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Signals Representation on Energetic Plane Based on Teager-Kaiser Energy Operator

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Abstract

Development of complicated machines and need for maintaining high efficiency and safety of their work is main reason for development of new and more reliable monitoring techniques. One of the main aims of condition monitoring is detection of early stage of failure and monitoring of its development [1]. Such techniques should be sensitive for change in diagnostic signal due to arise of failure.

Paper presents exemplary representations of signals types on energetic plane calculated using Teager-Kaiser energy operator (TKEO). First basic information on TKEO is presented. Next energetic plane is introduced and models of signals are showed. In final section of the paper example of model of signal containing disturbance related with mashing is presented. Teager-Kaiser energy operator, due to is properties, can be used for detection of transient events such as impulses resulting from disturbances of mating of teeth in gearboxes. Such a disturbance of mating is related with decrease of stiffness of given tooth due to crack or development of pitting [2]. Sensitivity of Teager-Kaiser energy operator allow for earlier detection of transient disturbances then use of raw data methods such as Hilbert transform demodulation.

Keywords: Teager-Kaiser energy operator, energetic plane, vibration signal, gearbox

1. Introduction

Constant development of machines, use of new, lighter materials, care for environment and socio-economical costs of machine failures are main reasons of making every effort to develop new and reliable techniques of determination of technical condition of machines and structures. Among these techniques one can list new techniques of signal processing [3, 4], application of sensors embedded in structure [5], use of different physical phenomena for detection of early stages of failures [6, 7]. Teager-Kaiser energy operator (TKEO) is signal processing technique which allow for calculation of energy waveform of diagnostic signal.

Change of technical condition of machine, or its component, due to arise and development of failure, is related with change of amplitude and frequency structure, or in general energetic structure, of vibration signal generated by working machine [1] This change of technical state of machine is related with change of energy dissipated in form of noise, vibrations or heat [8]. Early detection of change of energy can allow determining early phase of failure and monitoring its development. This can lead to choose of optimal maintenance strategy and allow for measureable savings.

At present number of signal processing techniques are used for detection of symptoms of change of technical state. However still new techniques are developed and applied in task of condition monitoring. Such technique is Teager-Kaiser energy operator. One might find publications presenting successful application of TKEO in different tasks of condition monitoring, such as: bearing fault diagnosis [9, 10] or detection faults of gears [11].

2. Teager-Kaiser energy operator

Teager-Kaiser energy operator is non-linear operator which applied to time signal calculates measure which can be interpreted as energy of this signal [12]. Publications [13-16] contain description and analysis of properties of TKEO. For continuous signals this operator has following form:

$$\Psi(x(t)) = \dot{x}^2(t) - x(t)\ddot{x}(t) \tag{1}$$

However for implementation of TKEO in numeric environments following discrete version is most common:

$$\Psi_d(x_n) = x_n{}^2 - x_{n-1}x_{n+1} \tag{2}$$

In publication authors will constantly refer to discrete signals and energy operator of discrete signals so for simplicity $\Psi(x)$ will be used as denotation of result of TKEO acting on analyzed signal x and will be called measure of energy of signal.

Teager-Kaiser energy operator acting on time signal x calculates waveform of measure of energy of analyzed signal. As a result one obtains new signal $\Psi(x)$ which is also a function of time. Number of publications presents results of implementation of TKEO as useful tool for bearing [9, 10] and gearbox failure detection [11].

An important issue, one should remember using Teager-Kaiser energy operator, is that $\Psi(x)$ is not energy in classical meaning but its measure. As pointed out in [12] $\Psi(x)$ can have negative values and its result can differ depending on parameters of analyzed signal.

For discrete single harmonic signal $x_n = A\cos(\Omega n)$, Teager-Kaiser energy operator calculates result (3) which for low values of $\Omega = 2\pi f / f_S$ estimates product of square of amplitude multiplied by square of frequency.

$$\Psi(x) = A^2 \sin^2(\Omega) \approx A^2\Omega^2 \tag{3}$$

This result depend both from amplitude and frequency structure of signal. Change of amplitude or frequency in signal will influence measure of energy of signal indicating change in monitored machine. Interesting property of Teager-Kaiser energy operator (2) is its sensitivity to sudden changes in analyzed signal due to transient disturbances such as impulses.

