Tchórzewska-Cieślak Barbara
Rzeszow University of Technology, Poland

Model of risk of water mains failure using fuzzy logic

Keywords

risk, safety, water supply, fuzzy logic

Abstract

Water supply network is an essential element of urban water supply systems. The operation of a water supply system is inseparably connected with a risk of failure. The main problem in the risk of failure analysis of water mains is the uncertainty of the information collected on the description of failure. In order to consider the uncertainty of information, the theory of fuzzy sets was used. The fuzzification of frequency, severity and the consequences of the incident scenario is basic input for fuzzy risk analysis. The presented model is part of a complex model of risk management of failures in water mains and can be used in practice in system operator's decision-making process. An adaptation of the fuzzy set theory to analyse risk of failure of water mains is not a standard approach. An effect of the analysis of different sources of risk can be used for the design of a more reliable safety system assurance.

1.Introduction

A water supply system (WSS) belongs to the critical infrastructure of cities, and it should be a priority task for waterworks and even for the local authorities to ensure the suitable level of its safety. Its aim is to supply consumers with a required amount of water, with a specific pressure and a specific quality, according to binding standards, and at an acceptable price. Modelling the risk of failure in water supply network consists of three main tasks:

- assessment (estimation) of the frequency/ probability of emergency scenarios (undesirable events),
- assessment (estimation) of various consequences of emergency scenarios (undesirable events),
- estimation of water mains protection level and the various types of protection minimizing the possible consequences of emergency scenarios (undesirable events).

The case that occurs most frequently in the risk analysis is a statistical uncertainty caused by the random nature of the studied phenomenon, the influence of external factors, as well as the time factor that determines a change of analysed undesirable event (failure) [7],[15],[18].

In many cases, data on failures of water mains are obtained from experts (water supply system operators, engineers or researchers).
These data are often imprecise and incomplete. The following data, among others, are necessary to perform risk analysis in the WSS [3],:

- data identifying the analysed object (e.g. water treatment station, distribution pipeline), the name and type of the object and its basic technical data,
- data about failures (undesirable events), repairs and other breaks in the WSS's operation (information about the date, time and duration of failure, and a description of the failure),
- data relating to the reasons behind the occurrence of undesirable events,
- data relating to the consequences of these events.

The main aim of this study is to present a risk analysis model using fuzzy set theory and the application of this theory in the risk management process in water network.

2. The risk of failure of water mains

Risk assessment includes the so called risk analysis, which is the process aimed at the determination of the consequences of failures (undesirable events) in the WSS, their extend, sources of their occurrence and the assessment of the risk levels. Haimes [4]-[6]

suggests that risk assessment concerns its reasons, as well as its likelihood and consequences.

Drinking water infrastructure system uncertainty or risk is defined as the likelihood or probability that the drinking water service fails to provide water on-demand to its customers [3].

For purposes of this paper, operational reliability of the WSS is defined as the ability to supply a constant flow of water for various groups of consumers, with a specific quality and a specific pressure, according to consumer demands, in specific operational conditions, at any or at a specific time.

Failure is defined as the event in which the system fails to function with respect to its desired objectives.

Safety of the WSS means the ability of the system to safely execute its functions in a given environment. The measure of WSS safety is risk.

Risk (r) is a function of the parameters: the probability or frequency (f_i) that representative emergency scenario occurs (RES), the magnitude of losses (C_j) caused by RES and the degree of sensitivity (E_k) to RES, according to equation (1).

$$r = \sum_{RES=1}^{N} (f_i \cdot C_j \cdot E_k) \qquad (1)$$

where:

RES- a series of the successive undesirable events (failures),

f_i - a point value depending on the frequency of RES or a single failure,

i- a number of the scale for the frequency,

C_j--a point value of losses caused by RES or a single failure,

j- a number of the scale for the losses,

E_k–a point value for the parameter of exposure (sensitivity of water mains) associated with RES or a single failure,

k- a number of the scale for the sensitivity,

N-number of *RES*.

