Drewno 2019, Vol. 62, No. 204
DOI: 10.12841/wood.1644-3985.275.12

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OPTIMAL DESIGN OF A FURNITURE FRAME BY REDUCING THE VOLUME OF WOOD

The aim of this study is to optimize the volume of a furniture frame to make it lighter, while still meeting the same load requirements. The finite element method (FEM) and MATLAB nonlinear programming were utilized for the optimization. A beech stool frame was taken as an example. First, the finite element model was set up and analyzed to obtain the internal forces of the stool frame, and the internal forces were investigated to obtain the maximum critical forces in each type of member. Then constraints were obtained to determine the limitations for minimizing the volume of the stool frame, which was defined as the objective function subject to constraints. Finally, the objective function and constraints were programmed and solved using MATLAB software, and as a result the minimum volume and dimensions of members were determined. An experimental test was conducted to determine whether the optimized stool was strong enough to carry the same loads. The results showed the optimized stool to be 58% lighter than the non-optimized version, while also satisfying the requirements of GB 10357-3. In conclusion, the method is suitable for the optimization of a furniture frame, making it lighter and reducing the manufacturer's material costs.

Keywords: furniture structure, stool frame, mortise-and-tenon joint, optimization, FEM, MATLAB

Introduction

Furniture is not merely an item of household and office supplies, but is also an industrial product. For consumers, comfortable, durable and serviceable furniture is a requirement of in daily life. For manufacturers, it is important not only to satisfy customers' requirements, but also to reduce production costs, and for this reason light and strong wood furniture and other wood structures are desired [Que et al. 2017; Hu et al. 2018a]. This problem has long been a concern of manufacturers of furniture and other wood products – for instance, windows [Jansna et al. 2013] and household furniture [Pakarinen and Asikainen 2001]. Solutions to this problem are concerned with how to optimize the structure of

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furniture to make it strong enough to carry the desired load with the minimum material. In practice, furniture is often excessively strong, and material is wasted from the standpoint of solid mechanics.

Many researchers and furniture designers have considered the following three approaches to the problem: 1) using modified wood materials and high--strength wood to replace natural wood materials used in furniture; 2) introducing advanced machines to reduce the wood materials during the processing process; 3) adding metal fasteners to make furniture frame more solid. Obviously, these three methods cannot solve this problem fundamentally from the standpoint of structure design. With the development of computer technology and finite element (FE) theory, it is more convenient now than in the past for researchers to analyze complex structures with FE software. Most furniture frames are made up of structural elements called indeterminate frames, which make it a rather tedious task to analyze the internal forces in the frame. However, using the finite element method (FEM), it has been possible to overcome this issue [Gustafsson 1997].

Many researchers have contributed to work in this area. First to be mentioned is the work of Eckelman [1966] and Eckelman and Suddarth [1969]. In their study, the stiffness method of matrix structural analysis was applied to furniture frame design, and modifications needed to treat frames with semi-rigid, elastically non-linear joints of finite size were developed. Gustafsson [1997] described how to analyze and design a chair using FEM. Aydin and Ergün [2016] studied chair frames with various stretcher positions by the finite element method (FEM), regarding the joint as rigid. The results showed that frames without stretchers yield more deformation, and the use of a stretcher reduced the stresses and deformations in the frames. Hu et al. [2018b] put forward an optimal design method to determine the best stretcher positions in a stool, based on FEM and a response surface method, treating a mortise-and-tenon joint as a semi-rigid joint. An integrated FEM optimization algorithm was proposed by Smardzewski and Gawroński [2001]. In another study by the same authors, a gradient optimization approach was proposed. An external penalty function was generated from constraints, and optimization was carried out on that basis [Smardzewski and Gawroński 2003]. In addition, the minimum weight of chair frames subject to required strength constraints was studied by Guray et al. [2015] using a logarithmic barrier method and gradient descent method.

The aim of the present study is to determine the optimum volume of a stool frame subjecting to a number of load requirements from GB 10357-3 [1989]. The finite element method (FEM) and MATLAB nonlinear programming were utilized to perform the optimization. In addition, the optimized stool was tested in accordance with GB 10357-3 to determine whether it complied with the requirements.

