ANALYSIS OF THE DETERMINATION OF THE ACCURACY PARAMETER
FOR DUAL RECEIVERS
BASED ON EGNOS SOLUTION IN AERIAL NAVIGATION

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Abstract: The paper presents the results of research on the determination of the accuracy parameter for European Geostationary Navigation Overlay System (EGNOS) positioning for a dual set of on-board global navigation satellite system (GNSS) receivers. The study focusses in particular on presenting a modified algorithm to determine the accuracy of EGNOS positioning for a mixed model with measurement weights. The mathematical algorithm considers the measurement weights as a function of the squared inverse and the inverse of the position dilution of precision (PDOP) geometrical coefficient. The research uses actual EGNOS measurement data recorded by two on-board GNSS receivers installed in a Diamond DA 20-C airplane. The calculations determined the accuracy of EGNOS positioning separately for each receiver and the resultant value for the set of two GNSS receivers. Based on the conducted tests, it was determined that the mixed model with measurement weights in the form of a function of the inverse square of the PDOP geometrical coefficient was the most efficient and that it improved the accuracy of EGNOS positioning by 37%–63% compared to the results of position errors calculated separately for each GNSS receiver.

Keywords: SBAS, EGNOS, accuracy, GNSS receivers, position errors

1. INTRODUCTION

Satellite-based augmentation system (SBAS) positioning systems enable the determination of the four main parameters of global navigation satellite system (GNSS) positioning in aviation, i.e. of the accuracy, continuity, availability and integrity parameters [1, 2]. Accuracy is understood as the comparison of the determined coordinates of the aerial vehicle with the reference trajectory of the flight. Thus, it may be stated that the accuracy of SBAS positioning in aerial navigation is the difference between the coordinates of the aerial vehicle determined using the SBAS solution and the reference position of the flight [3]. The availability parameter defines the period during which the SBAS system was functioning and enabled a navigation solution of the position of the aircraft without any interruptions or failures [4]. The continuity of SBAS positioning is defined as the capacity of the system to function without any unplanned failures on the route of the flight [5]. Finally, the integrity as a quality parameter of GNSS positioning is, in fact, a measure of the trust that can be placed on the measurement results obtained from the navigation solution [6]. If this definition is referred directly to aviation, integrity describes the level of trust in navigating both in the horizontal and vertical planes. Among the parameters of the quality of SBAS positioning, the accuracy is the most important, and it requires continuous tests and analyses for specific types of aviation operations.

2. SCIENTIFIC KNOWLEDGE ANALYSIS

The accuracy of SBAS positioning has been studied and analysed in numerous aviation experiments. In our part of the globe, these analyses focussed mainly on the functioning and operation of the European Geostationary Navigation Overlay System (EGNOS) system [7]. The institutions that have been actively involved in the research on the application of the EGNOS support system in Polish aviation since the beginning include the Polish Air Force University in Dęblin and the University of Warmia and Mazury in Olsztyn. In Poland, the first tests with use of the EGNOS system were started in 2003. At that time, the EGNOS system was in the EGNOS System Test Bed (ESTB) test phase [8]. The research determined the accuracy of EGNOS positioning for on-board GNSS receivers. The coordinates of the aerial vehicle from the EGNOS solution were compared to the reference position of the flight calculated with the RTK-OTF (Real Time Kinematic – On The Fly) differential technique [9]. It should be added that, during the realisation of test flights, numerous breakdowns in the functioning of the EGNOS system were noted, which also led to the deficiencies in the determination of the accuracy of EGNOS positioning in measurement epochs. Further aviation tests were conducted in 2007, when the accuracy of EGNOS positioning was analysed as part of the Open Service (OS) system of the EGNOS system [10]. The flight experiment analysed the accuracy of EGNOS positioning for various classes of GNSS navigation receivers. During the analyses, once again, interrup-

