Acoustic pressure of a circular plate vibrating in a finite baffle with fluid loading

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ABSTRACT

In this paper the problem of calculating the acoustic pressure around a thin circular plate is analysed. It was assumed that the plate was clamped at the circumference of the planar finite baffle and radiated into a lossless homogeneous medium. A model of the plate included internal dissipation in the plate's material and the influence of the acoustic wave radiated by the plate on its vibrations. The vibrations of the plate were forced by time-harmonic external pressure. The acoustic pressure was calculated on the basis of the known distribution of vibration velocity in a series of eigenfunctions, using properties of the oblate spheroidal coordinates. The result was obtained as a single series of spheroidal function products. The number of terms ensuring a required accuracy could be determined numerically.

INTRODUCTION

The acoustic radiation pressure tends to be significant if the interface area between the vibrating structure and the fluid is large, such as in case of a plate. The vibrating plate generates an acoustic pressure wave in the fluid. At the same time, the surrounding medium exerts pressure on the plate. The magnitude of this interaction depends on the nature of the surrounding medium. If the plate in question is immersed in „light" fluid such as air, the interaction effect can be neglected. On the other hand, the presence of relatively dense fluid has an influence on the distribution of acoustic field around the plate.

The interactions between sound and vibration have been the subject of several previous investigations ([3,4,8] and the overview given by Crighton [1]). The solution for the radiated acoustic pressure was given for a plate which has been placed in a planar rigid and unlimited baffle. However, the real acoustical systems often differ from this model because the radius of the source is comparable with linear dimensions of the baffle. If emitted waves are a few times shorter or longer than the

dimensions of the baffle. If emitted waves are a few times shorter or longer than the geometrical size of the source, then the finite dimension of the baffle has an influence on a directional characteristics of the system under consideration. In this case the fluid loading includes the diffracted field due to the edges.

In this paper, the numerical result for the far-field pressure pattern for the time - harmonic excited plate at low frequencies is presented. It was assumed that the plate clamped at the circumference was placed in a limited baffle and radiated into moderately „heavy" fluid.

ASSUMPTION OF THE ANALYSIS

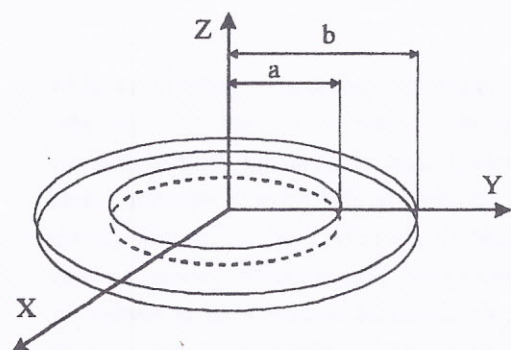Consider the fluid-plate configuration as illustrated in Fig. 1.

Fig. 1. A circular plate in a rigid baffle of a radius b.

A circular thin plate of a radius α and thickness h is surrounded by a lossless liquid medium with the static density ρ_0. It is assumed that the plate is made of a homogeneous isotropic material with density ρ, Poisson's ratio v, Young's modulus E. The plate is clamped in a flat, rigid and finite baffle of a radius b and is excited to vibration by an external time-harmonic force:

$$F(r,\varphi,t) = F_0\, e(-i\omega t), \qquad (1)$$

where F_0=const for $0<r<a$.

Taking into account only linear, harmonic and axially-symmetric vibrations of the plate in a steady state, as well as influence of a radiated wave on vibrations of the plate and an internal damping inside the plate's material, the plate differential equation of motion can be described as follows [7,8]:

$$BV^4 w(r,\phi,t) + m\frac{\partial^2 w(r,\phi,t)}{\partial t^2} +$$

$$+ R\frac{\partial}{\partial t}[\nabla^4 w(r,\phi,t)] = \qquad (2)$$

$$= F(r,\phi,t) - \rho_0\frac{\partial \Psi(r,\phi,0,t)}{\partial t}$$

where $B = Eh^3/12(1-v^2)$ is bending stiffness, $w(r,\phi,t)$ - transverse dislocation of points on the surface of the plate, m - mass of the plate, R - coefficient of internal damping, $\Psi(r,\phi,0,t)$ - acoustic potential on the surface of the plate, related with acoustic pressure p in the fluid by the equation

$$p = \rho_0 i\omega \Psi e(-i\omega t) \qquad (3)$$

and satisfies the Helmholtz equation

$$(\nabla^2 + k_0^2)\Psi = 0, \qquad (4)$$

with the condition

$$\frac{\partial \Psi}{\partial n}\bigg|_{n=0} = -i\omega w(r,\phi) = v(r,\phi), \qquad (5)$$

k_0 denotes the acoustic wavenumber at frequency ω, $k_p = (m\omega^2/B)^{1/4}$ will denote the structural wavenumber in the vaccum at frequency ω.

