

A WINDOW BASED METHOD TO REDUCE THE END-EFFECT IN EMPIRICAL MODE DECOMPOSITION

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Abstract

Empirical Mode Decomposition technique (EMD) is a recent development in non-stationary and non-linear data analysis. It is an algorithm which adaptively decomposes the signal in the sum of Intrinsic Mode Functions (IMFs) from which the instantaneous frequency can be easily computed. EMD has proven its effectiveness but is still affected from various problems. One of these is the “end-effect”, a phenomenon occurring at the start and at the end of the data due to the splines fitting on which the EMD is based. Various techniques have been tried to overcome the end-effect, like different data extension or mirroring procedures at the data boundary. In this paper we made use of the IMFs orthogonality property to apply a symmetrical window to the data before EMD for end-effect reduction. Subsequently the IMFs are post-processed to compensate for data alteration due to windowing. The simulations show that IMFs obtained with this method are of better quality near the data boundaries while remaining almost identical to classical EMD ones.

Keywords: Empirical Mode Decomposition, Intrinsic Mode Functions, end-effect problem, windowing.

1. INTRODUCTION

The Empirical Mode Decomposition (EMD) is a signal processing method firstly developed by N.E. Huang [5] particularly suited for non-linear and non-stationary data analysis. It aims to decompose the signal in the sum of Intrinsic Mode Functions (IMF) rather than sinusoidal functions (as Fourier transform does) or other a priori chosen expansion basis. The IMFs represent single oscillatory modes leading to meaningful instantaneous frequency estimates, so far allowing a better insight on the physical processes involved in the data under analysis. In fact, an IMF is a function defined in [5] as follows:

- 1) in the whole dataset, the number of extrema and the number of zero-crossings must either equal or differ at most by one;
- 2) at any point, the mean value of the envelope defined by the local maxima and the envelope defined by the local minima is zero.

In another words an IMF is almost symmetric with a unique local frequency. The properties of the IMF also set the convergence criterion of EMD. The EMD procedure is described as following:

- 1) find out the local maxima and the local minima of the signal $s(t)$ firstly, then the upper (lower) envelope can be get as the cubic spline interpolation of the maxima (minima).
- 2) Compute the mean envelope $m(t)$ as the average of the upper envelope and the lower one. Let $h = s(t) - m(t)$ be the new signal and repeat the procedure above up to h satisfies the IMF definition, then set $c_i = h$.
- 3) Separate the IMF from the signal, $r_{i+1} = s(t) - c_i$.

- 4) Let r_{i+1} be the new signal now (i.e. $s(t) = r_{i+1}$), repeat the sifting until the final residue $r_n(t)$ has at most one extreme or it is a constant or a monotonic function. Thus:

$$s(t) = \sum_{i=1}^n c_i(t) + r_n(t) \quad (1)$$

The obtained IMFs form a local orthogonal basis as stressed in [5], although orthogonality can't be theoretically proved. In fact, the EMD method is totally data adaptive - leading to the aforementioned advantages in non-linear and non-stationary data analysis - but still lacks of a firm mathematical background. This is the main cause of two of the principal problems of EMD process, which are the convergence criterion and the end-effect, the latter being treated in this paper. The convergence criterion declares if the result of the last completed sifting is an IMF or not. In the EMD literature various criteria are proposed and here we recall some of the principal ones. In [5] is made use of a Cauchy type convergence criterion which stops the sifting process at iteration k when the parameter SD_k is less than a predetermined value. SD_k is defined as follows:

$$SD_k = \frac{\sum_{t=0}^T [h_{k-1}(t) - h_k(t)]^2}{\sum_{t=0}^T [h_{k-1}(t)]^2} \quad (2)$$

where k is the actual iteration number, t is the signal sample index and T is the signal's total samples number. Typically, the stopping values for SD lie in the 0.2 - 0.3 range. In this paper we made use of this

stopping criterion, and the SD stopping value had been set to 0.2. In [2] the Mean Value criterion is illustrated, where the SD value is simply the mean envelope $m(t)$ at current iteration and must be smaller than a predetermined value everywhere to stop sifting. In [6] the Fixed Siftings criterion is proposed, where the sifting process is stopped after a given number of iterations (typically around 10). In [4] the S -number criterion is illustrated, where the sifting is stopped when for S consecutive iterations the signal's number of extrema and zero crossings stay the same and are equal or differ at most by one. Typical values for S are in the 4 - 8 range. The next sections will discuss the end-effect problem in details, and propose a windowing method to reduce it. The reduction of the end-effect problem is the main target and novelty of this paper.