To analyse risk defined in this way the matrix methods can be used [13]-[14]. According to equation (1) the qualitative risk matrix was developed, assuming a descriptive point scale for the particular risk parameters. Depending on the frequency of a given failure the point weights for the parameter *f* are presented in *Table 1*.

Table1. Criteria for a descriptive point scale for the parameter f_i.*(i=1,2,3,4,5)*.

Point weight f_i	Probability of failure	The average frequency of failure [1/year]
f_1=1	improbability, once in 10 years and less often	0.1
f_2=2	very low probability, once in (5÷10) years	0.2
f_3=3	low probability, once in (2÷5)years	0.5
f_4=4	medium probability, once in (0.5÷1) year	1
f_5=5	probability, once in(1÷6) months	2
f_6=6	high probability, once a (¼ ÷ 1) month	12
f_7=7	very high probability, once a week and more often	56

The criteria and the point weights for the assumed descriptive point scale for the parameter of losses C_j and sensitivity E_k are presented in *Tables 2* and *Table 3*.

Table 2. Criteria for a descriptive point scale for the parameter C_j, *(j=1,2,3)*.

Point weight C_j	Description
C_1=1	small losses : • perceptible organoleptic changes in water, • isolated consumer complaints, • financial losses up to $5 \cdot 10^3$EUR
C_2=2	medium losses: • considerable organoleptic problems (odour, changed colour and turbidity), • consumer health problems, numerous complaints, • information in local public media, financial losses up to 10^5 EUR
C_3=3	large losses: • endangered people require hospitalisation, • professional rescue teams involved, serious toxic effects in test organisms, • information in nationwide media, financial losses over 10^5 EUR

Table 3. Criteria for a descriptive point scale for the parameter E_k, $(k=\{1,2,3\})$.

Point weight E_k	Description
$E_1=1$	small sensitivity to failure (high resistance): • the network in the ring system, • the ability to cut off the damaged section of the network by means of gates (for repair), • the ability to avoid interruptions in water supply to customers, • full monitoring of water mains, continuous measurements of pressure and flow rate at strategic points of the network covering the entire area of water supply, utilising SCADA and GIS software, the possibility to remote control network hydraulic parameters
$E_2=2$	medium sensitivity to failure: • the network in the radial or mixed system, • the possibility to cut off the damaged section of the network by means of gates, but the network capacity limits water supply to customers, • water mains standard monitoring, measurements of pressure and flow rate
$E_3=3$	large sensitivity to failure (low resistance): • the network in the radial system, • the inability to cut off the damaged section of the network by means of gates (for repair) without interrupting water supply to customers, • limited water mains monitoring

The risk *r* calculated from equation (1), for a single RES, takes values within the range [1÷63].
Risk values are presented in *Table 4*, according to the three-parameter matrix and equation (1) [2],[19].

Table 4. Risk value according to Equation 1 (the risk matrix).

f_i	$E_k=1$			$E_k=2$			$E_k=3$		
	C_j			C_j			C_j		
	1	2	3	1	2	3	1	2	3
	r								
1	1	2	3	2	4	6	3	6	9
2	2	4	6	4	8	12	6	12	18
3	3	6	9	6	12	18	9	18	27
4	4	8	12	8	16	24	12	24	36
5	5	10	15	10	20	30	15	30	45
6	6	12	18	12	24	36	18	36	54
7	7	14	21	14	28	42	21	42	63

Table 5 presents the five-step scale of risk .

Table 5. Scale of risk.