3. Energetic plane – examples of signals

Concept of energetic plane assumes observation of diagnostic signal represented in coordinates of measure of energy, expressed by equation 3, and velocity of change of measure of energy. Nomenclature of Lie brackets allows for calculation of higher order energetic operators as it is presented in [15]. From point of view of concept of energetic plane most interesting is energetic operator of order 3:

$$\Upsilon_3(x) = [x, x^{(2)}] = \dot{x}\ddot{x} - x\dddot{x} = d\big(\Psi(x)\big)/dt \tag{4}$$

which can be treated as velocity of change of measure of energy of signal x.

To present representation of few types of signals on energetic plane following absic models were created:

- single harmonic signal:

$$x(t) = A\cos(2\pi ft + \phi) \tag{5}$$

- signal with amplitude modulation

$$x_{AM}(t) = A*(1 + M*\cos(2\pi f_{AM}t))*\cos(2\pi ft + \phi) \tag{6}$$

- signal with frequency modulation

$$x_{FM}(t) = A\cos(2\pi ft + m*\cos(2\pi f_{FM}t)) + \phi) \tag{7}$$

- signal with amplitude and frequency modulation

$$x_{AMFM}(t) = x_{AM}(t)*\cos(2\pi ft + x_{FM}(t) + \phi) \tag{8}$$

where: A – amplitude of carrier signal, f – frequency of carrier signal, M – modulation depth, f_{AM} – frequency of amplitude modulation, m – modulation index, f_{FM} – frequency of frequency modulation, ϕ – initial phase.

Figures 1 to 4 present representation of mentioned above signals on energetic plane. All presented have same parameters of carrier signal, i.e. amplitude A, frequency f, phase ϕ, and differ only with values of modulation depth and index. As one may see, for single harmonic signal representation on energetic plane is constant and has form of a point. It is due to unchangeable amplitude and frequency of signal. Change of amplitude or frequency value would cause shifting of point along energy axis. Appearance of amplitude or frequency modulation change signal's representation significantly. Increase of modulation depth M or index m causes increase of size of curve being representation of the signal. It is worth to point out that however size of curve changes its general character is similar for given type of modulation: in case of AM – elliptic-like curve, in case of FM – curve creates characteristic loops. Occurrence of amplitude and frequency modulation creates complicated shape. This representation will depend most importantly from values of modulation parameters (M, f_{AM}, m, f_{FM}). However, also in this case, it can be seen that increase of parameters M and m will result in increase of size and centre of gravity of curve. Change of modulation frequencies will also change location and size of signal's representation on energetic plane.