2. THE EMD END-EFFECT

The end-effect is a phenomenon occurring in EMD due to splines fitting at the beginning and at the end of the data. The upper and lower envelope in the sifting process will swing at the two ends of the data sequence, due to the lack of constraints for the spline definition. If we adopt the classical mathematic definition of local extrema in a limited set, a boundary data point is always a local maximum or minimum, depending on the following data point (or the preceding one if the data point considered is the last one). Therefore, there is always one spline of the two envelopes to be calculated (upper and lower) which lacks of definition between the first (or last) local extrema and the data boundary. This results in a swing of the envelope splines (and consequently of the mean spline) that will gradually pollute inside the whole data sequence making the result seriously distorted, particularly the lowest frequency IMFs (which are the last extracted by the sifting process). As previously mentioned, this IMFs corruption is called end-effect in the EMD jargon. Researchers have been developing several techniques to reduce the end-effect, the most of them dealing with signal extension outside the endpoints in order to resolve the splines uncertain definition at the boundaries. Direct data mirroring [7] is an operation that chooses two endpoints of a signal as the mirror positions, expands it beyond the two mirrors in the outside directions, and obtains a new periodic signal with a length of two times the original one. Since only the extrema are needed for envelopes calculation, the natural update of the mirroring method aims to the correct estimation of the data extrema beyond the boundaries by means of existing signal processing tools. In [1] this is achieved by virtue of a neural network estimation, while in [8] the extrema extension is done by the use of a Support Vector regression at both endpoints. In [10] the Ratio Boundary Extension technique is proposed, an approach that couples the mirror expansion with the extrapolation prediction of regression function to the treatment of end-effects, with extrema estimation by means of a quadratic

interpolation on the near-endpoints extrema. Other works are concerned with applications of Auto-Regressive and Moving-Average (ARMA) modeling [3], polynomial regression and so on. In our view, all of these kinds of signal processing tend to loose the philosophical approach of EMD which is the source of its powerful outcomes. In fact, the aforementioned signal processing tools have been developed on some starting hypotheses on the data that the original EMD doesn't guess (due to its algorithmic form), thus potentially losing the non-stationary/non-linear adaptability of EMD. Moreover, the end-effect problem could still remain due to the fact that even if extended perfectly, a digitized signal has always two endpoints to deal with. In our view the original EMD approach (i.e.: no a priori hypothesis on the data) is conserved, and in the next section we propose a signal symmetrical windowing in order to reduce the end-effect.

3. A WINDOW BASED METHOD TO REDUCE EMD END-EFFECT

The basic idea is similar to the one proposed in [9], where the signal is symmetrical windowed in order to have always the endpoints with zero value. By doing this, during the sifting process the upper and lower envelope are forced to have the first and the last point with zero value, resolving the uncertainty of the envelope value at the endpoints. Unlike in [9] where a "flat top" and classic cosine windows were used, such as Hanning and Hamming, we made use of a triangular window (also called Bartlett window) in order to give less weight to the signal near the boundaries and no significance at all at the endpoints, where the windows is zero valued. This approach simulates the perspective of the human eye watching the horizon: the further one watch, the less detail is retrieved (i.e.: the less certainty on what is being looked at). In our case, the "observer" is placed in the middle of the signal and his horizons are the endpoints, by virtue of the windowing. Furthermore, if $w(t)$ is the window, we rewrite Eq. 1 for the windowed signal as:

$$w(t)s(t) = w(t) \left(\sum_{i=1}^n c_i(t) + r_n(t) \right) \quad (3)$$

Since the EMD process separates the IMFs simply by differentiation, and IMFs are locally orthogonal, we thought that we could extract the IMFs of the windowed signal and subsequently multiply them by $1/w(t)$, obviously rejecting the endpoints where the window reciprocal is infinite (i.e.: multiply both sides of Eq. 3 by $1/w(t)$ where possible). By doing this, we should be able to retrieve the original signal's IMFs. To prove the effectiveness of this procedure, we tested it on simulated data and the results are compared with the standard EMD without any end-effect compensation.

