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Research on information content of light field on the image plane illuminated by incoherent light source

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A novel approach for the measurement of information content of light field on the image plane illuminated by an incoherent light source has been proposed according to Shannon's information theory. We put forward a hypothetical concept named "coherent wavelet source" on an incoherent light source which can form an independent coherent signal on the object surface based on the expression of mutual intensity on the object surface. As a result, the number of independent coherent signals on the object surface can be made out by dividing the whole illumination source on a coherent wavelet source size. Meanwhile, the maximum number of degree of freedom of light field which can reach the image plane is given based on the structure of imaging system. Concrete algorithm for the information content of the imaging light illuminated by a 1D linear light source and a 2D circular light source are presented.

Keywords: information content, mutual intention, degree of freedom, coherent wavelet source, independent coherent signal.

1. Introduction

Optics is one of the most basic and effective means for people to obtain information. Therefore, it is one of the most fundamental problems in optical phenomena and optical transmission for the quantitative and scientific description of light field information. All parameters involved in the light field, such as light intensity, amplitude, phase, are the factors that affect the information content of light field. That is, the change of any parameter will change the information content of light field directly. So, various coherent states of lighting source must be taken into account for the measurement of the information content of light field.

At present, a series of studies have been carried out on the propagation of light field with various coherent states in optical systems – for instance, the propagation of partially coherent light in the optical system [1–3], the evolution of the optical statistical properties in the turbulent atmosphere [4, 5] and transmission characteristics through complex media such as single crystal, optical fiber and so on [6, 7]. However, the research on information content of imaging light field mainly focuses on the optical system under coherent light illumination [8, 9]. As a matter of fact, the light field distribution on the illumination surface illuminated by a real light source at a certain distance is always partially coherent. And this illumination mode exists in microscope and optical information processing systems widely. So it is particularly important for the measurement of information of light field with a partially coherent state illuminated by a real light source. There are also a few studies on information content of partially coherent light [10, 11]. For example, the Shannon entropy and mutual information-based degrees of partially coherent light with Gaussian fluctuations have been analyzed in [10, 11]. But the concrete algorithm for the information content of the imaging light is based on the complex mathematical deduction, which results in the difficulty in calculating the information content of an actual optical system. In this paper, the imaging information of the object light passing through an imaging system illuminated by the extended quasi-monochromatic incoherent light is studied, and a novel approach to the measurement of information content of light field has been proposed.

Firstly, a hypothetical concept named "coherent wavelet source" (an incoherent light source with maximum allowed size) related to the incoherent light source has been put forward which can meet completely coherent lighting on the whole object surface based on the expression of mutual intensity on the object surface. And then the whole incoherent light source is divided into a lot of independent coherent wavelet sources which form independent coherent signals of the same number on the object surface. Lastly, combined with the structure of imaging optical system, the information content of the light field on the image plane can be achieved based on the information theory of Shannon.

2. The information content of the imaging light field

According to the information theory of Shannon [12], the information content I transmitted by a light field can be expressed as

$$I = N_{\mathrm{DOF}} \log_2(1 + m) \tag{1}$$

where m is a signal-to-noise ratio (SNR) of the light field, and N_{DOF} is the total degree of freedom of the light field. Figure 1 has showed the relationship between the information content I and SNR. As can be seen from Fig. 1, the information content is rapidly increasing when SNR increases from 0 to 100. But when SNR is more than 100, the contribution of SNR to the information content increases only a little. Therefore, for the optical system with high SNR, the information content is mainly determined by the total degree of freedom N_{DOF}, including the degrees of freedom of space N_{s}, wavelength N_{c}, time N_{t}, and polarization N_{p}, and their relationship can be expressed as

$$N_{\mathrm{DOF}} = N_{\mathrm{s}} N_{\mathrm{c}} N_{\mathrm{t}} N_{\mathrm{p}} \tag{2}$$

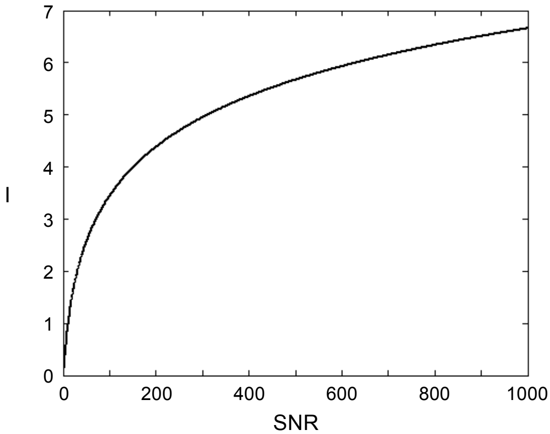

Fig. 1. The relationship between information content I and SNR.