Scale r_l	Risk category	Point scale range
1	negligible risk (Nr)	[1÷6]
2	tolerable risk (Tr)	(6÷10]
3	controlled risk (Cr)	(10÷20]
4	intolerable risk(Ir)	(20÷27]
5	unacceptable risk(Ur)	(27÷63]

3. The use of fuzzy set theory in the analysis of risk of water mains failure

The notion of fuzzy sets was introduced in 1965 by LA Zadeh of the University of Berkeley [21]. Unlike in the classical set theory, the limit of the fuzzy set is not precisely determined, but there is a gradual transition from non-membership of elements in a set, through their partial membership, to membership. This gradual transition is described by the so called membership function $\mu_{A,}$ where A is a set of fuzzy numbers. Risk analysis is based largely on expert opinions and uses such linguistic terms as small losses, high risk and can be described by means of fuzzy sets [1], [8]-[11],[16]-[17]. For risk analysis of water mains failure the membership function class type *t* (a triangular function) according to equation (2), the membership function class type γ according to equation (3) and the membership function class type *L* according to equation (4), were proposed [2],[[20]-[22].

$$\mu_A(x,a,b,c)=\begin{cases} 0 & for \quad x \le a \\ \dfrac{x-a}{b-a} & for \ a \le x \le b \\ \dfrac{c-x}{c-b} & for \ b \le x \le c \\ 0 & for \quad x \ge c \end{cases} \tag{2}$$

$$\mu_A(x,a,b)=\begin{cases} 0 & for \quad x \le a \\ \dfrac{x-a}{b-a} & for \ a \le x \le b \\ 1 & for \quad x > b \end{cases} \tag{3}$$

$$\mu_A(x,a,b)=\begin{cases}1 & \text{for} \quad x \le a \\ \frac{b-x}{b-a} & \text{for } a < x \le b \\ 0 & \text{for} \quad x > b\end{cases} \quad (4)$$

where:
x- variable, parameter value,
μ_A- the membership function of variable x in the fuzzy set A ,
a, b, c -the membership function parameters (minimal, median (central) and maximum value of fuzzy number),

For the frequency parameter the set of possible linguistic characterization is defined as:

$\overline{F}=\{f_i\}$, i={1,2,3,4,5,6,7}.

Table 6 shows the linguistic characterization, type and parameters of membership function.

Table 6. The linguistic characterization, type and parameters of membership function, for *f* parameter, $\overline{F}=\{f_1,f_2,f_3,f_4,f_5,f_6,f_7\}$.

i	linguistic characterization	type of membership function	membership function parameters		
			a	b	c
1	improbability	type *L, acc.to eq.(4)*	0.1	0.2	-
2	very low probability	triangular *t, acc.to eq.(2)*	0.1	0.2	0.5
3	low probability	triangular *t, acc.to eq.(2)*	0.2	0.5	1.0
4	medium probability	triangular *t, acc.to eq.(2)*	0.5	1.0	1.5
5	probability	triangular *t, acc.to eq.(2)*	1.0	1.5	2.0
6	high probability	triangular *t, acc.to eq.(2)*	1.5	2.0	12
7	very high probability	type γ, *acc.to eq.(3)*	2.0	12	-

Figure 1 shows forms of membership function for the *f* parameter.

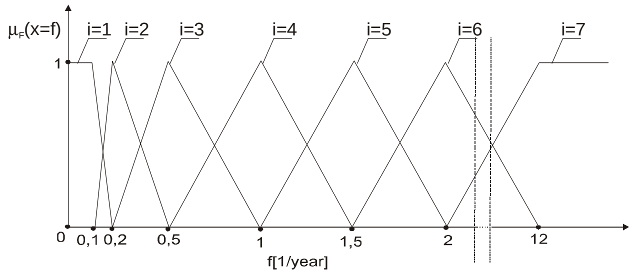

Figure 1. Form of membership function for the parameter f

For i=1 the membership function belongs to a class type *L*, and is defined as:

$$\mu_F(x=f,0.1,0.2)=\begin{cases}1 & \text{for} \quad x \le 0.1 \\ \frac{0.2-x}{0.1} & \text{for } 0.1 < x \le 0.2 \\ 0 & \text{for} \quad x > 0.2\end{cases}$$

For i=2 the membership function belongs to a class type *t* and is defined as:

$$\mu_F(x=f,0.1,0.2,0.5)=\begin{cases}0 & \text{for} \quad x \le 0.1 \\ \frac{x-0.1}{0.1} & \text{for } 0.1 \le x \le 0.2 \\ \frac{0.5-x}{0.3} & \text{for } 0.2 \le x \le 0.5 \\ 0 & \text{for} \quad x \ge 0.5\end{cases}$$

