis of the RANS equation. Then the added mass of the underwater objects with two different shapes is obtained. On the basis of the CFD method, Uhlman [13] studied the added mass law of the super-cavity vehicle under the influence of the waves and compared the conclusion with the experimental result. For the swaying objects, Xuan Huang [14] used CFD technique to simulate the flow field of cavity and studied the added mass of the natural cavity disc and the axisymmetric slender body. In the three methods above, the theoretical calculation method can only be used for the objects with

simple shape, which has a strong limitation. The experiment method is complex and time-consuming, especially when the size of object is large or the linetype of object is complex, which may bring the great difficulty to the experiment. Therefore, for the submarine-launched missile which moves fast and has more complex linear shape, the above method cannot be applied. In contrast, the numerical simulation method can be used to calculate the added mass of objects with arbitrary shape and size. And the speed and accuracy are only limited to the hardware configuration and the turbulence model. Thus, the method can be used to calculate the added mass of the submarine-launched missile. However, at present, the numerical calculation method is mainly used to study the added mass of the underwater moving objects. The study of the added mass of the water-exit process is still in the initial stage, and the relevant references are also less. The mainly reason for this situation is that, the added mass is constant during underwater motion, but in water-exit stage, the added mass has a time-varying characteristics and is not constant. So there is a certain degree of difficulty in the research of the problem.

For the problems that contain the modeling and the time-varying added mass calculation of the submarine-launched missile in water-exit process, combining with features of the water-exit motion, introducing the added mass changing rate, the water-exit dynamic model is established in the paper. Then, from the model, by using the method of restraining numerical simulated conditions, a new fast effective calculation strategy for the time-varying added mass is proposed, which is later verified by the example of the sphere. On this basis, the added mass of a kind of submarine-launched missile in water-exit process is calculated under the different conditions. Then, by using the calculation results, the water-exit motion of the missile is simulated based on the water-exit dynamic model, and meanwhile the influence of the added mass changing rate on the water-exit process is investigated. The related conclusions have the reference meaning for the water-exit study and motion prediction of the submarine-launched missile.

WATER-EXIT DYNAMIC MODEL

FORCE ANALYSIS OF WATER-EXIT PROCESS

The force analysis of the submarine-launched missile in water-exit process is shown as Figure 1. The reference coordinate system $O-xyz$ is the body coordinate system of the missile. The barycentre is set as the origin of the coordinate system. And the x axis coincides with the longitudinal axis of the missile and point to the head of the missile; the y axis is located in the longitudinal plane of the missile, and perpendicular to the x -axis and pointing upward. The z axis, x axis and y axis constitute the right hand system. The water-exit angle of the missile is θ . The distance between the head of the missile and the surface of water is l_1 . In water-exit

process, the missile is mainly affected by the interaction of gravity G , buoyancy B , fluid force F and derivative moment.

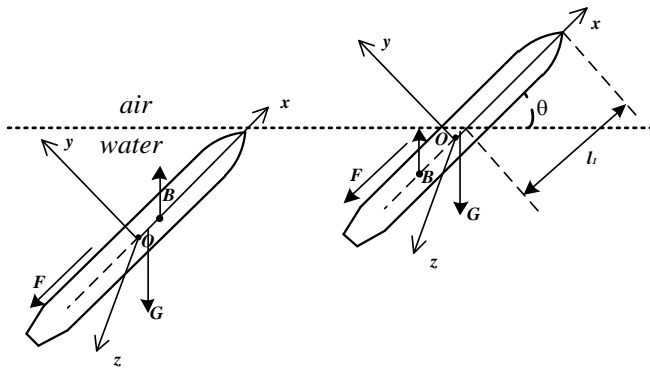


Fig. 1. Force analysis of water-exit process

FLUID FORCE F

When the submarine-launched missile exits the water, the fluid force is continuously distributed on its surface in a regular rule. The distribution rule is determined by the shape of the missile, the characteristics of fluid properties, and the motion state of missile. At present, it is difficult to directly obtain the fluid force of the moving object in fluid. Considering that the actual fluid are viscous fluid, in order to simply the problem, under the reasonable assumptions [15], the fluid force is decomposed into fluid force with viscousness and ideal fluid force without viscousness to be calculated respectively in this paper.

Effect of viscous fluid

Because the shape of the submarine-launched missile is symmetrical, there is almost no lift in the case of zero attack angle. Therefore, it is considered that the effect of the viscous fluid can be reflected in the resistance coefficient of the underwater moving missile. The effect mainly includes the drag generated by the motion of the missile and the force caused by the attack angle. Meanwhile, the velocity of the missile is at low speed and the speed changing range is small, so the influence of the Reynolds number changing on the drag coefficient is negligible [16]. In this paper, the product of the independent zero attack angle drag coefficient and the angle attack changing function is used to represent the resistance coefficient. So:

$$C_x = C_{x_0}(v) \cdot f_x(\alpha) \quad (1)$$

where, $C_{x_0}(v)$ is the drag coefficient function in the state of zero attack angle, v is the velocity of the missile, $f_x(\alpha)$ is the influence function of the attack angle α on the drag coefficient. Then $f_x(\alpha)$ is unfolded to the Taylor series expansion around $\alpha=0$.

$$f_x(\alpha) = f_x(0) + \sum_{n=1}^{\infty} \left(\frac{d^n f_x}{d\alpha^n} \right)_0 \frac{\alpha^n}{n!} \quad (2)$$

Because the magnitude of drag is independent with the positive and negative of α and it is an even function. So Equation (2) can be converted to Equation (3).

$$f_x(\alpha) = f_x(0) + \left(\frac{d^2 f_x}{d\alpha^2} \right)_0 \frac{\alpha^2}{2!} \quad (3)$$

It is easy to obtain that $f_x(0)=1$, and let $2k=(d^2 f_x/d\alpha^2)$. Then, Equation (3) can be simplified as:

$$f_x(\alpha) = 1 + k\alpha^2 \quad (4)$$

The drag coefficient is calculated as:

$$C_x = C_{x_0}(v) \cdot (1 + k\alpha^2) \quad (5)$$

where, the zero attack angle drag coefficient $C_{x_0}(v)$ and the coefficient k can be obtained by numerical calculation method. The speed range of the submarine-launched missile is small and the change of speed is not obvious. So $C_{x_0}(v)$ can be considered as a constant C_{x_0} .

In summary, the calculation formula of drag is:

$$F_\mu = C_x \cdot \frac{1}{2} \rho S v^2 \quad (6)$$

where, S is the immersion area of the missile in the water-exit process; ρ is the density of fluid.