Consider only static objects in the polarized light field, that is $N_t = 1$ and $N_p = 1$, then

$$N_{\mathrm{DOF}} = N_s N_c \tag{3}$$

and

$$N_c = \Delta\lambda\,\delta \tag{4}$$

where N_s represents the degree of freedom of the light field illuminated by the completely coherent light, and the $\Delta\lambda$ is wavelength bandwidth, δ is resolution of wavelength [13]. It follows that there is a number of independent coherent signals on the light field when illuminated by incoherent light or partially coherent light, and the number of independent coherent signals is exactly the degree of freedom of wavelength N_c of the light source.

For the convenience of the narrative, a schematic diagram for transmission of the degree of freedom of the light field in an optical imaging system has been shown in Fig. 2. In Fig. 2, the illumination field on the object surface A was produced by an extended quasi-monochromatic incoherent light source σ, and then, after it is transmitted or reflected by the object, the object light reaches the image plane A' passing through

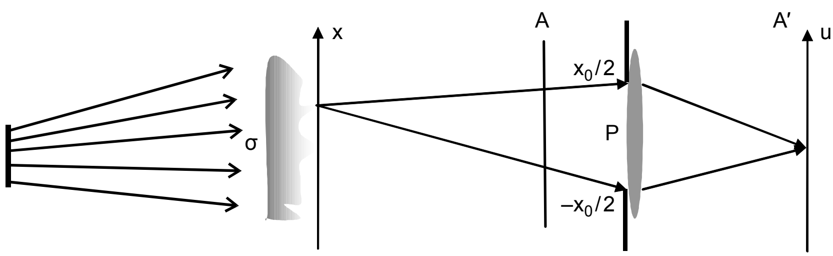

Fig. 2. Schematic diagram for transmission of the degree of freedom of the light field in an optical imaging system.

the pupil of x_0. So N_c, N_s and N_{DOF} in the Eq. (3) represent the number of independent coherent signals on the plane A, the degree of freedom of the light field on the plane A' under completely coherent illumination, and the total degree of freedom of the light field on the plane A' illuminated by incoherent light, respectively.

In the view of von Laue [14], the spatial degree of freedom of a coherent light field is proportional to the product of the area of the object (or image) and the frequency bandwidth of the optical system. Assuming that the spatial cut-off frequency in the x- and y-directions are $f_{x_{max}}$ and $f_{y_{max}}$, respectively, and the area of the object is S, the maximum number of the degree of freedom of the light field which can reach the image plane passing through the image system is

$$N_s = 4 f_x f_y S \tag{5}$$

Therefore, the number N_c of independent coherent signals of the light field distributed on the plane A is the key to obtain the information content of the light field on the image plane illuminated by incoherent light.

3. The mutual intensity of light field on the object surface under the incoherent illumination

In accordance with the partially coherent theories, the coherence degree of any two points in the light field can be represented by their mutual intensity [15].

Consider the mutual intensity of points P_1 and P_2 on a screen A illuminated by the extended quasi-monochromatic incoherent light source σ as shown in Fig. 3. In Fig. 3, σ is taken to be a part of a plane parallel to screen A, and the medium between the source and the screen is assumed to be homogeneous. Let z_1 and z_2 be the distances between a typical point S on the light source and the points P_1 and P_2, respectively, and $\bar{k} = 2\pi/\lambda$