For i=3 the membership function belongs to a class type *t* and is defined as:

$$\mu_F(x=f,0.2,0.5,1.0)=\begin{cases}0 & \text{for} \quad x \le 0.2 \\ \frac{x-0.2}{0.3} & \text{for } 0.2 < x \le 0.5 \\ 2(1-x) & \text{for } 0.5 < x \le 1.0 \\ 0 & \text{for} \quad x > 5\end{cases}$$

For i=4 the membership function belongs to a class type *t* and is defined as

$$\mu_F(x=f,0.5,1.0,1.5)=\begin{cases}0 & \text{for} \quad x \le 0.5 \\ 2x-1 & \text{for } 0.5 < x \le 1.0 \\ 3-2x & \text{for } 1.0 < x \le 1.5 \\ 0 & \text{for} \quad x > 1.5\end{cases}$$

For i=5 the membership function belongs to a class type *t* and is defined as

$$\mu_F(x=f,1.0,1.5,2.0)=\begin{cases}0 & \text{for} \quad x \le 1.0 \\ 2x-2 & \text{for } 1.0 < x \le 1.5 \\ 4-2x & \text{for } 1.5 < x \le 2.0 \\ 0 & \text{for} \quad x > 2.0\end{cases}$$

For i=6 the membership function belongs to a class type *t* and is defined as:

$$\mu_F(x=f,1.5,2.0,12.0)=\begin{cases}0 & \text{for } x\le 1.5\\ 2x-3 & \text{for } 1.5<x\le 2.0\\ \dfrac{12-x}{10} & \text{for } 2.0<x\le 12.0\\ 0 & \text{for } x>12.0\end{cases}$$

For i=7 the membership function belongs to a class type γ and is defined as:

$$\mu_F(x=f,2.0,12.0)=\begin{cases}0 & \text{for } x\le 2.0\\ \dfrac{x-2}{10} & \text{for } 2.0<x\le 12.0\\ 1 & \text{for } x>12.0\end{cases}$$

For the losses parameter the set of possible linguistic characterization is defined as:

$\overline{C}=\{C_j\}$, j={1,2,3}.

Table 7 shows the linguistic characterization, type and parameters of membership function for C parameter.

Table 7. The linguistic characterization, type and parameters of membership function, for *C* parameter, $\overline{C}=\{C_1,C_2,C_3,\}$.

j	linguistic characterization	type of membership function	membership function parameters		
			a	b	c
1	small	triangular *t, acc. to eq.(2)*	0.0	0.0	1.5
2	medium	triangular *t, acc. to eq.(2)*	0.5	1.5	2.5
3	large	triangular *t, acc. to eq.(2)*	1.5	3.0	3.0

For the sensitivity parameter the set of possible linguistic characterization is defined as:

$\overline{E}=\{E_j\}$, j={1,2,3}.

Table 8 shows the possible linguistic characterization for the sensitivity parameter E, type and parameters of membership function.

Table 8. The linguistic characterization, type and parameters of membership function, for *E* parameter, $\overline{E}=\{E_1,E_2,E_3,\}$.

k	linguistic characterization	type of membership function	membership function parameters		
			a	b	c
1	small	triangular *t, acc. to eq.(2))*	0.0	0.0	1.5
2	medium	triangular *t, acc. to eq.(2)*	0.5	1.5	2.5
3	large	triangular *t, acc. to eq.(2)*	1.5	3.0	3.0

Figure 2 shows forms of membership function for the parameters *C* and *E*.