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DOI 10.2478/ama-2022-0043

tions in receiving the corrections from the EGNOS system were noted, which resulted in a deteriorated accuracy of EGNOS positioning. The subsequent aviation experiments conducted with the use of the EGNOS system took place in the years 2010–2011, when a new service was introduced in EGNOS positioning, i.e. the Safety of Life (SoL) service [11]. For example, Grzegorzewski et al. [12] presented the results of the accuracy of EGNOS positioning for test flights conducted in south-eastern Poland. The analysis of the results revealed a low accuracy of EGNOS positioning for flights performed in the area of Chełm. Additionally, the results for EGNOS positioning were worse than those for automated GPS (Global Positioning System) positioning. Further flight experiments with the EGNOS system are described in Fellner and Jafernik [13] and Fellner et al. [14]. These studies present the results of EGNOS positioning as part of the SBAS (Satellite Based Augmentation system) APV (Approach with Vertical Guidance) landing procedures for the airports in Katowice and Mielec.

Later, more studies were conducted to assess the accuracy of EGNOS positioning, for the GNSS reference station installed at the Olsztyn-Datki airport in north-eastern Poland [15]. The present research project aimed to present the selection of the best location and the manner of stabilising the station to monitor GNSS signals at the airport. A station that locally monitors the signal from GPS and EGNOS satellites will, in consequence, improve the safety during the landing of aerial vehicles that sue the GNSS approach procedures and will allow determining the quality of GPS/EGNOS positioning in aviation. Other research experiments with the use of the EGNOS system in aerial navigation were conducted in Dęblin and Olsztyn, where physical reference GNSS stations were installed in order to monitor the quality of GPS/EGNOS data [16, 17]. The studies involved calculating the accuracy and integrity of EGNOS positioning in real time. Similar research works were described in the studies of Felski and Nowak [18] and Jafernik [19]. These articles also presented the results of the accuracy of EGNOS positioning for GNSS reference stations installed in Polish airports. The calculations were performed in real time and in post-processing mode. The next reproach experiment is described in the research of Ciećko and Grunwald [20]. It comprised the analysis of the accuracy of EGNOS positioning in a trial flight test for the purposes of checking the requirements of en-route navigation and precise landing approach PA category I.

As far as research conducted in Europe is concerned, there is a significant body of research that deserves mention [21–26]. In these studies, the accuracy of EGNOS positioning was determined for the given landing approach procedure, mainly precise procedure PA (Precision Approach) category I or SBAS APV.

The analysis of the state of knowledge reveals the following:
- Since the beginning of research with the use of the EGNOS system in aviation, the accuracy parameter was the essential parameter to be determined in aerial navigation;
- The determined values of the accuracy parameter have changed with the development, modernisation and introduction of the new EGNOS positioning services, e.g. based on data from EGNOS satellites: PRN123, PRN126 and PRN136;
- The accuracy of EGNOS positioning in flight experiments was determined and calculated for the EGNOS solution from a single receiver;
- The analysis of the state of knowledge shows that the topic of analysing the accuracy of EGNOS positioning was very important, which is reflected in the number of research projects.

In reference to the analysis of the state of knowledge, the existing scope of research may be extended to include the following elements:
- The accuracy of EGNOS positioning in flight tests should be determined based on a multi-receiver EGNOS solution;
- The research on the accuracy of EGNOS positioning in flight tests should use at least two GNSS receivers with the EGNOS tracking function;
- If at least two GNSS receivers are used, various mathematical models should be applied to enable the determination of the resultant EGNOS positioning accuracy.

This article presents the strategy of determining the parameter of accuracy for EGNOS positioning for two GNSS receivers. For this purpose, two different models of determining the EGNOS positioning accuracy were presented and applied in practice. The numerical calculations were based on a mixed model for various measurement weights to determine the accuracy for a set of two GNSS receivers. The obtained research results revealed that the application of the mixed model in calculations significantly improved the accuracy of EGNOS positioning.