The effect of the viscous fluid can be expressed as:

$$\begin{cases} F_{\mu x} = F_\mu \cdot \cos \alpha \\ F_{\mu y} = -F_\mu \cdot \sin \alpha \\ M_{\mu z} = -F_{\mu y} \cdot \frac{1}{2} x_a \end{cases} \quad (7)$$

Ideal fluid force

The ideal fluid is the incompressible, non-viscous fluid. The ideal fluid force can be obtained by the momentum and momentum moment theorem:

$$\begin{cases} -F_i = dQ_f / dt + \omega \times Q_f \\ -M_i = dK_f / dt + \omega \times K_f + v \times Q_f \end{cases} \quad (8)$$

where, F_i and M_i respectively represent main force and main moment of ideal fluid to the missile; Q_f and K_f respectively represent the momentum and momentum moment of ideal

fluid for the missile; ω and v respectively represent the rotating angular velocity and velocity of the missile.

According to the theory of potential flow, introducing the added mass matrix, considering the axisymmetric characteristics of the missile, the force of ideal fluid is obtained as follows:

$$\begin{cases} F_{ix} = -\lambda_{11}\dot{v}_x + \omega_z (\lambda_{22}v_y + \lambda_{26}\omega_z) \\ F_{iy} = -\lambda_{22}\dot{v}_y - \lambda_{26}\dot{\omega}_z - \omega_z \lambda_{11}v_x \\ M_{iz} = -\lambda_{62}\dot{v}_y - \lambda_{66}\dot{\omega}_z + v_y \lambda_{11}v_x - v_x (\lambda_{22}v_y + \lambda_{26}\omega_z) \end{cases} \quad (9)$$

where, $\lambda_{11}, \lambda_{22}, \lambda_{26} = \lambda_{62}, \lambda_{66}$ is added mass coefficients, which respectively represent vertical added mass, lateral added mass, added static moment, added inertia moment.

Unlike the traditional objects that only motion underwater, the added mass of submarine-launched missiles in water-exit process is time-varying. Therefore, on the basis of Equation (9), the influence factor of the added mass changing rate is introduced in the paper and the ideal fluid force for the missile in the water-exit process is obtained as follows:

$$\begin{cases} F_{ix} = -\lambda_{11}\dot{v}_x - \dot{\lambda}_{11}v_x + \omega_z (\lambda_{22}v_y + \lambda_{26}\omega_z) \\ F_{iy} = -\lambda_{22}\dot{v}_y - \dot{\lambda}_{22}v_y - \lambda_{26}\dot{\omega}_z - \dot{\lambda}_{26}\omega_z - \omega_z \lambda_{11}v_x \\ M_{iz} = -\lambda_{62}\dot{v}_y - \dot{\lambda}_{62}v_y - \lambda_{66}\dot{\omega}_z - \dot{\lambda}_{66}\omega_z + v_y \lambda_{11}v_x - v_x (\lambda_{22}v_y + \lambda_{26}\omega_z) \end{cases} \quad (10)$$

It can be inferred that the added mass is an important parameter to describe the effect of the ideal fluid. Its calculation result is related to the estimation of the effect of the fluid to the missile in the modeling process, which can directly affect the accuracy of the model. At present, the added mass of the objects with simple shape can be calculated by the slender body section theory. However, the object studied in this paper is the submarine-launched missile with a more complex linear shape. And there is greater error by using the slender body tangent theory to calculate the added mass of the missile in water-exit process. Therefore, in the following, the water dynamics model of the submarine-launched missile is combined with the software Fluent to study the added mass calculation strategy in water-exit process.

OTHER PARAMETERS

Buoyancy B changes with the immersion depth of the missile. The influence of the surface heave is not considered and Buoyancy is obtained as follows:

$$B = \pi\rho g \int_0^{L-x_a} R^2(x)dx \quad (11)$$

In water-exit process, the location of buoyant centre of the missile changes constantly. And its location can be obtained as follows:

$$x_{b0} = \frac{\pi\rho g \int_0^{L-l_1} R^2(x)xdx}{B} \quad (12)$$

The location of barycentre is as follows:

$$x_0 = \frac{\int_0^L R^2(x)xdx}{\int_0^L R^2(x)dx} \quad (13)$$

And the immersion area of the missile in water-exit process is as follows:

$$S = 2\pi \int_0^{L-l_1} R(x)dx \quad (14)$$

The rotary inertia of the missile is as follows:

$$J = \pi\rho_0 \int_0^L R^2(x) \left[1/4 R^2(x) + (x - x_0)^2 \right] dx \quad (15)$$

In Equations (11) - (15), $R(x)$ is the radius of the missile that changes with the x axis in body coordinate system.

WATER-EXIT DYNAMIC MODEL

In summary, according to the relevant parameters and the force conditions of the submarine-launched missile, the dynamic model of the missile in water-exit process is established as follows:

$$\begin{cases} F_{ix} + F_{\mu x} + (B - G) \sin \theta + T = m(\dot{v}_x - v_y \omega_z - y_c \dot{\omega}_z) \\ F_{iy} + F_{\mu y} + (B - G) \cos \theta = m(\dot{v}_y + v_x \omega_z - y_c \omega_z^2) \\ F_{iz} + F_{\mu z} - Bx_b \cos \theta = J \cdot \dot{\omega}_z - my_c \dot{v}_y - my_c v_y \omega_z \end{cases} \quad (16)$$

where, the subscripts x and y respectively represent the corresponding components of the axis; x_c, y_c and z_c are barycentric coordinates.

CALCULATION STRATEGY OF ADDED MASS

Considering that the added mass is only decided by the shape of the object and has nothing to do with the motion law of the missile, the study makes the following settings for Equation (9):

1) Let $v_y = 0, \omega_z = 0$. The direction of \dot{v}_x is along with the positive direction of x axis and the missile moves in the x axis direction with the increased speed at this moment. So F_{ix} in Equation (9) can be simplified into:

$$F_{ix+} = -\lambda_{11}\dot{v}_x - \dot{\lambda}_{11}v_x \quad (17)$$

The direction of \dot{v}_x is set along the negative direction of x axis, so there are:

$$F_{ix-} = \lambda_{11}\dot{v}_x - \dot{\lambda}_{11}v_x \quad (18)$$

From Equations (17) and (18), it is obtained as follows:

$$\begin{cases} \lambda_{11} = (F_{ix-} - F_{ix+}) / 2\dot{v}_x \\ \dot{\lambda}_{11} = -(F_{ix-} + F_{ix+}) / 2v_x \end{cases} \quad (19)$$