Materials and methods

All specimens were made from beech (*Fagus orientalis* L.), purchased from a local commercial supplier (Nanjing, China). Referring to ASTM D 4442 [2001], the average density of the beech wood was 0.692 g/cm^3, and its moisture content was conditioned to and maintained at 10.8% before and during the experiment. The physical and mechanical properties of the beech wood were determined at the initial stages of the research [Hu and Guan 2017a, 2017b] and are given in Table 1. These parameters were input into the ABAQUS FEM software to simulate elastic behaviors of beech. Besides, proportional limits were used to judge whether the furniture structural members retained their elasticity.

Table 1. Mechanical properties of beech

Mechanical properties	Modulus of elasticity (MPa)			Poisson ratio (dimensionless)						Tangential modulus (MPa)	
	E_L	E_R	E_T	v_{LR}	v_{LT}	v_{RT}	v_{TR}	v_{TL}	v_{RL}	G_{LR}	G_{LT}
	12205	1858	774	0.502	0.705	0.526	0.373	0.038	0.078	899	595
Proportional limit (MPa)	Compression (σ_C)			Shear (τ_S)						Bending (σ_B)	
	53.62			5.00						50.55	

Preparation of specimens

A stool with mortise-and-tenon joints was taken as an example. It was composed of four legs, four rails and four stretchers. Aydin and Ergün [2016] studied chair frames with various stretcher positions using the by FEM. The results showed that frames without stretchers yield more deformation, and the use of a stretcher reduced the stresses and deformations in the frames. In this study, the example stool had four stretchers, two at the front and back (X) of the stool and two at the sides (Y). Dimensions of these members and their coordinates are shown in figure 1a, and dimensions of the mortise and tenon are shown in figure. 1b. The tenon fit in its width direction (i.e., the difference between tenon width and mortise height as shown in figure. 1b) was 0.2 mm. The tenon fit in its thickness direction (i.e., the difference between tenon thickness and mortise width) was a negative constant of 0.2 mm. The mortise measured 12 mm high × 10 mm wide × 15 mm deep.

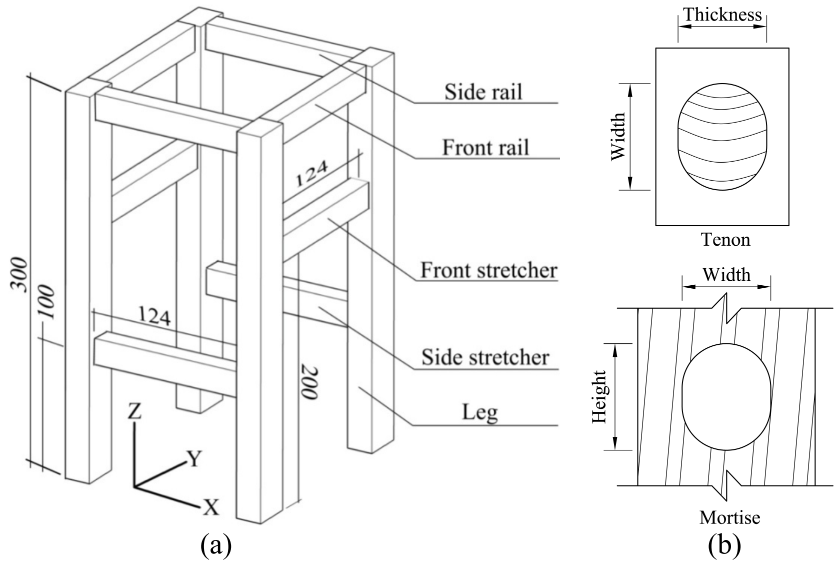

Fig. 1. Dimensions of stool frame

In setting up the FEM, the members of the stool were regarded as straight beams with different sections, and the members had rigid joints with the Beam31 element in ABAQUS. The dimensions, local axis and grain orientations of the legs, rails and stretchers are shown in figure 2. The parameters (*a*, *b*, *c*, *d*, *e* and *f*) are variables, and their initial values are 30, 20, 20, 15, 20 and 15 mm respectively.