4. SIMULATIONS AND DISCUSSION OF THE RESULTS

The window based method proposed had been tested on different simulated signals, all of which have a sampling frequency of 10 kHz and a timespan of 5 seconds. We tested the triangular window, described by the following Eq. 4:

$$w(t) = 1 - \left| \frac{t-a}{a} \right| \quad (4)$$

where $a = T/2$ is half the length of the signal. The first signal tested is given by the following Eq. 5, and is reported in Fig. 1a along with its windowed version (Fig. 1b).

$$s_1(t) = \sin(2\pi \cdot 0.5 \cdot t) \cdot \sin(2\pi \cdot 20 \cdot t) + 0.1 \cdot \cos(2\pi \cdot 10 \cdot t) + t(2-t) \quad (5)$$

$s_1(t)$ is the sum of an amplitude modulated (AM) sine, a cosine and a parabolic term: this is done to evaluate the effect of windowing when the data is composed of simple oscillating functions and a

nonlinear “trend”. In this case, applying the direct mirror extension will fail due to the parabolic term induced boundaries asymmetry. From this signal the standard EMD extracts three IMFs (Fig. 2a – Fig. 2c)) while the proposed method with triangular window extract 4 IMFs (Fig. 2d – Fig. 2g). It can be seen how in this case the windowing doesn’t corrupt the IMFs far from the endpoints while some swing still occurs. It is also noticeable how the second IMF from the standard EMD (Fig. 2b) suffers the end-effect probably due to some mode-mixing with the nonlinear (parabolic) trend, while the proposed method doesn’t fail (Fig. 2e). In fact, the trend extracted by the standard EMD (Fig. 2c) is distorted near the right endpoint to compensate for the previous IMF corruption whilst the window method is not affected by the cited phenomenon, but the latter spreads the trend in two components (Fig. 2f – Fig. 2g). Finally, notice that the triangular window discontinuity at the center point is reflected in the residual IMFs (Fig. 2f – Fig. 2g) extracted by the proposed method.

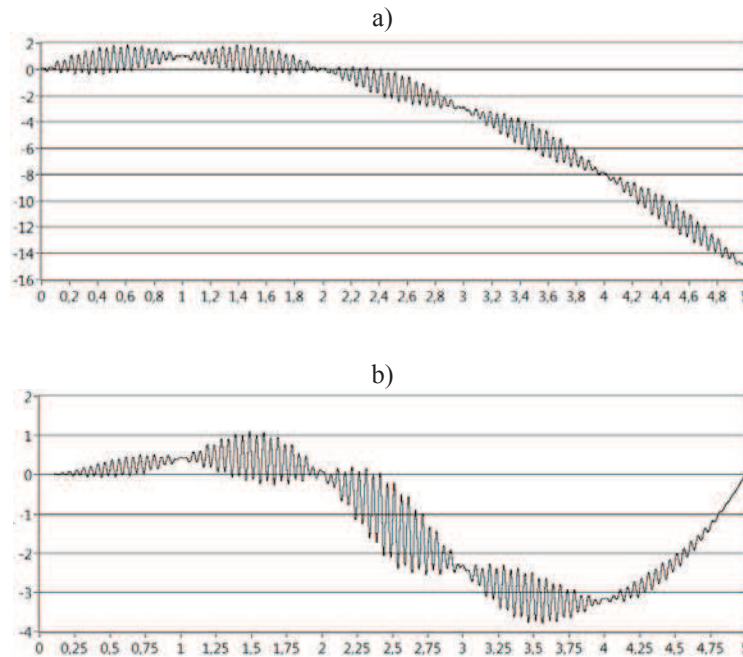


Figure 1. $s_1(t)$ (a) and its triangular windowed version (b)