2) Let $v_x = 0, \omega_z = 0$. The direction of \dot{v}_y is along with the positive direction of y axis and the missile moves in the y axis direction with the increased speed at the moment. So F_{iy} and M_{iz} in Equation (9) can be simplified into:

$$\begin{cases} F_{iy+} = -\lambda_{22}\dot{v}_y - \dot{\lambda}_{22}v_y \\ M_{iy+} = -\lambda_{26}\dot{v}_y - \dot{\lambda}_{26}v_y \end{cases} \quad (20)$$

The direction of \dot{v}_y is set along the negative direction of y axis, so there are:

$$\begin{cases} F_{iy-} = \lambda_{22}\dot{v}_y - \dot{\lambda}_{22}v_y \\ M_{iy-} = \lambda_{26}\dot{v}_y - \dot{\lambda}_{26}v_y \end{cases} \quad (21)$$

From Equations (20) and (21), it is obtained as follows:

$$\begin{cases} \lambda_{22} = (F_{iy-} - F_{iy+}) / 2\dot{v}_y \\ \dot{\lambda}_{22} = -(F_{iy-} + F_{iy+}) / 2v_y \\ \lambda_{26} = (M_{iy-} - M_{iy+}) / 2\dot{v}_y \\ \dot{\lambda}_{26} = -(M_{iy-} + M_{iy+}) / 2v_y \end{cases} \quad (22)$$

3) Let $v_x = 0, v_y = 0$. The direction of $\dot{\omega}_z$ is along with the counterclockwise direction of z axis and the missile rotates around the y axis direction at the moment. So M_{iz} in Equation (9) can be simplified into:

$$M_{iz+} = -\lambda_{66}\dot{\omega}_z - \dot{\lambda}_{66}\omega_z \quad (23)$$

The direction of $\dot{\omega}_z$ is set along the clockwise direction of z axis, so there are:

$$M_{iz-} = \lambda_{66}\dot{\omega}_z - \dot{\lambda}_{66}\omega_z \quad (24)$$

From Equation (23) and (24), it is obtained as follows:

$$\begin{cases} \lambda_{66} = (M_{iz-} - M_{iz+}) / 2\dot{\omega}_z \\ \dot{\lambda}_{66} = -(M_{iz-} + M_{iz+}) / 2\omega_z \end{cases} \quad (25)$$

where, $F_{x+}, F_{x-}, F_{y+}, F_{y-}$ respectively represent subjected fluid forces to the missile when the it move in positive direction of x axis, in negative direction of x axis, in positive direction of y axis and in negative direction of y axis. $M_{y+}, M_{y-}, M_{z+}, M_{z-}$ respectively represent subjected fluid force moments to the missile when it move in positive direction and negative direction of y axis and rotate around clockwise direction and counter clockwise direction of z axis. So the calculation formulas of the added mass and the added mass changing rate of the submarine-launched missile in water-exit process are as follows:

$$\begin{cases} \lambda_{11} = (F_{ix-} - F_{ix+}) / 2\dot{v}_x \\ \lambda_{22} = (F_{iy-} - F_{iy+}) / 2\dot{v}_y \\ \lambda_{26} = (M_{iy-} - M_{iy+}) / 2\dot{v}_y \\ \lambda_{66} = (M_{iz-} - M_{iz+}) / 2\dot{\omega}_z \end{cases} \begin{cases} \dot{\lambda}_{11} = -(F_{ix-} + F_{ix+}) / 2v_x \\ \dot{\lambda}_{22} = -(F_{iy-} + F_{iy+}) / 2v_y \\ \dot{\lambda}_{26} = -(M_{iy-} + M_{iy+}) / 2v_y \\ \dot{\lambda}_{66} = -(M_{iz-} + M_{iz+}) / 2\omega_z \end{cases} \quad (26)$$

where, known from the related knowledge of fluid mechanics, $F_{x+}, F_{x-}, F_{y+}, F_{y-}$ can be obtained by the surface integral of the intensity of pressure on surface of the missile and $M_{y+}, M_{y-}, M_{z+}, M_{z-}$ can be obtained by the surface integral and distance integral of the intensity of pressure on surface of the missile.

In summary, the calculation strategy of the added mass can be described as follows: Firstly, through the CFD software, the flow field numerical model is established and the two groups of the motion law in same velocity and inverse acceleration is set, which is $v, \dot{v}, \omega, \dot{\omega}$ with $v, -\dot{v}, \omega, -\dot{\omega}$. Then, $F_{x+}, F_{x-}, F_{y+}, F_{y-}, M_{y+}, M_{y-}, M_{z+}, M_{z-}$ in the two conditions are calculated. Finally, the corresponding added mass is calculated by Equation (25). From the calculation process above, as long as the environment of flow field under different motion parameters is determined, by the proposed computation strategy in the paper, without solving the complex potential function and considering the influence of the free surface, the added mass of any shape at any time in water-exit process can be calculated by the fluid simulation software. Therefore, the calculation strategy can be used to calculate the added mass of the submarine-launched missile in water-exit process.

NUMERICAL SIMULATION

The water-exit process involving two kinds of fluids is a complex non-linear unsteady multiphase coupling problem that contains free surface. As the interface between water and air, free surface is a special interface. On the hand, it is the boundary of the flow field and a necessary condition for solution of flow field. On other hand, its location is not predicted in advance, but as the part of the solution to the problem given by the solution process. Therefore, it is very difficult to use the numerical method to simulate the problem of two-phase flow with free surface. In order to simulate the two-phase flow with free surface in the paper, the VOF method is applied to capture the fluctuation of free surface [17]. In VOF method, different groups of fluid share a set of momentum

equations, the control domain is established both for water and air, and the free surface is captured by solving the added equations. In the calculation process, the volume fraction of each fluid component is recorded in every flow unit of the entire flow field. The method can make a better description for the phenomenon such as the fluctuation and roll of free surface and also has a great advantage in dealing with complex free surface flow problems. At the same time, the calculation cost of the method is relatively small, and the requirements for the hardware especially the memory size is relatively low. In addition, in order to adapt to the shape of fluid that changes with the motion of the missile in the water-exit process and obtain a good numerical solution, the dynamic mesh method is applied in the paper.

CONTROL EQUATION

The following three-dimensional control equation is used:

$$\nabla \cdot \bar{U} = 0 \quad (27)$$

$$\frac{\partial(\rho\bar{U})}{\partial t} + \nabla \cdot (\rho\bar{U} \times \bar{U}) = -\nabla P + \nabla \times (\mu \nabla \times \bar{U}) + \rho g + F_{SV} \quad (28)$$

where, $\bar{U} = (u, v, \omega)$ represent velocity vector of the fluid mass point in three directions of x, y, z axis; ρ, μ respectively represent the density of fluid and dynamic viscosity coefficient; g is the gravitational acceleration, F_{SV} is the equivalent volume force form of the surface tension, which obtained by the phase function.