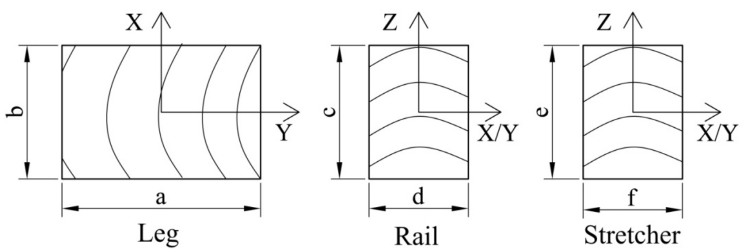

Fig. 2. Dimensions, local axis and grain orientations of beams

Finite element model

The finite element model of the stool is shown in figure 3, with cross-sections of beams hidden. Node numbers are shown in black with a capital N, and element numbers in red with a capital E. According to GB 10357-3-1989 (Test of mechanical properties of furniture strength and durability of chairs and stools), four load cases were applied to the stool model. These were: I vertical load test on the rail; II front to backload test on the rail; III side thrust load on the rail; IV thrust load on the corner (table 2). When load cases II, III and IV were applied individually, load case I was also applied as the equilibrium load. In addition, the freedoms of the feet (N2, N23, N24 and N25) were all constrained in the X, Y and Z directions.

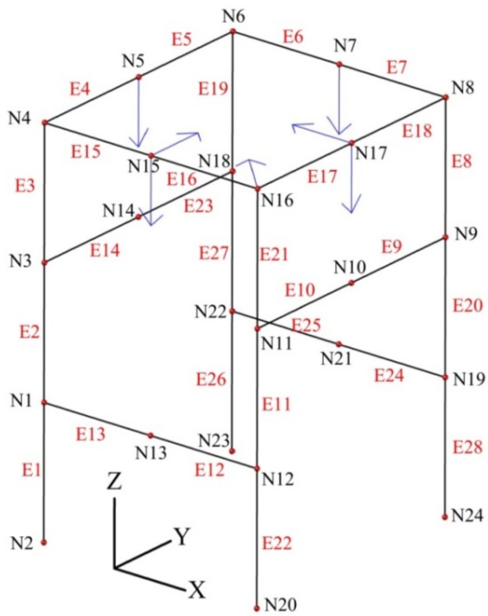

Fig. 3. Numbering of nodes and elements of the stool

Table 2. Load cases applied to the stool

Load case	Application node/nodes	Magnitude (N)	Direction
I	N5, N7, N15, N17	2000	–Z
II	N17	760	–X
III	N15	760	Y
IV	N16	760	X-Y corner 45°

Optimization methods

ABAQUS 14.1 FEM software and MATLAB 7.0 were utilized to optimize the cross-sectional dimensions of the beams. A flow chart of the optimization process is shown in figure 4. First of all, an objective function is defined; in this study, it was expected that the stool would be strong enough to carry the applied loads with the minimum volume. Secondly, the responding constraints were needed to limit the objective function. The axial force, shear force and bending moment of the beams were extracted from the FEM result, and then they were converted to the corresponding stress according to equations (1), (2) and (3), and compared with the proportional limit stress of beech. The maximum stress of a beam must be smaller than proportional limit stress of beech itself, to ensure that the structure of the stool satisfies the requirements of GB 10357-3-1989. Finally, the solution to the objective function with constraints was obtained by MATLAB programming.