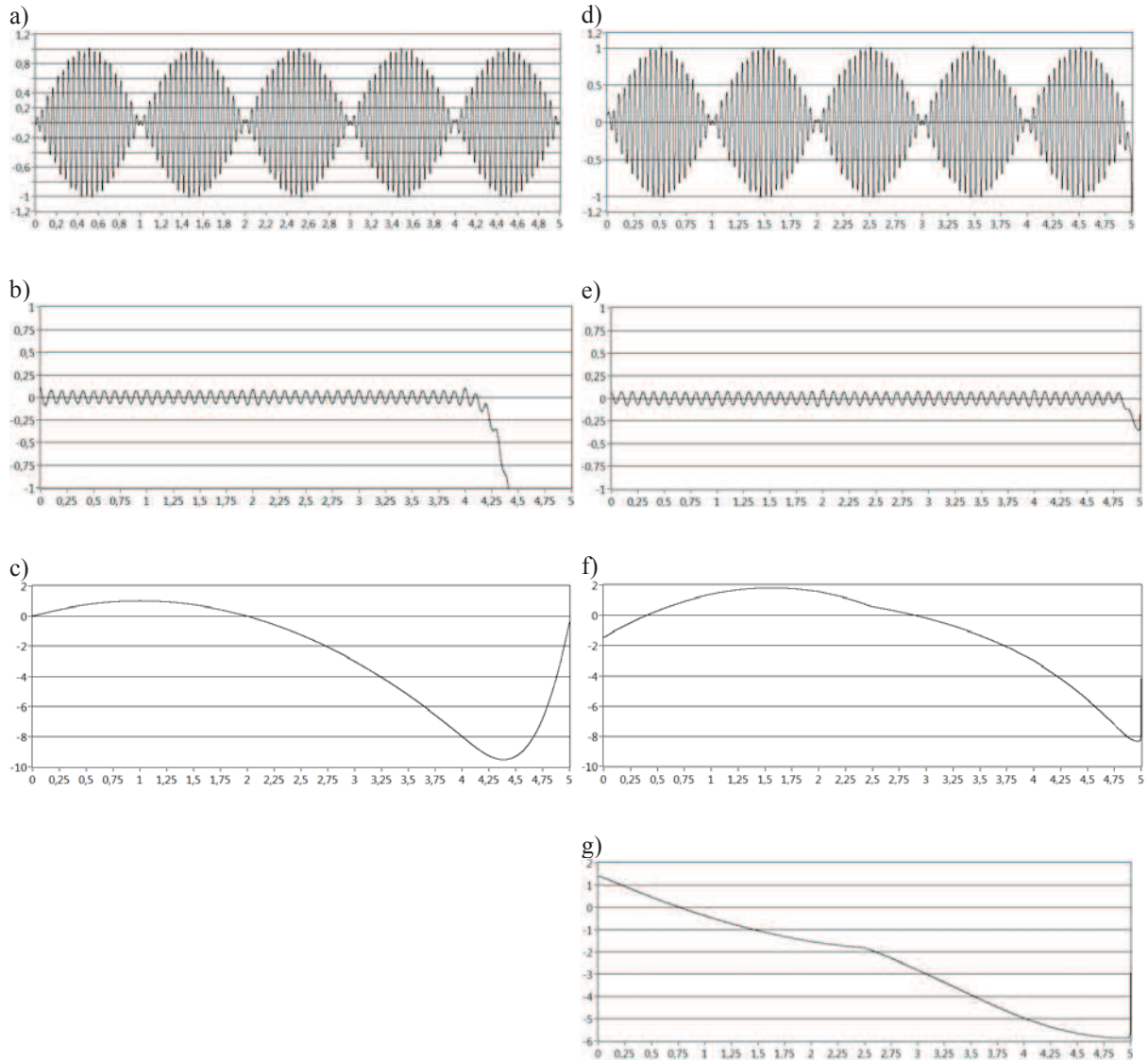


Figure 2. In the left column (a-c) the IMFs extracted by the standard EMD from $s_1(t)$, whilst in the right column (d-g) the corresponding IMFs extracted by the proposed windowing method