Summarising, the main author's contribution to the work is as follows:
- development of an integration model of the EGNOS solution for two GNSS receivers,
- implementation of a linear combination model based on weighting factors,
- application of selected different weighting factors,
- demonstration of the effectiveness of the proposed model for determining the accuracy of the EGNOS solution for two GNSS receivers,
- implementation of the developed algorithm for GPS and EGNOS kinematic data from an aviation experiment.

The forthcoming portions of the article are classifiable as follows: The third section presents the research method, the fourth the research test, the fifth the research results, the sixth discusses the results and the final section presents the conclusions.

3. RESEARCH METHOD

The research methodology was based on two mathematical models that enable the determination of the accuracy parameter of EGNOS positioning for a measurement system consisting of two GNSS receivers. The first mathematical model concerns the determination of the accuracy of EGNOS positioning for the mixed model that is based on the measurement weights (α, β), as shown in Eq. (1):

$$\begin{cases} dB = \alpha \cdot dB_{Rx1} + \beta \cdot dB_{Rx2} \\ dL = \alpha \cdot dL_{Rx1} + \beta \cdot dL_{Rx2} \\ dh = \alpha \cdot dh_{Rx1} + \beta \cdot dh_{Rx2} \end{cases} \qquad (1)$$

where (dB, dL, dh) are position errors, resultant value of the accuracy of EGNOS positioning, $Rx1$ is GNSS receiver 1, $Rx2$ is GNSS receiver 2, α is measurement weight for receiver $Rx1$, $\alpha = \frac{1}{PDOP_{Rx1}^2}$, $PDOP_{Rx1}$ is value of the position dilution of precision (PDOP) geometrical coefficient [27] for receiver $Rx1$, β is measurement weight for receiver $Rx2$, $\beta = \frac{1}{PDOP_{Rx2}^2}$, $PDOP_{Rx2}$ is value of the PDOP geometrical coefficient for receiver $Rx2$, $(dB_{Rx1}, dL_{Rx1}, dh_{Rx1})$ are position

errors [28], positioning accuracy determined from a single EGNOS solution for receiver $Rx1$, and $(dB_{Rx2}, dL_{Rx2}, dh_{Rx2})$ are position errors [28], positioning accuracy determined from a single EGNOS solution for receiver $Rx2$.

Eq. (1) describes an algorithm of the mixed model for the determination of the accuracy parameter of EGNOS positioning. In Eq. (1), measurement weights (α, β) are used for the model of integrating the values of accuracy of EGNOS positioning for a single GNSS receiver. The measurement weights (α, β) were calculated as a function of the inverse square of the PDOP geometrical coefficient determined for a single EGNOS solution from a single GNSS receiver. The mathematical model (1) ensures a linear combination of the position errors $(dB_{Rx1}, dL_{Rx1}, dh_{Rx1})$ determined for receiver $Rx1$ and the position errors $(dB_{Rx2}, dL_{Rx2}, dh_{Rx2})$ determined for receiver $Rx2$. As a result, Eq. (1) will finally enable the determination of the resultant accuracy of EGNOS positioning for two GNSS receivers.

The second mathematical solution comprises a mixed model that uses measurement weights (γ, δ) to determine the accuracy of EGNOS positioning, as shown in Eq. (2):

$$\begin{cases} dB = \gamma \cdot dB_{Rx1} + \delta \cdot dB_{Rx2} \\ dL = \gamma \cdot dL_{Rx1} + \delta \cdot dL_{Rx2} \\ dh = \gamma \cdot dh_{Rx1} + \delta \cdot dh_{Rx2} \end{cases} \quad (2)$$

where γ is measurement weight for receiver $Rx1$, $\gamma = \frac{1}{PDOP_{Rx1}}$, δ is measurement weight for receiver $Rx2$ and $\delta = \frac{1}{PDOP_{Rx2}}$.