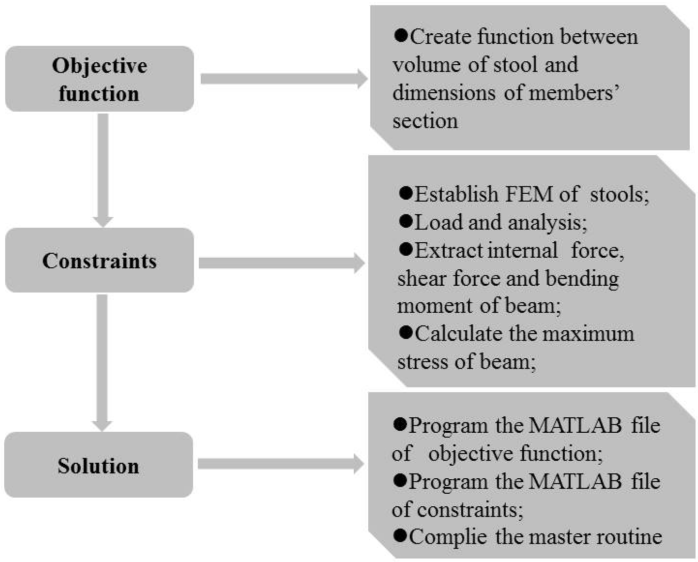

Fig. 4. Optimization flow chart

$$\sigma_L = \frac{F_L}{S} \tag{1}$$

$$\sigma_{\max} = \frac{6M}{th^2} \tag{2}$$

$$\tau_{\max} = \frac{3F_S}{2th} \tag{3}$$

where σ_L is the axis compression stress (MPa), $\sigma_{\max}$ is the maximum bending stress (MPa), $\tau_{\max}$ is the maximum shear stress (MPa), F_L is the axis compression force (N), S is the area of the beam section (mm^2), M is the maximum bending moment (N·mm), h is the beam width (mm), t is the beam thickness (mm), and F_S is the shear force (N).

Testing methods

Two groups of stools were machined, one non-optimized (fig. 5a) and one optimized by the method described in this paper (fig. 5b). The dimensions of the mortise-and-tenon joints were constant. Nine stools were measured for each group, and specimens are shown in figure 5. Then experimental tests were carried out in accordance with GB13057-3 to measure the maximum mechanical capabilities of the stools. In addition, the withdrawal capacity and bending moment of the T-shaped joint at the stretcher and leg of the non-optimized stool were measured according to the method described by Derikvand et al. [2013, 2014].

(a) (b)

Fig. 5. Non-optimized and optimized stools

Results and discussion

Finite element analysis

According to the FEM described above, axial force, shear force and beam bending moment were determined; values are given in Appendix 1. These data indicate that the axial force, shear force and bending moment of all members of the stool under the four load cases.

Objective function

Because the goal was to achieve the minimum volume of the stool, the objective function was expressed as equation (4). The variables *a, b, c, d, e* and *f* are the width and thickness of a leg, rail and stretcher respectively, the lengths of these elements being 300, 124 and 124 mm respectively.

$$V = 4*300\ a\times b + 4*124\ c\times d + 4*124\ e\times f \quad (4)$$

Constraints on the objective function

According to the FEM results, the axial force, shear force and bending moment of all members were obtained (Appendix 1). The internal forces of beams in different load cases were obtained, and the maximum value was chosen to calculate the corresponding stress based on equations (1), (2) and (3). For safety, the maximum stress of all members should be smaller than the proportional limit stress of beech (i.e. $\sigma_L \leq \sigma_C$, $\sigma_L + \sigma_{\max} \leq \sigma_B$ and $\tau_{\max} \leq \tau_S$). Then the constraints of the objective function were obtained as shown in table 3.

Table 3. Constraints on the objective function

	Axial stress	Bending stress	Shear stress
Leg	779/(*a**b*)≤53.62	779/(*a**b*)+6*8914/(*b**a*^2)+6*12488/(*a**b*^2)≤50.55	3*223/(2*b*a^2)≤5 3*239/(2*a*b^2)≤5
Rail	325/(*c**d*)≤53.62	98/(*c**d*)+6*6441/(*d**c*^2)+6*17179/(*c**d*^2)≤50.55	3*513/(2*d*c^2)≤5 3*382/(2*c*d^2)≤5
Stretcher	52/(*e**f*)≤53.62	52/(*e**f*)+6*671/(*f**e*^2)+6*20736/(*e**f*^2)≤50.55	3*405/(2*f*e^2)≤5 3*15/(2*e*f^2)≤5