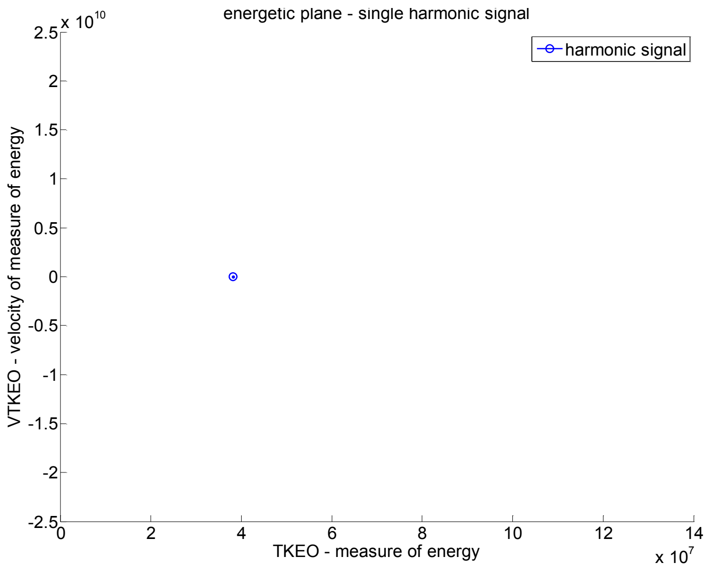

Figure 1. Single harmonic signal on energetic plane

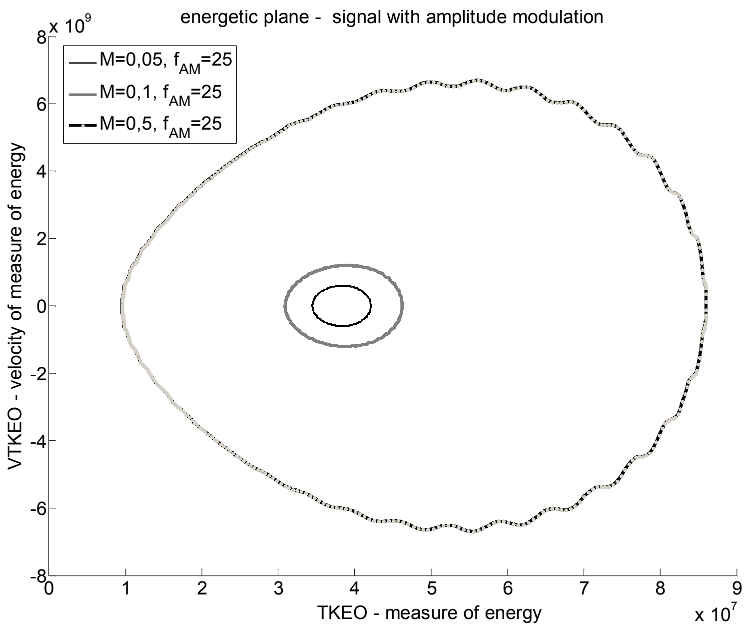

Figure 2. Signals with amplitude modulation with different depth of modulation on energetic plane

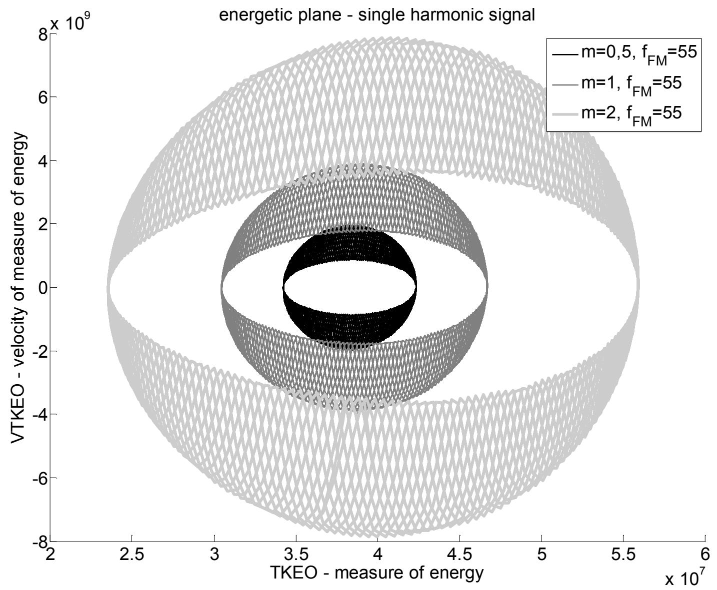

Figure 3. Signals with frequency modulation with different index of modulation on energetic plane

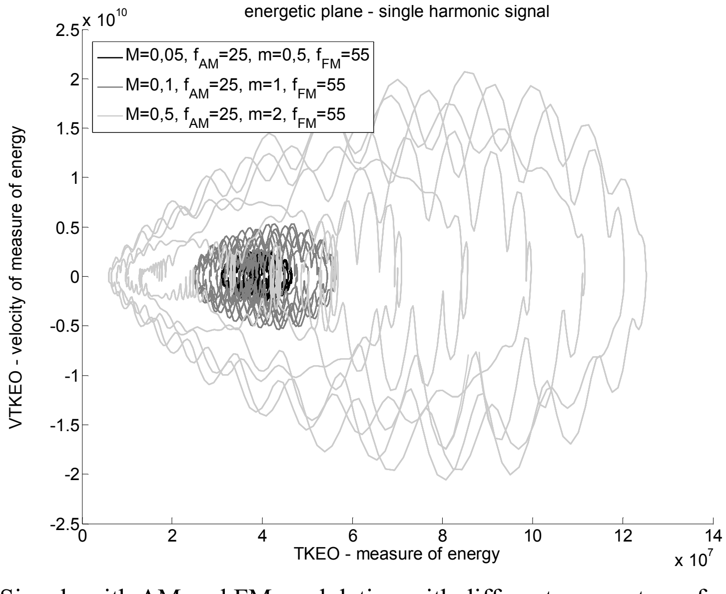

Figure 4. Signals with AM and FM modulation with different parameters of modulation on energetic plane

During operation of gearbox, progressing degradation process causes appearance of local disturbances in signal. This can be caused by coming in to contact of tooth with lower stiffness. Such disturbance can have form of transient change of depth of modulation [17, 2]. To analyze usability of energetic plane based on calculation of Teager-Kaiser energy operator for detection of transient events in signal, model was created allowing occurrence of transient disturbance of depth of amplitude modulation. The purpose of the model is to model local and transient change of meshing of tooth in gearbox. For purpose of test, AM signal was generated with bell-shape disturbance of depth of modulation M. Such shape of disturbance was chosen in order to perform change of parameter as smooth as possible. Duration of disturbance was chosen in such way it would correspond to time of mesh of pair of teeth. Parameters of signal were constant except modulation depth which for one tooth increased 20% from its nominal value. Figure 5 presents comparison of representation of signal with and without disturbance. Change is easily observable.