Eq. (2) describes an algorithm of the weighted average model for the determination of the accuracy parameter of EGNOS positioning. In Eq. (2), measurement weights (γ, δ) are used for the model of integrating the values of accuracy of EGNOS positioning for a single GNSS receiver. The (γ, δ) measurement weights were calculated as the inverse of the PDOP geometrical coefficients determined for a single EGNOS solution and a single GNSS receiver. Similarly to Eq. (1), the mathematical model (2) provides a linear combination of the position errors $(dB_{Rx1}, dL_{Rx1}, dh_{Rx1})$ determined for receiver $Rx1$ and the position errors $(dB_{Rx2}, dL_{Rx2}, dh_{Rx2})$ determined for receiver $Rx2$. Based on that, Eq. (2) enables the determination of the resultant accuracy of EGNOS positioning for two GNSS receivers in another way.

4. RESEARCH TEST

The presented algorithm for the determination of the value of accuracy of EGNOS positioning with the use of two GNSS receivers has been verified and tested during a flight experiment. The experiment was conducted in north-eastern Poland in the autumn of 2020. The flight experiment comprised a test flight with a Diamond DA 20-C aircraft. Figs. 1 and 2 present the horizontal and vertical trajectories of the flight of the aircraft, respectively. The test flight on the route Olsztyn-Suwałki-Olsztyn lasted approximately 4 h. Two geodesic receivers (one manufactured by Septentrio (manufactory: Belgium) and another by Trimble (manufactory: USA) were installed on board the aircraft. For the Septentrio receiver, the AT1675-29 PolaNt* GG satellite antenna was used, and for the Trimble receiver, the GA830 type antenna was used [29]. These receivers recorded GNSS data at 1-s intervals. The collected GNSS data enabled determining the coordinates of the airplane from the EGNOS solution for each of the receivers separately [30] and then computation of the position errors, i.e. in this case, the parameters $(dB_{Rx1}, dL_{Rx1}, dh_{Rx1})$ and $(dB_{Rx2}, dL_{Rx2}, dh_{Rx2})$. The position errors $(dB_{Rx1}, dL_{Rx1}, dh_{Rx1})$ and $(dB_{Rx2}, dL_{Rx2}, dh_{Rx2})$ were determined based on the comparison of the coordinates of the aircraft obtained from GPS solution with EGNOS corrections and the reference position of the flight calculated with the use of the RTK-OTF differential technique [8, 10]. The aircraft position was estimated based on GPS data as a GNSS system and also EGNOS corrections as a SBAS system [31, 32].
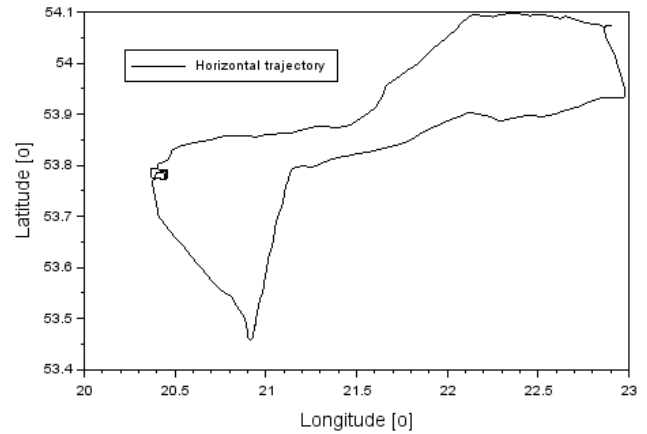

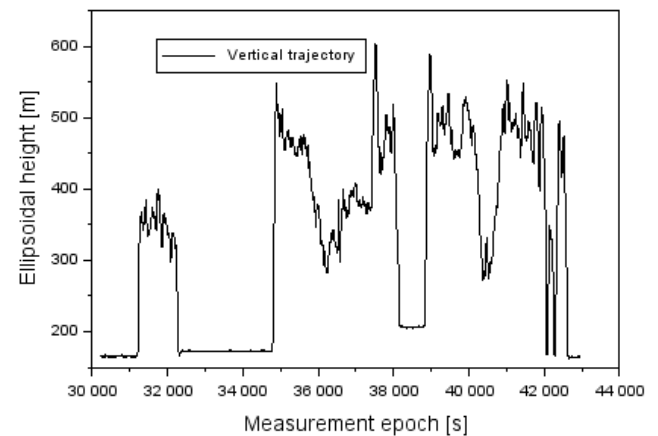**Fig. 1.** The horizontal trajectory of aircraft (own study)