VOF continuous equation

$$\frac{\partial F_q}{\partial t} + v_q \cdot \nabla F_q = \frac{S_{F_q}}{\rho_q} \quad (29)$$

The term at the right end of the equation is 0, and the solution equation can be written as:

$$\frac{\partial F}{\partial t} + \frac{\partial(uF)}{\partial x} + \frac{\partial(vF)}{\partial y} + \frac{\partial(\omega F)}{\partial z} = 0 \quad (30)$$

The constraint equation is:

$$\sum_{q=1}^2 F_q = 1 \quad (31)$$

Considering the viscosity of water, the control equation is closed by using the turbulence model 'standard $k - \varepsilon$ model' which has a good stability.

Turbulence kinetic energy equation

$$\frac{\partial}{\partial t}(\rho k) + \frac{\partial}{\partial x_i}(\rho k u_i) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_k} \right) \frac{\partial k}{\partial x_j} \right] + G_k + G_b - \rho \varepsilon - Y_M + S_k \quad (32)$$

Diffusion equation ε

$$\frac{\partial}{\partial t}(\rho \varepsilon) + \frac{\partial}{\partial x_i}(\rho \varepsilon u_i) = \frac{\partial}{\partial x_j} \left[\left(\mu + \frac{\mu_t}{\sigma_\varepsilon} \right) \frac{\partial \varepsilon}{\partial x_j} \right] + C_{1\varepsilon} \frac{\varepsilon}{k} (G_k + C_{3\varepsilon} G_b) - C_{2\varepsilon} \rho \frac{\varepsilon^2}{k} + S_\varepsilon \quad (33)$$

DYNAMIC MESH TECHNOLOGY

In the numerical calculation, the whole calculation domain is composed of meshes. The motion of the objects means the motion of the boundary in the object surface. That is to say, the motion conditions need to be defined on the boundary conditions. In the paper, UDF self-programming statement is introduced into the numerical calculation to define the motion speed of the meshes, and the water-exit motion of objects in specific direction is achieved. Where, the motion speed of the mesh for each current time step is the sums of the speed of the mesh for the last time step and the speed increment that caused by subjected two phase fluids pressure, viscous fluid force and own gravity for current time step. In the process of the water-exit motion, some of the meshes are constantly compressed and some of the meshes are constantly stretched. The distortion caused by deformation can affect the accuracy of the calculation. When the distortion reaches a certain extent, the negative volume meshes will be produced, which can lead to failure of the calculation. In this case, dynamic mesh technology need be used to define the motion and update of the mesh.

However, at present, in the main dynamic mesh update method, the local reconstruction method easily leads to non-converge of the calculation due to the poor quality of the rebuild meshes, and the dynamic stratification method is not suitable for the flow field area of the submarine-launched missile with a more complex shape in the paper^[18]. Therefore, in order to obtain the good numerical solution and ensure the quality of dynamic mesh, especially in the case of simulating the completely non-linear free surface change caused by water-exit process, combining the two methods above, a new update method for the dynamic mesh is used.

In the water-exit process, the local flow field area around the water-exit object is set as the area subjected by new dynamic mesh update method and called the motion area, and other calculation area keep still. The speed of the object is assigned to the entire local dynamic meshes area and the front and back boundary of the area is restricted to keep still. The dynamic stratification update of the meshes in this area is carried out after the motion of this area. The meshes nearby the surface of the object only move as the moving area and the complex update operation for the meshes is not performed. The location of the mesh change is transferred to the plane boundary in the front and back of local dynamic mesh. This operation ensures that the mesh around the object

does not change and the mesh quality and quantity meet the requirement in the simulation of water-exit process. Besides, as same with the dynamic stratification method, the new updating method is also required to define an ideal height value for the adjacent mesh layer of the update boundary of the dynamic mesh. When the mesh in this layer is being stretched, the height of the mesh is allowed to increase until meeting the Equation (1), and its division form is the same as the dynamic stratification method [19].

$$h_{\min} > (1 + \alpha)h_{id} \quad (34)$$

where, h_{\min} is the minimum height of the nth layer mesh, h_{id} is the ideal height of the mesh, and α is mesh cutting factor.

In this paper, the new method transfers the dynamic mesh problem at complex boundaries to a simple boundary. Therefore, for this method, on the one hand, the dynamic mesh technology that suit for the most fluid field calculation of the objects with complex shape such as the submarine-launched missile is achieved; on the other hand, the updating time of the meshes is effectively reduced. Meanwhile, for the calculation area in which the dynamic meshes continuously updates, the number of meshes is ensured to strictly keep constant so that the calculation speed cannot be reduce by the increase of the mesh.

EXAMPLE VALIDATION

The added mass of the sphere has a theoretical analytical value and also has more reliable experimental data. Therefore, based on the proposed added mass calculation method in the paper, with the means of numerical simulation, the added mass of the sphere in water-exit process is calculated by simulation to verify the reliability of the added mass calculation method.

Construct the calculation domain and divide the mesh

The overall calculation domain is a cuboid whose size is 20 * 10 * 10. Considering the curved face shape of the sphere (Diameter D = 1m) and that the unstructured meshes can better adapt to the change of the curved face, triangular meshes are used as the surface meshes of the model, and tetrahedron and hexahedron are used as the calculation domain. In order to ensure the speed and accuracy of calculation and reduce the number of dynamic meshes, the paper divided the whole calculation domain into the static and changing two areas. And when dividing meshes, under the criteria of quality priority, the tetrahedron meshes is used for the static area and the hexahedron meshes is used for the changing area. In the calculation process, the meshes in the static area remain unchanged and the meshes in the changing area are updated in real time according to the motion of the boundary.