In addition, the parameters a, b, c, d, e and f should be smaller than their initial values, and so the group of inequalities (5) must be satisfied. These dimensions were also subject to the requirements expressed in the group of inequalities (6) according to general knowledge of furniture structure design.

$$\begin{cases} 0<a<30 \\ 0<b<20 \\ 0<c<20 \\ 0<d<15 \\ 0<e<20 \\ 0<f<15 \end{cases} \tag{5}$$

$$\begin{cases} a \geqslant b \\ c \geqslant d \\ e \geqslant f \\ b \geqslant d \\ d \geqslant f \end{cases} \tag{6}$$

MATLAB programming

The objective function and constraints are all nonlinear, giving rise to a typical nonlinear programming problem. MATLAB 7.0 was applied to solve this; the code is given in Appendix 2. First of all, the objective function and constraints were defined, and then a master routine was written to call the objective function and constraints. After the calculations were performed, a solution was obtained. The obtained values of the parameters *a, b, c, d, e* and *f* were 14.02, 14.02, 14.32, 14.02, 13.67 and 13.67 mm respectively. The volume after optimization is $4.2818e^5$ mm^3, compared with $10.1786e^5$ mm^3 before optimization. The volume of the stool was thus reduced by 58%, while maintaining its capacity to carry the same loads.

Experimental results

The results for each load case are presented in table 4. The optimized stool was capable of carrying the loads defined in GB 10375-3, while the non-optimized stool was stronger than required by GB 10375-3. In other words, the non-optimized stool wasted a significant amount of wood and increased the cost of materials. The results demonstrate that the optimization method can be used to make a stool lighter while still capable of carrying the required loads.

Table 4. Comparison between experimental and standard stools

Load case	I (N)	II (N)	III (N)	IV(N)
GB10357-3	2000	760	760	760
Optimized	5073(10.5)	1534(6.4)	2087(11.3)	1368(7.5)
Non-optimized	17864(12.5)	6694(8.6)	8973(15.4)	3431(4.6)

In the finite element model used in this study the stool joint was regarded as rigid, which is not consistent with the real structure of the stool, which had semi-rigid mortise-and-tenon joints. However, if the mortise-and-tenon joint was strong enough to carry the applied loads and was not damaged (i.e., there was no relative displacement, rotation or torsion between mortise and tenon), then the joint can be regarded as rigid to some extent. Therefore, the strength of the joint was measured by performing a withdrawal and bending test with a T-shaped specimen joined by the same mortise-and-tenon joint as the stool. The experimental results showed that the withdrawal force of the joint was 568 (56) N and the bending moment was 30687 (2864) N·mm. The maximum axial forces and bending moments of the stretcher (71.16 N, 20736.90 N·mm) and rail (325.38 N, 17178.80 N·mm) in the FEM were all far smaller than the experimental values. As a result, in this study it was reasonable to regard the mortise-and-tenon joint as a rigid joint to some degree.

Conclusions

The results suggest that the method used can reduce by nearly 58% the volume of wood in the stool frame used in this study, and the frame can be designed between the minimum and maximum dimensions of the beam cross-sections (*a, b, c, d, e* and *f*). In conclusion, the method proposed in this study has been shown to be capable of optimizing the structure of a furniture frame with mortise-and-tenon joints, making it much lighter while still capable of carrying the same loads. If this method can be used in the wood furniture industry, it will enable furniture enterprises to save on costs of wood material. However, the method should be investigated in depth to improve its practicality and accuracy.

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List of standards

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Acknowledgement

This study was supported by Scientific Research Foundation of Nanjing Forestry University (GXL2019074), A Project Funded by the National First-class Disciplines (PNFN), and A Priority Academic Program Development of Jiangsu Higher Education Institutions (PAPD).