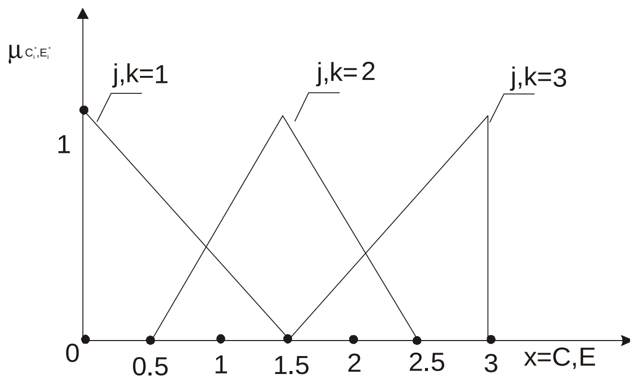

Figure 2. Form of membership function for the parameters E and C.

For j,k=1 the membership function belongs to a class type *t* and is defined as:

$$\mu_{\overline{C},\overline{E}}(x=C,E,0,0,1.5)=\begin{cases}0 & \text{for } x=0\\ \dfrac{1.5-x}{1.5} & \text{for } 0\le x\le 1.5\\ 0 & \text{for } x\ge 1.5\end{cases}$$

For j,k=2 the membership function belongs to a class type *t* and is defined as:

$$\mu_{\overline{C},\overline{E}}(x=C,E,0.5,1.5,2.5)=\begin{cases}0 & \text{for } x=0.5\\ x-0.5 & \text{for } 0.5\le x\le 1.5\\ 2.5-x & \text{for } 1.5\le x\le 2.5\\ 0 & \text{for } x\ge 2.5\end{cases}$$

For j,k=3 the membership function belongs to a class type *t* and is defined as:

$$\mu_{\overline{C,E}}(x=C,E,1.5,3,3)=\begin{cases}0 & for \quad x=1.5\\ \dfrac{x-1.5}{1.5} & for\ 1.5\le x\le 3.0\\ 0 & for \quad x\ge 3.0\end{cases}$$

For risk the set of possible linguistic characterization is defined as:

$\overline{R}=\{r_l\}$, $l=\{1,2,3,4,5\}$.

Table 9 shows the linguistic characterization of risk, type and parameters of membership function.

Table9. The linguistic characterization, type and parameters of membership function, for risk, $\overline{R}=\{r_1,r_2,r_3,r_4,r_5\}$.

l	linguistic characterization	type of membership function	membership function parameters		
			a	b	c
1	negligible risk (Nr)	triangular *t, acc eq.(2)*	0.0	0.0	15.75
2	tolerable risk (Tr)	triangular *t, acc eq.(2)*	0.0	15.75	31.5
3	controlled risk (Cr)	triangular *t, acc eq.(2)*	15.75	31.5	47.25
4	intolerable risk(Ir)	triangular *t, acc eq.(2)*	31.5	47.25	63
5	unacceptable risk(Ur)	triangular *t, acc eq.(2)*	47.25	63	63

Figure 3 shows forms of membership function for risk.

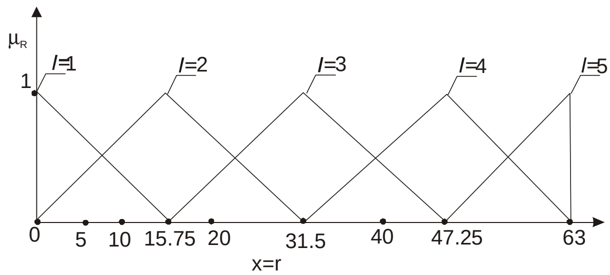

Figure 3. Form of membership function for risk r..

For l=1 the membership function belongs to a class type t and is defined as:

$$\mu_R(x=r,0,0,15.75)=\begin{cases}0 & for \quad x=0\\ \dfrac{15.75-x}{15.75} & for\ 0\le x\le 15.75\\ 0 & for \quad x\ge 15.75\end{cases}$$

For l=2,3,4 the membership function belongs to a class type t and is defined as:

$$\mu_R(x=r,a,b,c)=\begin{cases}0 & for \quad 0\le x\le 15.75(l-2),\\ \dfrac{x}{15.75}-(l-2) & for\ 15.75(l-2)\le x\le 15.75(l-1)\\ l-\dfrac{x}{15.75} & for\ 15.75(l-1)\le x\le 15.75\cdot l\\ 0 & for \quad 15.75\cdot l\le x\le 63\end{cases}$$