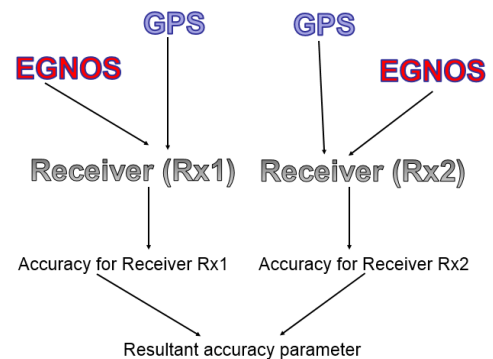**Fig. 2.** The vertical trajectory of aircraft (own study)

Fig. 3. The flowchart of presented mathematical algorithm (own study). EGNOS, European Geostationary Navigation Overlay System

The final parameters $(dB_{Rx1}, dL_{Rx1}, dh_{Rx1})$ and $(dB_{Rx2}, dL_{Rx2}, dh_{Rx2})$ define the accuracy of GPS + EGNOS

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DOI 10.2478/ama-2022-0043

positioning for each GNSS receiver separately. The subsequent stage consisted in the development and practical application of the algorithms (1) and (2) for the purposes of determining the resultant accuracy of GPS + EGNOS positioning for a set of two GNSS receivers. For this purpose, a digital application was developed in the Scilab v.6.0.0 programming language [33], which was developed by writing the source codes for Eqs (1) and (2). As a result, the positioning accuracy was calculated for the mixed model, the weighted average model and the arithmetic average model. The results of the conducted numerical analyses are presented in Section 5. Fig. 3 shows the final flowchart of the computational algorithm developed for Eqs (1) and (2).

5. RESEARCH RESULTS

The presentation of the test results begins with presenting the values of position errors $(dB_{Rx1}, dL_{Rx1}, dh_{Rx1})$ and $(dB_{Rx2}, dL_{Rx2}, dh_{Rx2})$ obtained separately for each GNSS receiver. Fig. 4 presents the values of the parameters $(dB_{Rx1}, dL_{Rx1}, dh_{Rx1})$. In the presented diagrams, the Trimble receiver is referred to as $Rx1$. The values of the position errors are as follows: for the B (Latitude) component, from -2.37 m to $+1.15$ m; for the L (Longitude) component, from -2.08 m to $+1.88$ m; and for the h (ellipsoidal height) component, from -2.47 m to $+5.64$ m. It is worth noting that since the epoch of 36,000 s the positioning accuracy for the $Rx1$ receiver has been decreasing. This is due to a decrease in the number of tracked GPS satellites, which in turn affects the availability of EGNOS corrections for GPS satellites. From the epoch of 30,000 s to 36,000 s, the number of GPS satellites ranged from 6 to 13, and from the epoch of 36,000 s to the end of the experiment, it dropped sharply from 10 to 7. The smaller the number of tracked GPS satellites, the larger the increase in the PDOP geometric coefficient, as shown in Fig. 6.
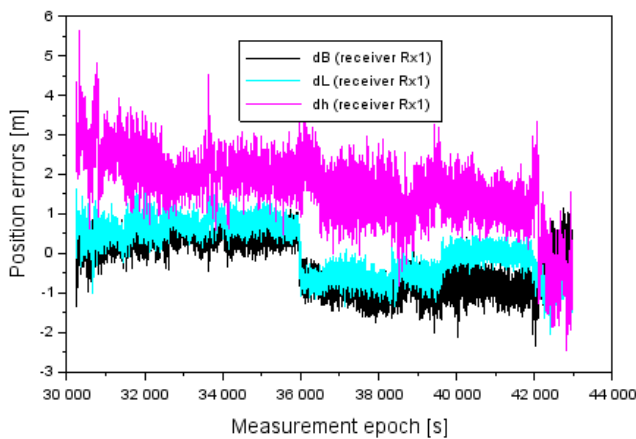