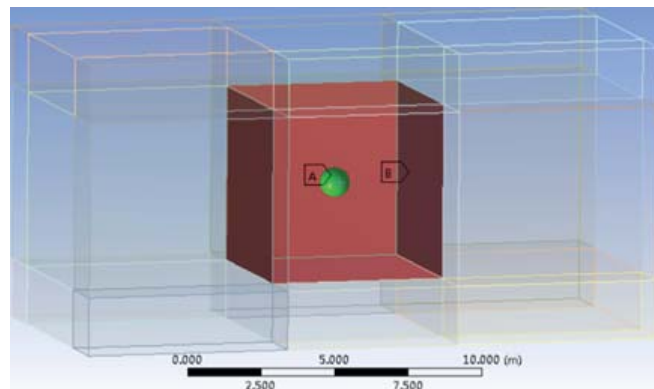


Fig. 2. Construction of the calculation domain

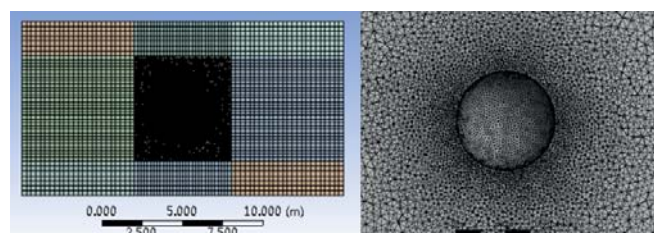


Fig. 3. Results of dividing the meshes

Results of calculation

In the initial stage of the numerical calculation, the free surface is set at location that is 2m away from the centre of the sphere (coordinates Y = 2m), and the initial velocity $V=[1,2]$ and acceleration $a=\{[3,-3];[10,-10]\}$ is assembled into four group of motion laws to calculate. By the calculation strategy in the paper, vertical and horizontal added mass are obtained as shows in Table 1.

Tab. 1. Calculation results of added mass in one step

Calculated conditon	VA[1,3,-3]	VA[1,10,-10]	VA[2,3,-3]	VA[2,10,-10]
Theoretical value(kg)	261.79939	261.79939	261.79939	261.79939
Vertical added mass(kg)	259.97665	259.96674	260.03994	260.10492
Relative error(%)	0.69624	0.70002	0.67206	0.64724
Lateral added mass(kg)	/	260.37681	/	260.46103
Relative error(%)	/	0.54339	/	0.51122

It is easy to see from Table 1 that the calculation results of vertical and lateral added mass in different combinations and the theoretical values are very close. The maximum relative error is 0.70002% and the minimum one is 0.51122%, which indicates that the calculation strategy is correct and effective. Meanwhile, the initial velocity and acceleration do not affect the calculation results, and the error is mainly from the mesh and turbulence model.

CALCULATION OF ADDED MASS IN WATER-EXIT PROCESS

On the basis of the proposed added mass calculation method in the paper, the added mass of the submarine-launched missile in water-exit process is investigated. And in the study, the shape parameters of the missile are selected from [20]. The specific geometric model is shown in Figure 4.

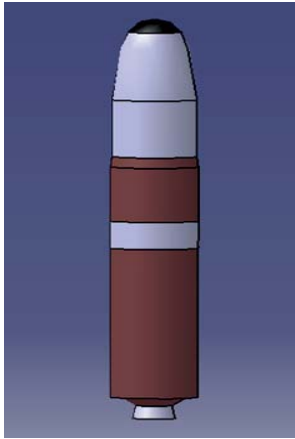


Fig. 4. Construction of submarine-launched missiles

The combinations $[v, \dot{v}, \omega, \dot{\omega}] = [1, 5, 0.1, 1]$ and $[v, \dot{v}, \omega, \dot{\omega}] = [1, 5, 0.1, 1]$ are selected. Then calculation is performed referring to the setting of the added mass in Section 3.3. In order to facilitate comparison, as shown in Equation (35), the immersion depth and added mass in water-exit process are dimensionless.

$$\bar{H} = H / L, \bar{\lambda} = \lambda / \rho V(L) \quad (35)$$

where, H is the length of the part of the missile that immersed underwater; L is the characteristic length of the missile; λ is the added mass; ρ is the density of water; V is the volume of the missile. Figure 5 is the calculation domain of numerical simulation for the missile.

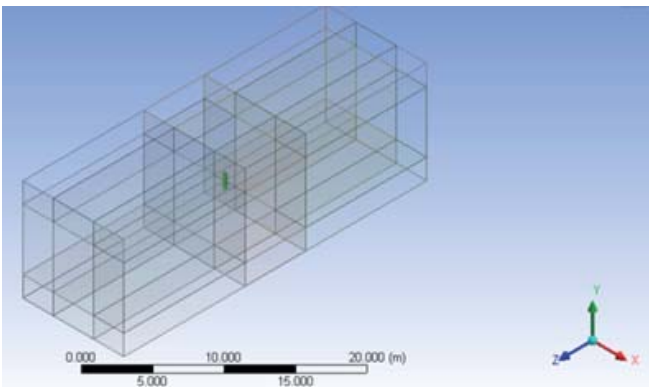


Fig. 5. Calculation domain

When the water-exit inclination angle of the missile is 90° , 60° and 45° , the mesh is divided as Figures 6-8 shown, and the calculation results of the added mass in water-exit process are shown in Tables 2-4. It can be seen from the tables that although the set positions of the initial surface are same in calculation, non-dimensional immersion depths are different under the different inclination angle. In order to facilitate the comparative analysis of the added mass change law in water-exit process under the different angles, Figures 9-12 are drawn.

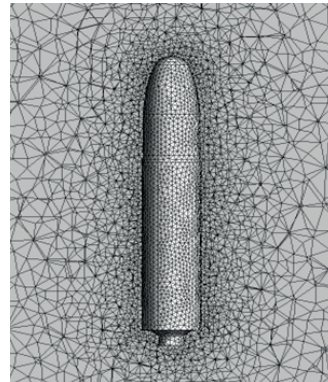


Figure 6. Inclination angle of 90°

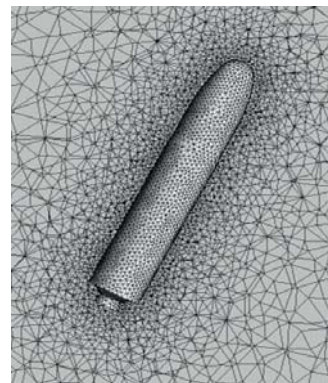


Figure 7. Inclination angle of 60°

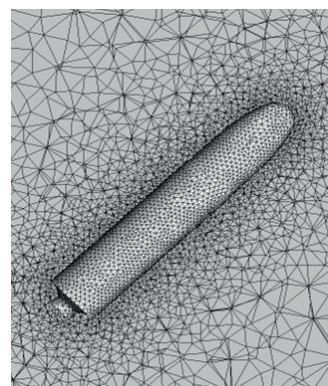


Figure 8. Inclination angle of 45°

Tab. 2. The added mass at inclination angle of 90°

non-dimensional immersion depth	λ_{11}	λ_{66}	λ_{22}	λ_{26}
0.06667	0.01864	0.00696	0.01026	0.00427
0.2	0.042	0.01637	0.07041	0.02728
0.4	0.054	0.03517	0.23544	0.07206
0.53333	0.05615	0.04015	0.3787	0.0924
0.66667	0.0574	0.04127	0.53349	0.09616
0.86667	0.05859	0.05014	0.7769	0.06462
0.93333	0.06038	0.05565	0.85279	0.04632