For l=5 the membership function belongs to a class type t and is defined as:

$$\mu_R(x=r,47.25,63,63)=\begin{cases}0 & for \quad x\le 47.25\\ \dfrac{x-47.25}{15.75} & for\ 47.25<x<63\\ 0 & for \quad x\ge 63\end{cases}$$

4. The decision model

Decision-making tools help in the selection of prudent, technically feasible, and scientifically justifiable actions to protect the environment and human health in a cost-effective way [16]-[19].

A fuzzy decision model calculates the output value based on the multiple input values. The model does not analyse the exact values of the arguments, only their degrees of membership in fuzzy sets, and on this basis the output value, being a base for the decision making process, is determined.

The Mamdani – Zadeh [12] type decision model was proposed for risk management process of water mains failure[7].

The input base of the proposed model consists of three values of risk parameters: the frequency of failure in water mains $x_1(f_i)$, losses associated with the occurrence of failure $x_2(C_j)$, and a degree of exposure (resistance) to failure $x_3(E_k)$.

The output of the model, which allows making an operational decision, is the index risk value for water mains failure $y\ (r_l)$.

The model consists of the following main blocks:

- The fuzzification block, which converts a vector of numbers (the crisp input values of risk parameters) into a vector of degrees of membership (a singleton method was used) [11].

- The block of rules, which provides a knowledge base for qualitative knowledge and consists in determining the relationship between the particular parameters of the model.

A rule base determines the relationships between the inputs and outputs of a system using linguistic *antecedent* and *consequent* propositions in a set of IF-THEN rules. The rule base of a complex system usually requires a large number of rules to describe the behaviour of a system for all possible values of the input variables. The base of rules contains the

logical rules which determine cause and effect relationship between the particular risk parameters in water mains.
Based on the risk matrix shown in Table 4, the base of rules was determined. It is a set of rules:
$Rl = (Rl_1, Rl_2, \ldots Rl_{63})$, in a general form:

If frequency is f_i and possible consequences are C_j and sensitivity is E_k then risk is r_l

where: if – premise, then- conclusion.

• The inference block– the determination of a fuzzy conclusion model in form of the resulting membership function. In this block all rules whose premises are satisfied, are activated.
Generally speaking, on the basis of premises we find the appropriate fuzzy set, which is the conclusion of the adopted fuzzy rules (*FRl*). After the aggregation of rules into five groups (five categories of risk), the global rule is determined as follows:

$$FRl(f_i,C_j,E_k;r_l)=FRl_1(f_i,C_j,E_k;r_l)\Delta FRl_2(f_i,C_j,E_k;r_l)\ldots \Delta FRl_M(f_i,C_j,E_k;r_l)$$

where Δ is an operator (S-norms) [20].

In the process of aggregation a degree of fulfilment of each rule is calculated based on the degree of fulfilment of its premises. For this purpose, the fuzzy logic operations: (*and, or*), are used. Based on the presented base of rules, the inference min-max, using the operator S-norms and T-norms, was proposed [2],[20]. The aggregating output membership function of a resultant output fuzzy risk category is expressed as:

$$\mu_R(r_l) = \max_m\{\min \mu_F^m(f_i), \mu_C^m(C_j), \mu_E^m(E_k), \mu_R^m(r_l)\}$$

where *m* is the number of rules, *i* the number of fuzzy frequency sets, *j* the number of loss parameter sets, *k* the number of sensitivity and *l* is the number of fuzzy risk sets.