The second simulated signal $s_2(t)$ is given by the following Eq. 6 and it is illustrated in Fig. 3a along with its windowed counterpart (Fig 3b):

$$s_2(t) = \sin(2\pi \cdot 0.5 \cdot t) \cdot \sin(2\pi \cdot 20 \cdot t) + \sin(2\pi \cdot e^t) + t(2-t) \quad (6)$$

$s_2(t)$ embodies an AM sine, an exponential chirp and a parabolic term in order to evaluate the performance of the algorithm in presence of a strongly nonlinear component. Since in the proposed method we

implemented the standard EMD we expect also strong mode-mixing in the IMFs due to the chirp. In the EMD jargon the mode-mixing denotes the split of a signal component among two or more IMFs. A characteristic example is indeed the chirp signal, its frequency components cover a wide band spectrum and they consequently appear in different IMFs. In Fig. 4 are reported the results of the simulation; standard EMD extracts four IMFs and also does windowed EMD. It can be seen how the IMFs behavior is very similar between the two methods also in presence of strong mode-mixing and nonlinear components.

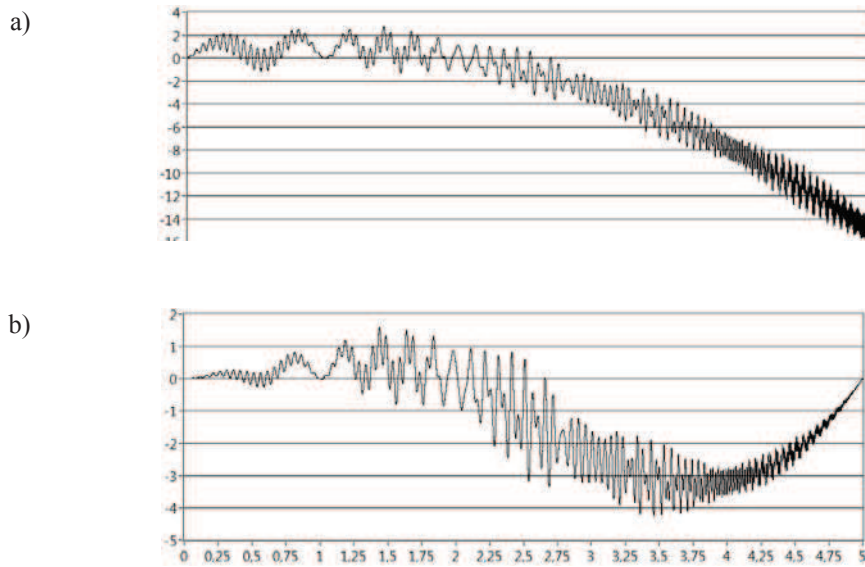


Figure 3. Simulated signal $s_2(t)$ (a) and its triangular windowed version (b)