Tab. 3. The added mass at inclination angle of 60°

non-dimensional immersion depth	λ_{11}	λ_{66}	λ_{22}	λ_{26}
0.06376	0.01844	0.00629	0.01563	0.00961
0.20232	0.03879	0.02572	0.0805	0.04484
0.53333	0.05513	0.05861	0.37927	0.13346
0.86435	0.05827	0.07624	0.76438	0.0948
0.93363	0.06258	0.08403	0.83012	0.07436
1.30313	0.12307	0.10008	0.94742	0.03449
2.07293	0.12796	0.10043	0.95595	0.03311

Tab. 4. The added mass at inclination angle of 45°

non-dimensional immersion depth	λ_{11}	λ_{66}	λ_{22}	λ_{26}
0.07136	0.02041	0.01123	0.03264	0.01888
0.20335	0.03572	0.02736	0.0974	0.05066
0.53333	0.05345	0.05587	0.3809	0.12454
0.86332	0.05849	0.07691	0.74088	0.09224
0.92931	0.06381	0.08178	0.79419	0.07955
1.47614	0.12464	0.10015	0.9472	0.03415
2.41895	0.12804	0.10043	0.95585	0.03306

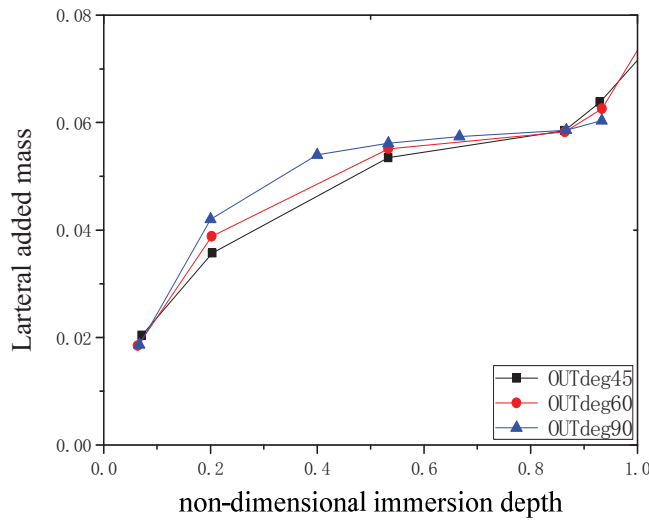


Fig. 9. Lateral added mass

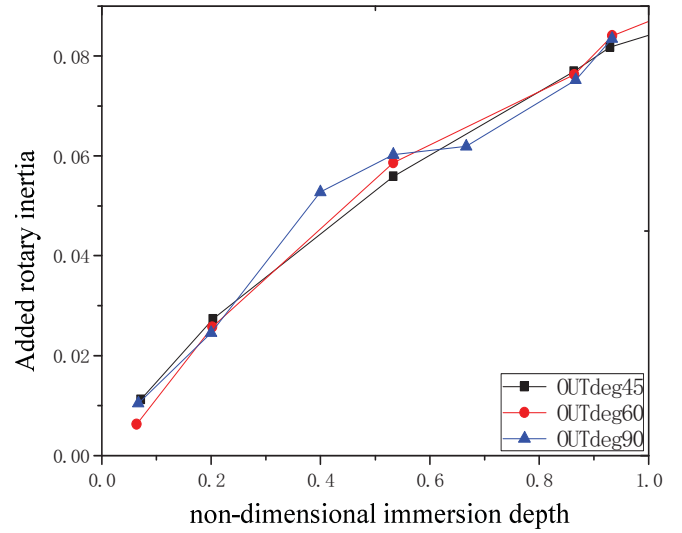


Fig. 10. Rotary inertia

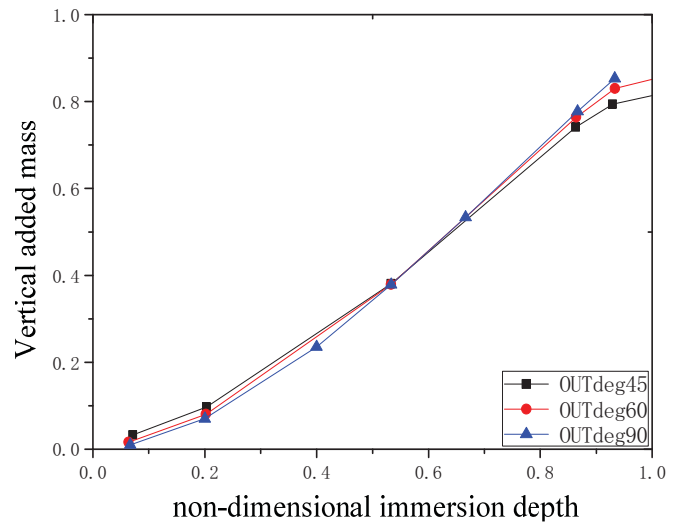


Fig. 11. Vertical added mass

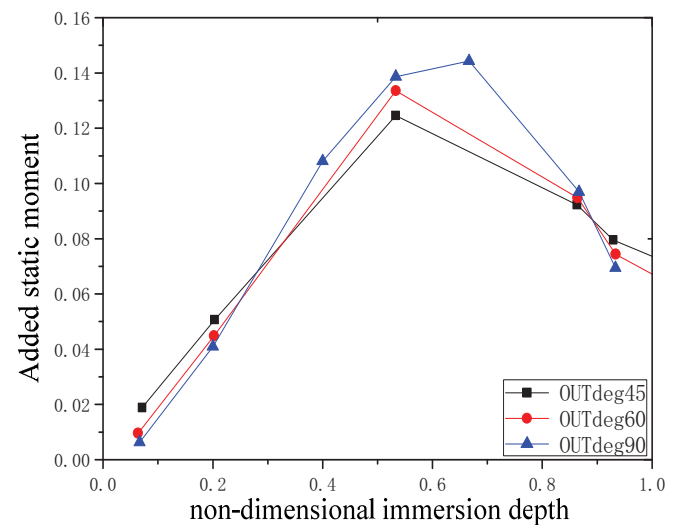


Fig. 12. Added static moment

The calculation results of lateral added mass are shown as Figure 9. It can be seen from the figure, in the water-exit process under the different inclination angles, with the non-dimensional immersion depth changing from 1 to 0, the lateral added mass firstly drops sharply and then its downward trend slows down, but the downward trend becomes bigger at the end of water-exit process. In water-exit process, with the same non-dimensional immersion depth, the lateral added mass of 90° is higher than the lateral added mass of 60° and 45°, and from the view of magnitude order, the maximum does not exceed 0.07.