• The defuzzification block, whose aim is to obtain a specific value of risk.
This process is the final stage of the model and provides the basis for the water supply system operator's decision-making process. For example, if the risk value corresponds to the category of unacceptable risk an operator undertakes some measures to reduce risk of failure (water mains modernization). The transformation of fuzzy set into not fuzzy value (determined) can be performed by various methods[2]. For the proposed model the centre of gravity method was used [2]:

$$r = \frac{\sum_{l=1}^{5} r_l \cdot \mu_l(r_l)}{\sum_{l=1}^{5} \mu_l(r_l)} \qquad (5)$$

A diagram of the proposed model is shown in *Figure 4*

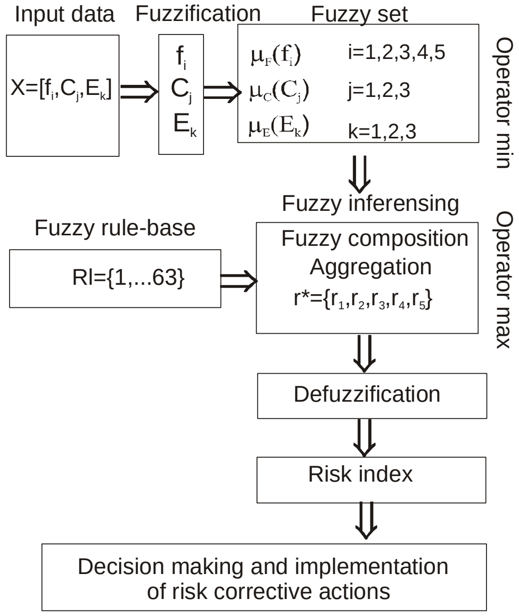

Figure 4. A diagram of the decision model for the risk analysis of water mains

5.The application example

Using the operating data of water supply system in a city with population of 200 thousand, the risk analysis of water mains failure, using the fuzzy software, was performed. Data on water main failures for five years of water supply network operation were collected and analysed in terms of frequency of failures and their consequences.,
Risk assessment was performed for three diameter ranges:

- up to ϕ150mm,
- ϕ(150-400)mm,
- ϕ> 400 mm.

In *Table 10* the results of the analysis are presented.

Table 10. Risk assessment for the analysed water mains.

ϕ [mm]	f_{medium}	C (est.)	E (est.)	Risk value	Risk category
up to 150	11.4	0.51	2.47	38.70	Ur
150-400	3.95	1.17	1,63	29.20	Ur

≥ 400	0,36	2.32	0.27	17.40	Cr

6. Conclusions

- Modern operation of urban water supply systems requires expanded safety management systems, based on models of risk management. It is caused by the fact that water supply systems belong to the critical infrastructure, which affects the health of the consumers.
- Now, however, new trends in the technical management of water supply systems are heavily focused on security issues related to the design, operation and management of these systems. Of particular interest in the future will be the evaluation of risk and reliability issues of the various components, subsystems and the systems as a whole, from the viewpoint of their susceptibility to terrorism.
- Effective risk management requires operators to monitor actively the entire water supply system, to develop emergency plans, to response fast in emergency, and to be able to analyse and assess risk.
- One of the methods to deal with uncertainties in risk assessment is a fuzzy logic where fuzzy sets are a fundamental issue.
- In the study the application of fuzzy logic theory to analyse risk of failure of water mains was proposed. In case of having inaccurate or various (eg from different experts) data on particular risk parameters, there is the possibility to describe them by a linguistic variables.
- In contrast to the traditional risk analysis, all variables of the risk parameters (according to equation (1)) are expressed in fuzzy sets defined by appropriate membership functions
- The probability or frequency of failures and their possible consequences can be defined as fuzzy values, particularly when they are estimated and not precisely determined, which often occurs at the analysis of failures in water supply network.
- The decision model presented in the study, based on assumptions of Mamdani's fuzzy modelling, may be used in practice in water mains as an element of a complex management of risk of failures of water mains.
- A certain limitation of the proposed method is the need to develop a database of the rules, based on the knowledge of experts, whose opinions on the assumed criteria may differ from each other.
- In order to develop the complete and most reliable database of the rules (the knowledge base), as much information as possible about failures of water mains, their possible consequences and causes, should be collected.
- Proposed method provides an alternative to other methods for assessing and managing risk of water network failure (subjective probability theory, mathematical theory of records) and its use is justified if you have a subjective assessment of risk parameters.