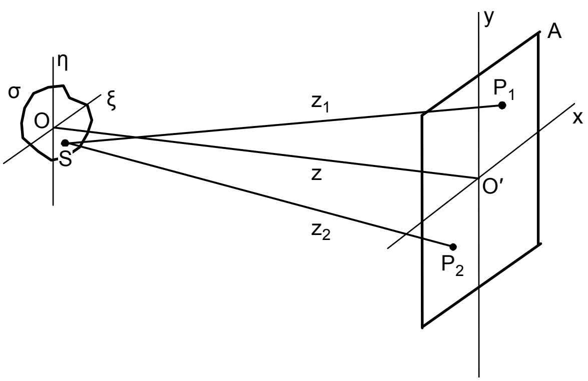

Fig. 3. Illustrating the distribution of mutual intensity of the light field under incoherent illumination (see text for explanation).

denotes the wave number in the medium. Based on the theory of Van Cittert–Zernike, the mutual intensity of points P_1 and P_2 on the screen A is

$$J(P_1, P_2) = \int_\sigma I(\xi, \eta) \frac{\exp\left[i\bar{k}(z_1 - z_2)\right]}{z_1 z_2} \mathrm{d}(\xi, \eta) \tag{6}$$

where (ξ, η) is the coordinate of the sample point S on the light source. Assuming that the linear dimensions of σ and the distance between P_1 and P_2 are all much smaller than the distance z between the source and the screen A, the Eq. (6) can be rewritten in the form

$$J_{12} = \iint_\sigma I(\xi, \eta) \exp\left[-i\bar{k}\left(\frac{x_1 - x_2}{z}\xi + \frac{y_1 - y_2}{z}\eta\right)\right] \mathrm{d}\xi \, \mathrm{d}\eta \tag{7}$$

And the corresponding complex degree of coherence is

$$j_{12} = \frac{\iint_\sigma I(\xi, \eta) \exp\left[-i\bar{k}\left(\frac{x_1 - x_2}{z}\xi + \frac{y_1 - y_2}{z}\eta\right)\right] \mathrm{d}\xi \, \mathrm{d}\eta}{\iint_\sigma I(\xi, \eta) \mathrm{d}\xi \, \mathrm{d}\eta} \tag{8}$$

Obviously, it can be seen from Eq. (8) that incoherent light becomes partially coherent after propagating a certain distance and the coherence degree of any two points on the screen A is related to the wavelength, the propagation distance and the size of the light source. When the wavelength and propagation distance are constant, the smaller the linear dimensions of the light source are, the greater the coherence degree of any two points on the screen A is. Therefore, it can be inferred that approximated completely coherent illumination can be formed on a certain area of illumination surface when the size of an incoherent light source is small enough.

4. Determination of the number N_c of independent coherent signals of the light field distributed on the object surface

From the above analysis, the light field on the object surface is partially coherent when illuminated by an incoherent light. It means that there are several independent coherent signals distributed on the object surface and each independent coherent signal will propagate independently to the image plane. In the following part a way is shown to obtain the number of independent coherent signals distributed on the object surface.

The mutual intensity of any two points on the object surface, which has been illuminated by the incoherent light is given in Eq. (8). So, thinking reversely, if completely coherent illumination is required to be formed on the whole object surface, the size of the incoherent light source must be limited to a certain value. As a result, the value is exactly the maximum allowed size for a little incoherent light source which can meet completely coherent illumination on the whole object surface. For the description convenience, the little incoherent light source with the maximum allowed size is called the coherent wavelet source. Since each coherent wavelet source forms completely coherent illumination on the object surface, and the light field formed by different coherent wavelet source is independent of each other, a coherent wavelet source corresponds to an independent coherent signal. So the number of independent coherent signals on the object surface is the same as the number of coherent wavelet sources, and the number of coherent wavelet sources can be made out by dividing the whole illumination source by a coherent wavelet source size.

Take a 1D linear light source and a 2D circular light source for examples as follows.

4.1. One-dimensional linear light source

For a uniform 1D linear light source of width $2a$ with its center at O (see Fig. 3), the complex degree of coherence of point P_1 and P_2 is, according to the Eq. (8), given by

$$j(x_1, x_2) = \frac{1}{2a}\int_{-a}^{a} \exp(-i\bar{k}P\xi)\mathrm{d}\xi = -\frac{1}{kpa}\sin\left[\frac{ka}{z}(x_1 - x_2)\right] \tag{9}$$

where $\bar{k} = 2\pi/\lambda$, $P = (x_1 - x_2)/z$, and z is the distance from the light source to the object, x_1 and x_2 are coordinates of P_1 and P_2, respectively.