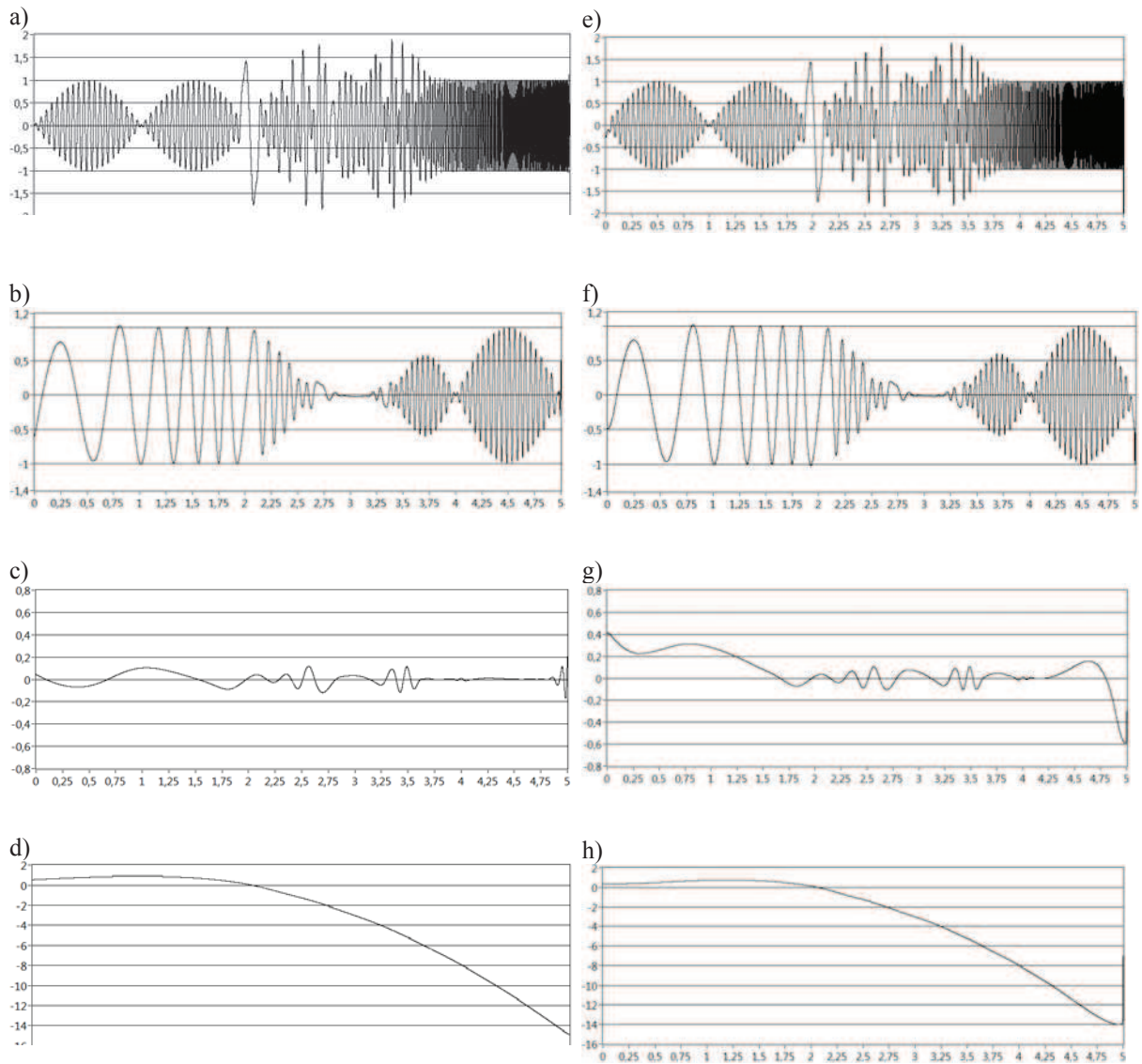


Figure 4. In the left column (a-d) the IMFs extracted by the standard EMD from $s_2(t)$, whilst in the right column (e-h) the corresponding IMFs extracted by the proposed windowing method

The main difference in the two IMF sets is in the third IMF since in the window method's one (Fig. 4g) there is more swing at the beginning than the standard EMD counterpart (Fig. 4c): however, this IMF seems to be a mode-mixing product of EMD.

The last simulated signal tested, $s_3(t)$, is reported in Fig. 5a along with its windowed version (Fig. 5b) and is given by the following Eq. 7:

$$s_3(t) = \sin(2\pi \cdot 0.1 \cdot t) \cdot \sin(2\pi \cdot 20 \cdot t) + e^{1-t} \cos(2\pi \cdot 10 \cdot t) + t(2-t)(4-t) \quad (7)$$

$s_3(t)$ embodies an AM sine, a fading cosine function and a polynomial function acting as a nonlinear trend; this is done in order to compare the performances of the proposed algorithm in case of this kind of signal mixture from the ability of signal's component retrieval point of view. The obtained IMFs are reported in Fig. 6.