Appendix 1: FEM results

	Element	Axial force (N)	Shear X-Y (N)	Shear X-Z (N)	Bending YY (Nmm)
Load case I					
Front rail	E17	-76.78	250.48	-0.49	-10.59
Side rail	E16	-27.60	-250.15	0.29	-4.07
Front stretcher	E10	71.16	0	0	-0.21
Side stretcher	E12	29.35	0	0	-0.15
Leg	E21	-500.05	27.59	-76.91	5476.62
Load case II					
Front rail	E4	17.71	249.95	0.73	1583.99
	E17	-97.70	-248.41	382.01*	6440.82*
Side rail	E15	-219.07	-513.15	-58.33	521.69
	E16	-217.95	-11.74	-46.36	4678.25
Front stretcher	E10	37.78	0	0	-1028.45*
	E14	40.40	0	0	-109.49
Side stretcher	E12	32.13	-384.72	-0.69	106.32
	E13	32.12	-384.72	-0.69	44.20
Leg	E3	-761.43	223.41*	-41.26	2818.85
	E21	-236.65	164.12	-39.37	2851.98
Load case III					
Front rail	E17	-233.16	-62.09	44.55	5757.75
	E18	-234.09	439.69	44.45	1307.60
Side rail	E7	20.85	-249.93	-0.50	-800.42
	E16	-71.69	248.40	-381.64	-8194.60
Front stretcher	E9	51.96*	405.77	4.13	256.95
	E10	51.88	405.78*	4.14	670.63
Side stretcher	E12	23.25	0	0	-22.26
	E24	28.92	0	0	-7.51
Leg	E8	-687.76	-23.80	238.95*	-12149.70
	E21	-310.32	-27.35	149.329	-6183.75
Load case IV					
Front rail	E17	-324.56	-99.05	-100.80	-4943.27
	E18	-325.38*	403.44	-102.53	5229.70
Side rail	E15	-281.56	-448.22	120.83	5539.07
	E16	-280.64	53.59	119.00	-5257.49
Front stretcher	E14	44.54	-275.43	-14.93	744.13
	E23	44.57	-275.43	-14.80	-742.24
Front stretcher	E24	27.58	232.05	9.44	410.66
	E25	27.58	232.06	9.20	-428.19
Leg	E19	-778.96*	130.73	165.55	8914.10*
	E21	-148.45	127.70	122.82	-4207.29

Note: negative values represent the directions of force or moment; * indicates the maximum values for the leg, rail, and stretcher in the four load cases.

Appendix 2: Program for optimizing the dimensions of a stool section

Master routine：

```
x0=[30;20;20;15;20;15];  %  The initial dimensions of beam sections;
A=[];b=[];
Aeq=[];beq=[];
vlb=[0;0;0;0;0;0];vub=[30;20;20;15;20;15];  % Range of parameters;
[x,fval]=fmincon('chair',x0,A,b,Aeq,beq,vlb,vub,'constraints');  % Call functions;
```

Objective function：

```
function f=chair(x)  % Define objective function;
f=1200*x(1)*x(2)+496*x(3)*x(4)+496*x(5)*x(6);
```

Constraints：

```
function [g,ceq]=constraints(x)   % Define constraints ；
g(1)=779/(x(1)*x(2))-53.62;
g(2)=325/(x(3)*x(4))-53.62;
g(3)=52/(x(5)*x(6))-53.62;
g(4)=779/(x(1)*x(2))+6*8914/(x(2)*x(1)^2)+6*12488/(x(1)*x(2)^2)-50.55;
g(5)=98/(x(3)*x(4))+6*6441/(x(4)*x(3)^2)+6*17179/(x(3)*x(4)^2)-50.55;
g(6)=52/(x(5)*x(6))+6*671/(x(6)*x(5)^2)+6*20736/(x(5)*x(6)^2)-50.55;
g(7)=3*223/(2*x(2)*x(1)^2)-5;
g(8)=3*239/(2*x(1)*x(2)^2)-5;
g(9)=3*513/(2*x(4)*x(3)^2)-5;
g(10)=3*382/(2*x(3)*x(4)^2)-5;
g(11)=3*405/(2*x(6)*x(5)^2)-5;
g(12)=3*15/(2*x(5)*x(6)^2)-5;
g(13)=x(2)-x(1);
g(14)=x(4)-x(3);
g(15)=x(6)-x(5);
g(16)=x(4)-x(2);
g(17)=x(6)-x(4);
ceq=[];
```

Submission date: 6.02.2018

Online publication date: 15.11.2019