Assuming that the complex degree of coherence of a complete coherence region is set to be in the range of $|j(x_1, x_2)| \le 0.88$ [15], that is, the maximum permissible deviation of the complex degree of coherence is 12%, the distance between x_1 and x_2 is

$$x_1 - x_2 = \frac{\lambda z}{2.4\pi a} \tag{10}$$

Reversely, the maximum length of the coherent wavelet source which can form coherent illumination on the whole object surface of length d in the x axis direction is

$$a'_{\max} = \frac{\lambda z}{2.4\pi d} \tag{11}$$

So the number of independent coherent signals on the object surface illuminated by 1D linear light source of width $2a$ is

$$N_{\mathrm{c}} = \frac{2a}{a'_{\max}} = \frac{4.8a\pi d}{z\bar{\lambda}} \tag{12}$$

4.2. Two-dimensional circular light source

For a uniform circular source of radius R with its center at O (see Fig. 3), the complex degree of coherence of point P_1 and P_2 is, according to Eq. (8), given by

$$j_{12}(P_1, P_2) = \frac{2J_1(v)}{v} \tag{13}$$

where

$$v = \frac{2\pi R}{\bar{\lambda} z}\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

and $J_1(\cdot)$ is the first order Bessel functions of the first kind, z is the distance from the light source to the object, and (x_1, y_1) and (x_2, y_2) are coordinates of P_1 and P_2, respectively.

According to Eq. (13), $|j_{12}(P_1, P_2)|$ decreases steadily from the value unity to zero when $v = 0$ to $v = 3.83$. That is, the coherence degree steadily decreases as the points P_1 and P_2 separated more and more. For example, $|j_{12}(P_1, P_2)|$ decreases to 0.88 when $v = 1$, *i.e.*, when

$$\left|\overline{P_1 P_2}\right| = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \frac{0.16\bar{\lambda} z}{R} \tag{14}$$

As the same process as above is in the case of 1D linear light source, regarding the maximum permissible deviation of the complex degree of coherence to be 12% [15], the diameter of the circular area on the object surface which is illuminated almost coherently by the extended quasi-monochromatic incoherent light is $0.16\bar{\lambda} z/R$.

Reversely, the maximum radius of the coherent wavelet source which can form coherent illumination on the whole object surface of diameter d can be obtained from Eq. (14), that is

$$r_{\max} = \frac{0.16\bar{\lambda} z}{d} \tag{15}$$

As a result, the amount of coherent wavelet sources on the circular light source is

$$N_c = \frac{\pi\rho^2}{\pi r^2} = \frac{\rho^2 d^2}{0.0256\bar{\lambda}^2 z^2} \tag{16}$$

and it is exactly the number of independent coherent signals on the object surface.

5. Conclusion

The research for the information content of light field is one of the most basic topics in all-optical phenomena or optical systems. A novel approach to the information content of light field on the image plane illuminated by incoherent light is provided. Based on Shannon's information theory, the number of degrees of freedom of light field on the image plane can be regarded as the product of the number of independent coherent signals formed on the object surface under the incoherent illumination and the number of degrees of freedom of the object itself transmitted through the optical system when illuminated by the completely coherent light source. Therefore, firstly, we put forward a hypothetical concept named coherent wavelet source on an incoherent light source which can form an independent coherent signal (completely coherent illumination) on the object surface based on the expression of mutual intensity on the object surface. And the number of independent coherent signals formed on the object surface can be obtained by dividing the whole illumination source by the size of coherent wavelet source since each coherent wavelet source corresponds to an independent coherent signal. Then combined with the spatial degree of freedom of object itself transmitted through the optical system when illuminated by the completely coherent light source, information content of the optical field illuminated by an incoherent light source can be achieved. The algorithms for the information content of the imaging light illuminated by a 1D linear light source and a 2D circular light source are also presented.

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