The calculation results of added rotary inertia are shown as Figure 10. It can be seen from the figure, in the water-exit process under the different inclination angles, with the non-dimensional immersion depth changing from 1 to 0, the added rotary inertia decreases with an approximately linear trend. The downward trend of the results of 90° have a slow-down area around the immersion depth of 0.5, and from the view of magnitude order, the maximum does not exceed 0.09.

The calculation results of vertical added mass are shown as Figure 11. It can be seen from the figure, with the decrease of the immersion depth, the vertical added mass basically decreases with an approximately linear trend. The consistency of the data under three inclination angles is so good that the data can be approximately considered as the same one. From the view of magnitude order, the maximum does not exceed 0.9 which is about 10 times of the lateral added mass and added rotary inertia.

The calculation results of added static moment are shown as Figure 12. It can be seen from the figure, with the decrease of the immersion depth, the added static moment firstly increase and then decrease. And there is a maximum near the immersion depth of 0.6. From the view of magnitude order, the maximum does not exceed 0.15.

SIMULATION OF WATER-EXIT PROCESS

The added mass is only related to the geometrical shape of the object. In water-exit process of the submarine launched missile, the added mass is only related to its immersion depth l and inclination angle θ . Therefore, in order to obtain the ideal fluid force required in the simulation of water-exit process, the above added mass calculation method can be used to obtain a considerable amount of added mass of the missile under different immersion depth l and inclination angle θ , and then a two-dimensional interpolation table of the water-exit whose row is immersion depth and column is inclination angle is established to express the added mass in water-exit process. Thus, according to the current immersion depth and inclination angle, the added mass of the missile in water-exit process can be obtained by interpolation. Then the ideal fluid force can be obtained by Equation (9). The added mass and the changing rate of the added mass in water-exit process are obtained by the equation below.

$$\begin{cases} \lambda_{11} = g_1(l, \theta) \\ \lambda_{22} = g_2(l, \theta) \\ \lambda_{26} = g_3(l, \theta) \\ \lambda_{66} = g_4(l, \theta) \end{cases} \begin{cases} \dot{\lambda}_{11} = g_5(l, \theta) \\ \dot{\lambda}_{22} = g_6(l, \theta) \\ \dot{\lambda}_{26} = g_7(l, \theta) \\ \dot{\lambda}_{66} = g_8(l, \theta) \end{cases} \quad (36)$$

Under the same condition, the methods of numerical simulation and theoretical calculation are both used to simulate the water-exit motion. In the two methods, the theoretical calculation is based on the established water-exit dynamic model of the submarine-launched missile. In addition, the required added mass parameters in the model are obtained by Equation (36). The set initial velocity v_{x0} is 10 m/s and v_{y0} is 0 m/s, the set initial water-exit inclination angle θ_0 is 45°, the set initial rotating angular velocity ω_{z0} is 0, the set initial attack angle α is 0°. In the whole process, the missile has no thrust, which means the thrust $T=0$. In order to eliminate the effects of free surface, the initial calculated position is set that the vertex of the head of the missile is at the location in the direction of the axis away from water surface in distance of two times of the missile diameter. Figure 13 shows the water-exit process that obtained by numerical simulation. And Figure 14 shows the water-exit process that obtained theoretical calculation.

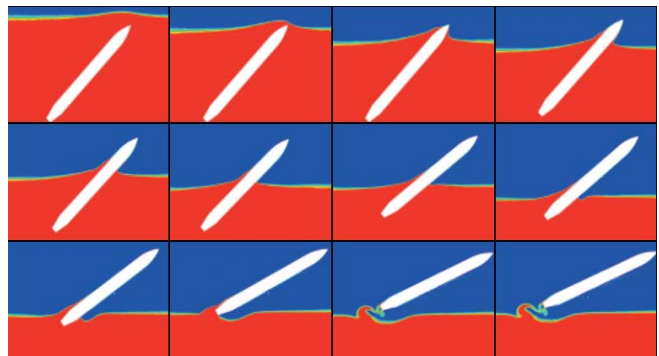


Fig. 13. Numerical simulated results of water-exit motion

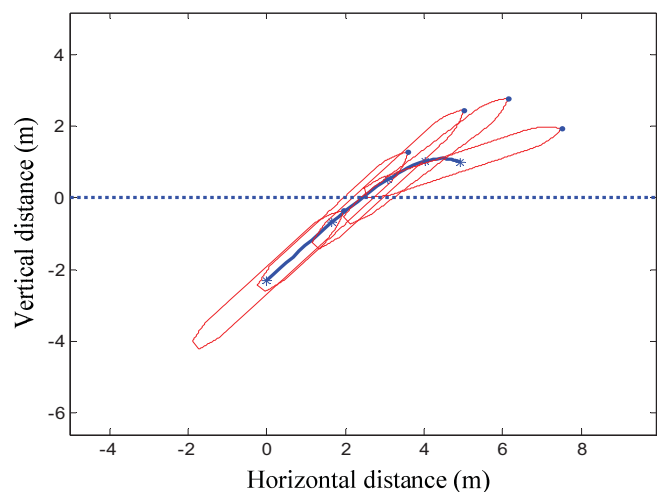


Fig. 14. Theoretical calculated results of water-exit motion

In Figure 14, the blue thick solid line is the trajectory of the barycentre motion of the missile, red fine solid line is the outline of the missile at all times, and the blue dotted line is the sea level.

It can be seen from Figure 13 and 14 that under the same working condition, the trajectory changing laws of the barycentre in water-exit process obtained by two methods are basically consistent. Therefore, on the one hand, the accuracy of the water-exit dynamics model of the submarine-launched missile established in the paper is verified. Thus, the established model can be used to predict and calculate the motion law of water-exit process in the future. On the other hand, the correctness of the proposed added mass calculation method in the paper is also verified.