Fig. 4. The position errors of EGNOS positioning for receiver Rx1 (own study)

Fig. 5 presents the results of the values of the parameters $(dB_{Rx2}, dL_{Rx2}, dh_{Rx2})$. In the presented diagrams, the Septentrio receiver is referred to as $Rx2$. The values of the position errors are as follows: for the B component, from -12.65 m to $+2.08$ m; for the L component, from -9.63 m to $+7.22$ m; and for the h component, from -1.38 m to $+14.34$ m. The comparison of the position error results for the receivers $Rx1$ and $Rx2$ reveals

that the divergence in position errors is significantly higher for the Septentrio receiver. This is particularly visible in the initial phase of the flight, when the accuracy of EGNOS positioning for receiver $Rx2$ falls below ±10 m. This is due to the low number of GPS satellites being tracked, i.e. only five satellites. This, in turn, affects the deterioration of the positioning conditions, hence the high values of the PDOP coefficient.

Fig. 6 presents the values of PDOP parameter for both GNSS receivers. In the initial measurement epochs, the values of the $PDOP$ coefficient for receiver $Rx2$ amount to almost 8.7, which results in a low accuracy of EGNOS positioning. For the other measurement epochs, the values of the $PDOP$ coefficient are lower than 2.5. In turn, the PDOP values for the $Rx1$ receiver range from 1.6 to 8.4. As for the $Rx2$ receiver, the highest PDOP values for the $Rx1$ receiver are visible in the initial phase of the experiment, where the number of GPS satellites is five. This means that the accuracy of EGNOS positioning decreases with the increase in the PDOP coefficient.
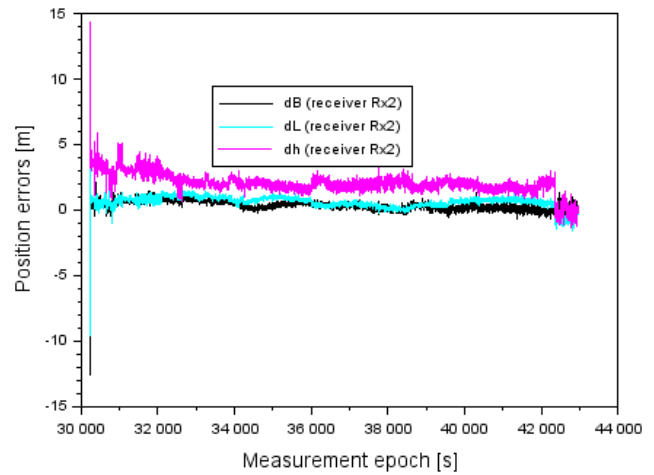

Fig. 5. The position errors of EGNOS positioning for receiver Rx2 (own study). EGNOS, European Geostationary Navigation Overlay System
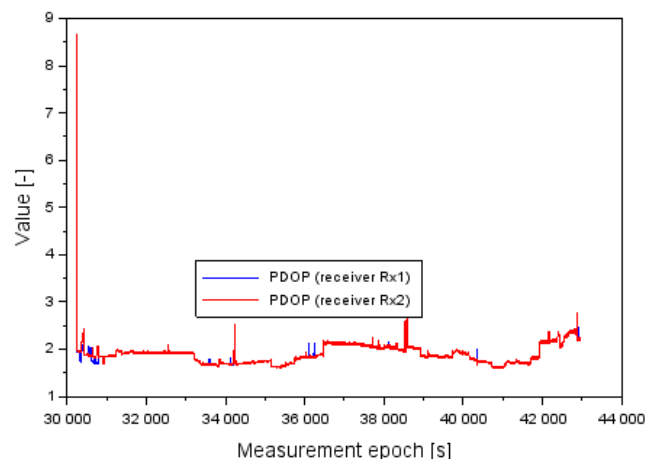