In this case again mode-mixing is present, as expected. The first IMFs are very similar, but the one gathered by the windowed EMD (Fig. 6e) offers

a more precise representation of the original component near the left endpoint in terms of its decaying amplitude dynamics. The second IMF from the standard EMD (Fig. 6b) shows what at a first glance could be judged as a swing resulting from the end-effect: in our view, the swing comes from mode-mixing of the fading cosine with the nonlinear (cubic) trend which is subsequently spread in the following IMFs (Fig. 6c – Fig. 6d). The second IMF, windowed version, (Fig. 6f) shows a similar effect but much more limited in amplitude: in fact, the following IMFs (Fig. 6g – Fig. 6h) represents in a better fashion the nonlinear trend. The latter is split in two components probably because of its dynamics (i.e.: oscillations) indeed the zero crossings in the third IMF (Fig. 6g) occur near $t=2$ and $t=4$ which are the real component's zeroes, while almost no trace of the same information could be retrieved from the standard EMD correspondent third and fourth IMF (Fig. 6c – Fig. 6d respectively).

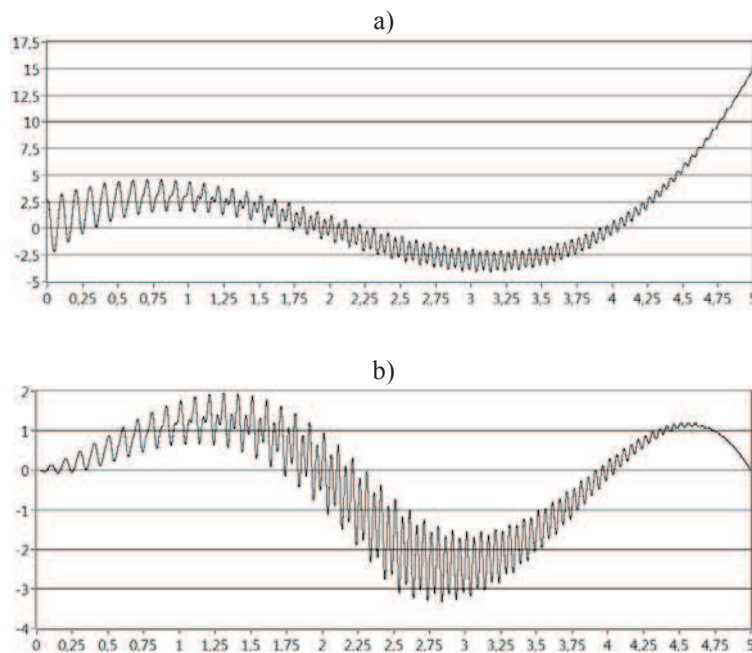


Figure 5. Simulated signal $s_3(t)$ (a) and its triangular windowed version (b)