In order to further investigate the validity of the added mass changing rate introduced in the modeling process of the submarine-launched missile, the influence of the added mass changing rate on the water-exit model of the missile is investigated by comparing the numerical simulated result with the simulated results obtained by solving the two theoretical models which respectively considers the added mass changing rate and don't consider the added mass changing rate. Figure 15 shows that the change of inclination angle of the missile in water-exit process under three different simulation conditions. In the figure, the red solid line represents the numerical simulation result, the blue solid line represents the simulation result obtained by solving the theoretical model that considers the added mass changing rate, and the blue dashed line represents the simulation result obtained by solving the theoretical model that don't consider the added mass changing rate. According to the comparison, before the missile touches the surface of water, the three simulation results are very similar, which indicates that the added mass changing rate has no effect on the accuracy of the model when the missile is completely immersed in water. As can be seen from Figure 15, in the water-exit process, after the head of the missile touching the surface of water, the decreasing trends of the inclination angles which reflected by the three simulated results are consistent. However, the simulation results of the theoretical model that considers the added mass changing rate are closer to the numerical simulation results. And by comparing, the simulation results of the theoretical model that don't considers the added mass changing rate is different from the other two simulation results for its smaller decreasing trend of the inclination angle. Therefore, it is inferred that the added mass changing rate can greatly affect the accuracy of the water-exit dynamics model so that it cannot be ignored in modeling process.

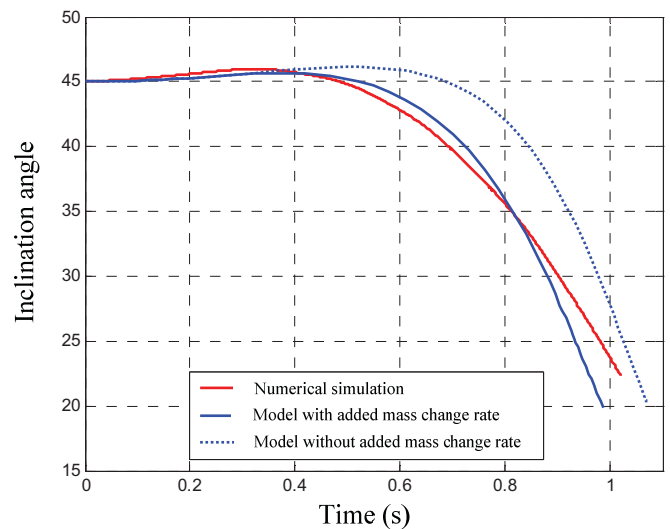


Fig. 15. Comparison of the water-exit inclination angle

Figure 16 is the comparison of the water-exit angular velocity of the missile under the three conditions. According to the comparison, in the process that the missile crosses the surface of water, the angular velocity of the model that don't consider the added mass changing rate is larger and the decrease of the inclination angle of the same model is relatively slow, which has a large deviation with the other two simulation results. In addition, it can be seen from the figure that there is still a little deviation between the numerical simulated result and theoretical result obtained by solving the water-exit model that don't consider the added mass changing rate.

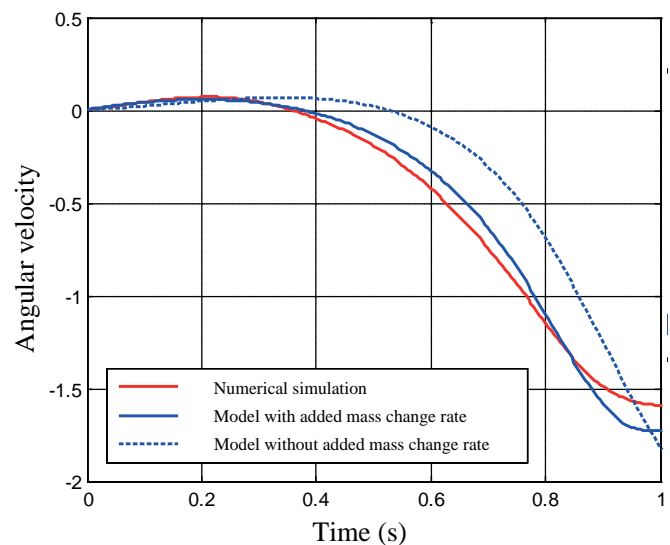


Fig. 16. Comparison of the water-exit angular velocity

Through analyzing, the reason of this phenomenon is that the calculation of the model ignores the disturbance on the surface of water such as cavity bubble and the surface heave in water-exit process. In addition, in the numerical

simulation, because of the choice of turbulence model, the setting of parameters and the partition of meshes, there is still some deviation between the numerical simulated result and the actual situation. In the future research work, the factors such as the disturbance on the surface of water will be considered to further improve the accuracy of the water-exit dynamic model.

In summary, the added mass change rating in water-exit process has a certain influence on water-exit motion. Therefore, the established theoretical model that considers the added mass changing rate in the paper can describe the water-exit process of the missile more accurately, which in the future will have a certain help for precisely studying the posture change of the submarine-launched missile in water-exit process and the flight trajectory of the missile after exiting from the water.

CONCLUSION

The calculation problem of the time-varying added mass in the water-exit dynamic model of the submarine-launched missile is investigated in the paper. In this study, the water-exit dynamic model that contains the added mass changing rate is established and a new calculation strategy based on Fluent numerical simulation and aimed at obtaining time-varying added mass is proposed. On the basis of the new strategy, the added mass of the sphere and the submarine-launched missile in different immersion depth are calculated and the change laws of the time-varying added mass is obtained. Then the calculated results is introduced into the water-exit dynamic model for simulation. Thus, the water-exit motion trajectory under the given condition is also obtained and the influence of the added mass changing rate on the water-exit process is investigated. The corresponding conclusions are as follows:

(1) In the water-exit process, with the non-dimensional immersion depth changing from 1 to 0, the lateral added mass firstly drops sharply and then its downward trend slows down. At the end of water-exit process, the downward trend becomes sharp again and increases as the inclination angle increases. The added rotary inertia decreases with an approximately linear trend and have a slow-down area in the downward trend around the immersion depth of 0.5 when the missile vertically exits from water. The vertical added mass decreases in an approximately linear trend. The consistency of the data under different inclination angles is so good that the data is approximately considered to be the same. The added static moment firstly increases and then decreases. And its maximum value is near the immersion depth of 0.6. From the view of magnitude order, the maximum value does not exceed 0.15.

(2) Through the example of the sphere and the comparison of the missile water-exit trajectory in the same conditions between the numerical simulated result and the theoretical calculated simulated results that based on water-exit dynamic model, on the one hand, the feasibility of the new calculation strategy proposed in this paper for calculating the added

mass of the more complex linear shape is verified; on the other hand, the accuracy of the water-exit dynamics model established in this paper is also verified.

(3) The added mass changing rate has a certain influence. And in the water-exit dynamics model of the missile, the added mass changing rate need to be considered and cannot be ignored.

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