SOLUTION OF THE HELMHOLTZ EQUATION

For the plate located in a finite baffle, the problem of determining the far-field acoustic pressure cannot be treated with a well known Rayleigh's formula. In this paper the solution of Eq.(4) in conjunction with (5) was obtained by using the method of separation of variables in the oblate spheroidal coordinate system (OSCS) [6]. Due to symmetry of radiated waves with respect to z axis, the following equation for outgoing waves was obtained:

$$\psi(\eta, \xi) = \sum_{l}^{\infty} A_l S_{0l}^{(1)}(-ih, \eta) R_{0l}^{(3)}(-ih, i\xi), \quad (6)$$

where $S_{0l}(-ih, \eta)$ denotes angular spheroidal function of the first kind, $R_{0l}^{(3)}(-ih, i\xi)$ - radial spheroidal function of the third kind and A_l - coefficients which can be derived from boundary conditions (5) by using an orthogonality property of angular spheroidal functions [2]

$$A_l = -\frac{b W_{nl}}{\dfrac{\partial R_{0l}^{(3)}(-ih, 0)}{\partial \xi} N_l} \quad (7)$$

where N_l denotes norm factor [2] and

$$W_{nl} = \int_{\eta_0}^{1} v(\eta) S_{0l}(-ih, \eta) \eta \, d\eta \quad (8)$$

the characteristic function in OSCS. The surface distribution of the normal velocity in oblate spheroidal coordinate system $v(\eta)$ is an unknown function. It can be determined by using a distribution of velocity in a series of eigenfunctions.

SOLUTION OF THE PLATE EQUATION OF MOTION

Using well-known formulae appropriate for harmonic phenomena and taking into account only axally-symmetric modes of the plate, the equation (1) can be expressed as [7,8]

$$(k_p^{-4} \nabla^4 - 1) v(r) - \varepsilon k_0 \psi(r, 0) = -\frac{i}{\omega m} f(r) \quad (9)$$

where

$$\varepsilon = \rho_0 c_0 / m\omega = \frac{0.5\rho_0}{\rho \dfrac{h}{2a} k_0 a} s \quad s=1,2,.. \quad (10)$$

denotes fluid loading parameter and $v(r)$ is normal velocity on the plate.

To reduce the partial differential equation (9) to a set of ordinary differential equation, let us present the normal velocity in the form of an infinite series of eigenfunctions

$$v(r) = \sum_{n}^{\infty} c_n v_n(r). \quad (11)$$

For the clamped circular plate the eigenfunctions $v_n(r)$ take the form

$$v_r(r) = v_{0n}[J_0(\gamma_n r / a) - \frac{J_0(\gamma_n)}{I_0(\gamma_n)} I_0(\gamma_n r / a)] \quad (12)$$

where $\gamma_n = k_n a$ and have an orthonormality property if

$$v_{0n} = 1 / (aJ_0(k_n a)) \quad (13)$$

Regarding the following equation

$$\nabla^4 v_n(r) = k_n^4 v_n(r) \quad (14)$$

as a result we obtain

$$\sum_n c_n (k_p^{-4} k_n^4 - 1) v_n(r) - \varepsilon k_0 \psi(r,0) =$$
$$= -\frac{i}{\omega m} \sum_n f_n v_n(r) \qquad (15)$$

The equation (15) will now be expressed in the oblate spheroidal coordinate system (OSCS) with the use of the following transformation [2]

$$r = b[(1 - \eta^2)(\xi_0^2 + 1)] \qquad (16)$$

Using properties of (OSCS) and assuming $\xi_0 = 0$, the obtained expression becomes appropriate for the plate in the finite baffle. The eigenfunctions take a form

$$v_r(\eta) = v_{0n}[J_0(\gamma_n \frac{b}{a}\sqrt{1-\eta^2}) +$$
$$-\frac{J_0(\gamma_n)}{I_0(\gamma_n)} I_0(\gamma_n \frac{b}{a}\sqrt{1-\eta^2})] \qquad (17)$$

and they remain orthonormal [5] if

$$v_{0n} = b / (a J_0(k_n a)) \quad . \qquad (18)$$

As a result of this transformation, the previous equation (9) turns into a system of linear algebraic equations [5]

$$c_m \left(\frac{k_m^4}{k_p^4} - 1\right) - \varepsilon b^2 \sum_n^\infty i\zeta_{mn} c_n = f_m \qquad (19)$$

where

$$f_m = -\frac{2i}{\omega m \gamma_m b^2} F_0 a^2 J_1(\gamma_m) \qquad (20)$$

is an expansion coefficient of the external excitation into the Fourier series and quantity

$$\zeta_{mn} = \sum_l \frac{W_{nl} W_{ml}}{N_l} ih \frac{R_{0l}^{(3)}(-ih,0)}{\partial R_{0l}^{(3)}(-ih,0) / \partial \xi} \qquad (21)$$

means normalised impedance of the plate.