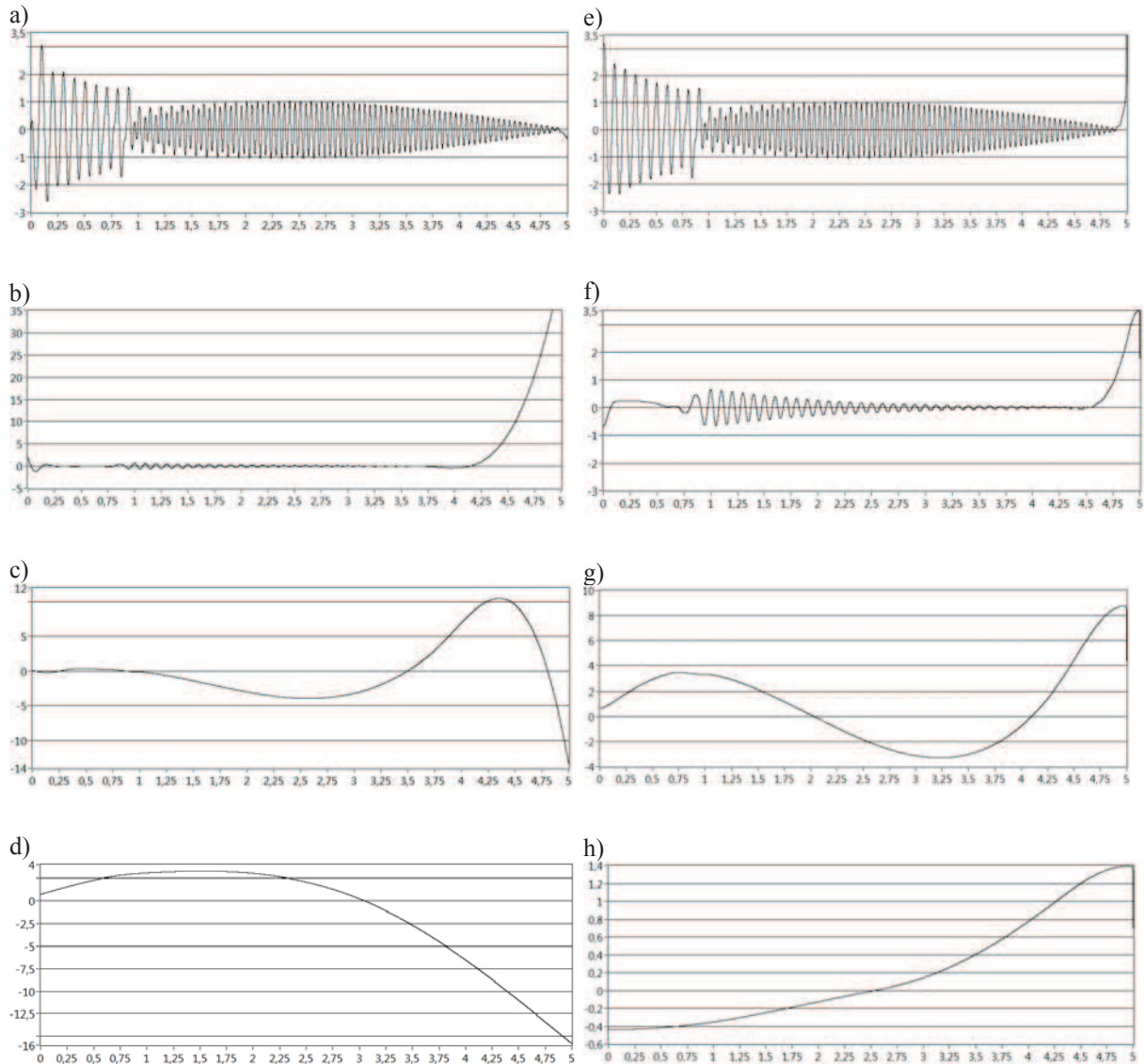


Figure 6. In the left column (a-d) the IMFs extracted by the standard EMD from $s_3(t)$, whilst in the right column (e-h) the corresponding IMFs extracted by the proposed windowing method

5. CONCLUSIONS

The presented signal windowing method for EMD end-effect reduction works well: cases have been showed where end-effect or mode-mixing induced swings in IMFs are strongly reduced. Swings still occur near the endpoint, but their amplitude is smaller than in standard EMD. In some cases, the proposed method produced better quality IMFs in presence of strong mode-mixing. The simulations performed (here we reported a few extracts because of available space) have shown in general a more precise representation of the low frequency component(s) particularly in case of asymmetry of the data. These results are obtained while maintaining the EMD data adaptability since no a priori assumption is made on the signal (or parts of it), in contrast with other techniques dealing with EMD end-effect which do (explicitly or implicitly). In our view, another important result comes from the use of the local orthogonality of the

IMFs which allows us to compensate for the signal windowing by multiplication by the window reciprocal after the IMF extraction. We think that this behavior of EMD should be taken in consideration by the researchers who are working on mathematical base for EMD. In fact, since EMD is defined only as an algorithm it was not certain if this operation (i.e., windowing and post EMD windowing compensating) could have produced consistent results. Moreover, we think that a mode-mixing avoiding technique could also improve the performance of windowed EMD particularly in presence of divergent trends near the endpoints.

6. ACKNOWLEDGEMENTS

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