References

[1] Braglia, M. & Frosolini, M., & Montanari, R. (2003). Fuzzy criticality assessment model for failure modes and effects analysis. *International Journal of Quality & Reliability Management*, 20(4), 503–524.

[2] Dubois, D., & Prade, H. (1980). *Fuzzy sets and systems: Theory and application*. New York: Academic Press.

[3] Ezell, B. & Farr, J. & Wiese, I. (2000). Infrastructure risk analysis of municipal water distribution system. *Journal of Infrastructure Systems*, ASCE. 6(3) ,118–122.

[4] Haimes, Y.Y., Moser, D. & Stakhin, E. Risk Based Decision Making in Water Resources *Journal of Infrastructure Systems*, ASCE, 12, 401-415.

[5] Haimes, Y.Y. (1998). *Risk Modeling, Assessment and Management*. Wiley, New York.

[6] Haimes, Y.Y. (2009). On the Complex definition of risk: a systems-based approach. *Risk Analisys* 29(12), 1647-1654.

[7] Hubbard D.W. (2009). *The failure of risk management*. Wiley. New York.2009

[8] Karwowski, W. & Mital, A. (1986). Potential applications of fuzzy sets in industrial safety engineering. *Fuzzy Sets System*. 19, 105–120.

[9] Kleiner, Y. & Rajani, B.B. & Sadiq, R. (2006). Failure risk management of buried infrastructure using fuzzy-based techniques. *Journal of Water Supply Research and Technology: Aqua*, 55 (2), 81-94.

[10] Kleiner, Y. (2004). A fuzzy based method of soil corrosivity evaluation for predicting water main deterioration. *Journal of Infrastructure Systems*, *ASCE*.10(4),149-156

[11] Lee, H.M. (1996). Applying fuzzy set theory to evaluate the rate of aggregative risk in software development. *Fuzzy Sets and Systems*, 79, 323-336

[12] Mamdani, E.H. (1977). Application of fuzzy logic to approximate reasoning using linguistic systems. *Fuzzy Sets and Systems*, 26, 1182-1191.

[13] Markowski, A. & Mannan, S. (2008). Fuzzy risk matrix. *Journal of Hazardous Materials, 59(1,)152-*
[14] Rak, J. & Tchórzewska-Cieślak, B. (2006). Review of matrix methods for risk assessment in water supply system. *Journal of Konbin*, 1(1), 67-76.
[15] Sadig, R., Kleiner, Y., & Rajani, B. (2007). Water quality failures in distribution networks-risk analysis using fuzzy logic and evidential reasoning. *Risk Analysis*.27(5),1381-1394.
[16] Sadiq, R., & Tesfamariam, S. (2009). Environmental decision-making under uncertainty using intuitionistic fuzzy analytic hierarchy process (IF-AHP). *Stochastic Environmental Research and Risk Assessment*, 23(1), 75-91.
[17] Shang-Lien, L. & Ruei-Shan, L. (2002). Diagnosing reservoir water quality using self-organizing maps and fuzzy theory. *Water Reserch*.36 ,265-2274.
[18] Tchórzewska-Cieślak B. (2007). Method of assessing of risk of failure in water supply system. European safety and reliability conference ESREL. *Risk, Reliability and Societal Safety*. Taylor & Francis, 2.1535-1539, Norway, Stavanger.
[19] Tchórzewska-Cieślak, B, (2009) Water supply system reliability management. *Environmental Protection Engineering*. 35 ,29-35.
[20] Yager, R.R. (2004). On determination of strength of belief for decision support under uncertainty-Part II: fusing strengths of belief. *Fuzzy sets and systems.* 142. 129-142.
[21] Zadeh, L. A. (1965). Fuzzy sets. *Information and Control*, 8, 338–353.
[22] Zio, E. (2009). *Computational Methods for Reliability and Risk Analysis.* Hardcover.1-250.