In order to determine expansion coefficients c_n the system (19) was solved numerically, what enables to find the velocity distribution on the surface of the plate.

CALCULATION OF THE ACOUSTIC PRESSURE

Basing our calculations on the relation (3) and expression (6) together with (7) and (8), the formula of the acoustic pressure of the circular plate with the finite baffle and fluid loading taken into consideration can be expressed as follows

$$p(\xi,\eta) = -2i\rho_0 hc \sum_{n=1}^\infty c_n *$$
$$\sum_{l=1}^\infty \frac{W_{nl}}{N_l \partial R_{0l}^{(3)}(-ih,0) / \partial \xi} S_{0l}(-ih,\eta) R_{0l}^{(3)}(-ih,i\xi) \qquad (22)$$

In the Frauhofer zone the following relationships are relevant

$$\xi \to \xi_\infty, \quad \eta \to \cos\theta$$
$$R_{0l}^{(3)}(-ih,i\xi)\big|_{\xi \to \xi_\infty} = (-1)^{l+1} \frac{e^{ih\xi_\infty}}{h\xi_\infty} \qquad (23)$$

Making use of the above conditions, the expression (22) can be substituted with the formula appropriate for the far - field

$$p(\xi_\infty, \theta) = -2i\rho_0 hc \sum_{n=1}^{\infty} c_n *$$

$$\sum_{l=1}^{\infty} \frac{(-1)^{l+i} \dfrac{e^{ih\xi_\infty}}{h\xi_\infty} W_{nl}}{N_l \partial R_{0l}^{(3)}(-ih,0)/\partial\xi} S_{0l}(-ih,\cos\theta)$$

$$(24)$$

With the use above formula the characteristics of acoustic pressure for θ= 0 were prepared. The calculations were performed for the following parameters of the plate : *h/2a=0.02,0.05; R=0.001, v=0.3* and for several values of ratio a/b.

CONCLUSION

The analysis presented in this paper has resulted in the expression for acoustic pressure given by a circular plate vibrating in a finite baffle with the fluid loading taken into consideration. The Fourier method was used to reach the solution in the form of a quickly convergent series. It was proved that it is enough to take only the first few terms in a practical calculation.

The plots of the acoustic pressure at θ=0 (at the main direction) were calculated for variable width of the baffle as a function of frequency for a few various values of the fluid loading parameter ε.

The performed calculation indicated that the frequency characteristic of the acoustic pressure depend on the width of the baffle (Fig.2). Furthermore, it was found that increase of the parameter characterising density of the surrounding medium caused the shift of the acoustic pressure maximums in the direction of lower frequencies (Fig. 3).
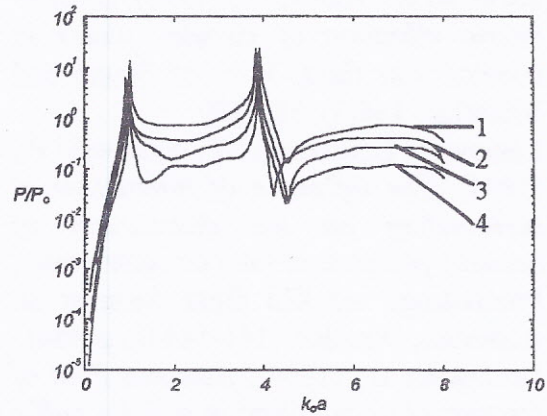

Fig.2. Log-linear plot of the normalised far-field pressure for a plate located in the finite baffle and immersed into fluid at s=1000, h/2a= 0.02; (1)- a/b=1, (2)-a/b=0.8, (3)-a/b=0.6, (4)- a/b=0.4
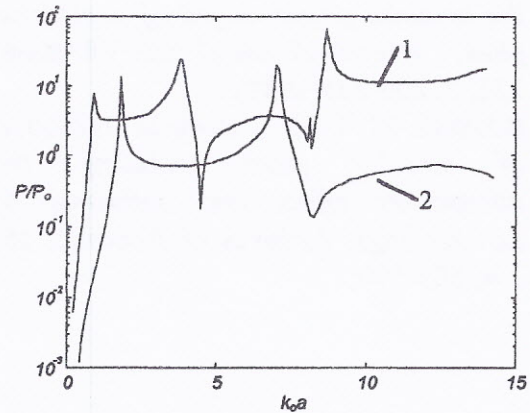

Fig.3. Log-linear plot of the normalised far-field pressure for a plate located in the finite baffle at a/b=0.8 and immersed into fluid (1)- „heavy" fluid, s=1000 ; (2)- „light" fluid, s=1; h/2a= 0.02;

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