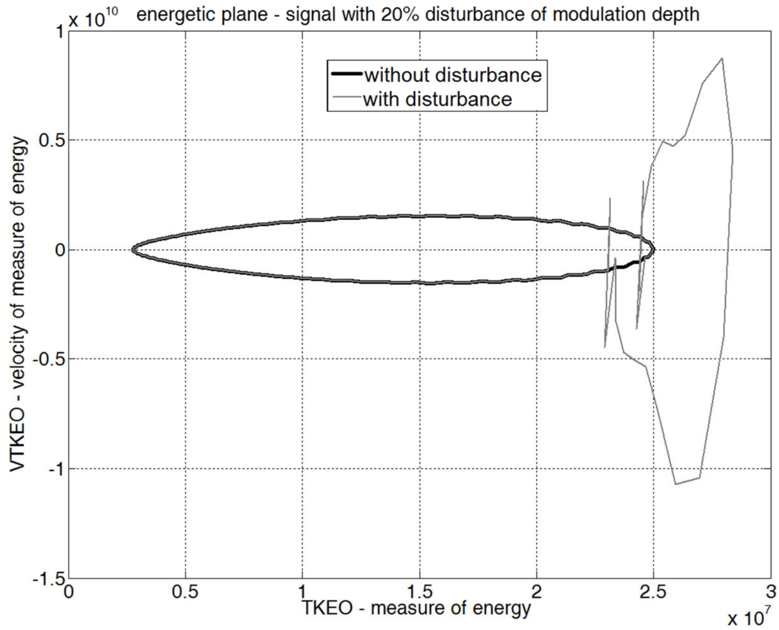

Figure 5. Signal with and without distribution of depth of AM, disturbance 20%

Change of size of signal representation due to disturbance of modulation parameter might be use as a diagnostic information testifying about change in meshing of gearbox. During modelling process other values of disturbance were also tested to analyse sensitivity of energetic plane to size of disturbance. According to observations it can be concluded that lowest realistic value of change of modulation depth which can be detected is between 2 and 5% of its nominal value.

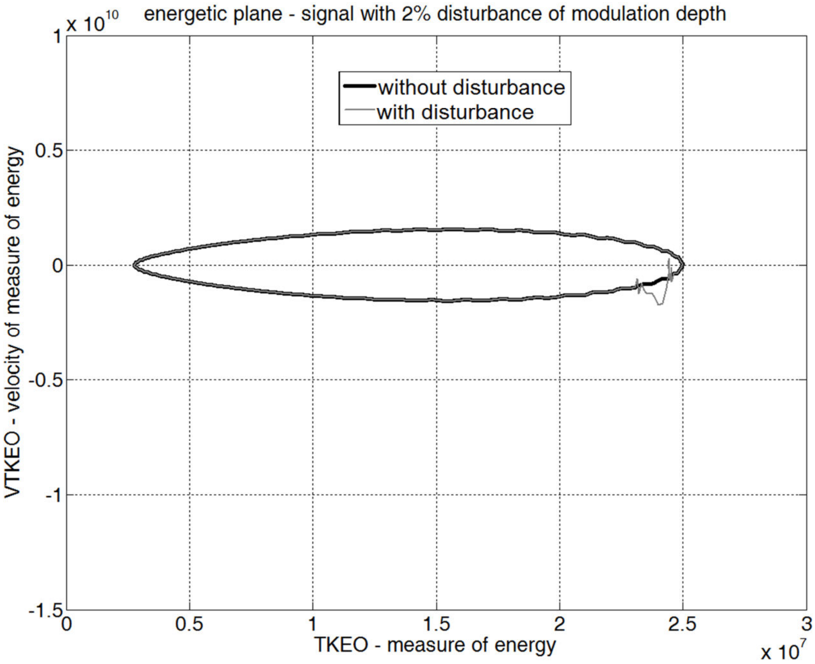

Figure 5. Signal with and without distribution of depth of AM, disturbance 2%

4. Conclusions

Paper presented concept of energetic plane which presents representation of signal in coordinates of measure of energy and value of velocity of change of measure of energy calculated using Teager-Kaiser energy operator. On this plane, given types of signals have similar representations which size depends from modulation parameters. Presented results shows usability of energetic plane in detection of transient events and disturbances of modulation parameters. Further research will be carried out in order to analyse usability of energetic plane in condition monitoring based on vibration signals from real gearbox.

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