Fig. 6. The values of PDOP parameters (own study). PDOP, position dilution of precision

The further analyses of the obtained results included the values of the weight coefficients (α, β) for Eq. (1), which are presented in Fig. 7. The values of the α coefficient range from 0.014 to 0.394, while the values of the β coefficient range from 0.013 to

0.395. One may notice that the values of these coefficients (α,β) decrease with the increase in the value of the $PDOP$. These relations may also be reversed: as the $PDOP$ geometrical coefficient decreases, the weight coefficients (α, β) increase.

Fig. 8 presents the determined values of the accuracy of EGNOS positioning for two receivers, calculated from Eq. (1). The values of the positioning errors for component B ranged from −0.58 m to +0.64 m while the values of positioning errors for the L component ranged from −0.62 m to +1.07 m; and finally, the values of the positioning error for the h component ranged from −0.54 m to +2.69 m. The results presented in Fig. 8 demonstrate that the best accuracy results were obtained for the B component, while the worst ones for the vertical component h.
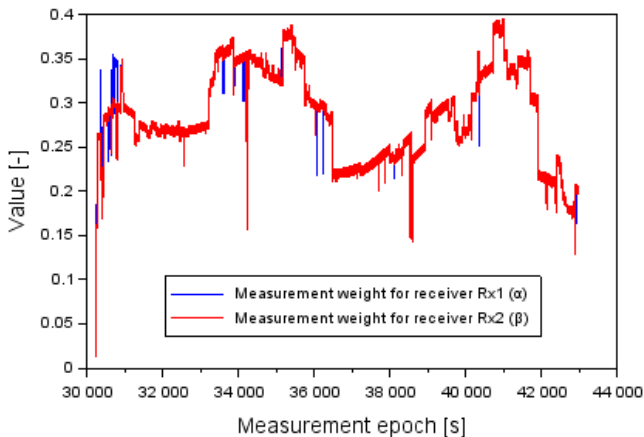

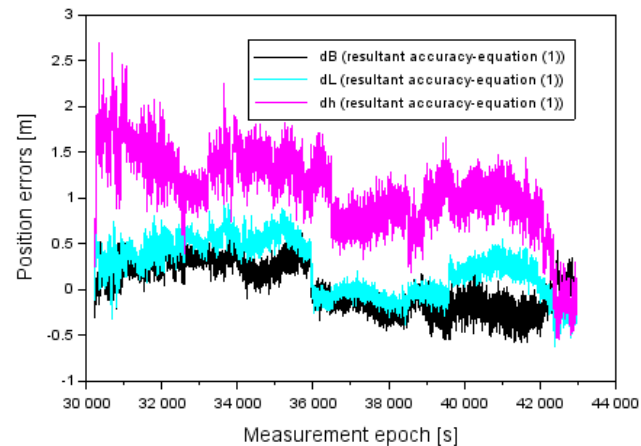**Fig. 7.** The values of measurement weights (α, β) (own study)

Fig. 8. The resultant accuracy of EGNOS dual receivers' solution based on Eq. (1) (own study). EGNOS, European Geostationary Navigation Overlay System

Fig. 9 presents the values of the weight coefficients (γ, δ) for Eq. (2). The values of the γ coefficient ranged from 0.019 to 0.628, while those of the δ coefficient ranged from 0.015 to 0.628. It should be noted that the change in the (γ, δ) coefficient depends on the value of the $PDOP$ parameter.

Fig. 10 presents the determined values of the accuracy of EGNOS positioning for two receivers, calculated from Eq. (2). The values of the positioning errors for component B ranged from −1.58 m to +1.22 m while the values of positioning errors for the L component ranged from −1.46 m to +1.80 m; and finally, the values of the positioning error for the h component ranged from −1.18 m to +5.07 m. The results presented in Fig. 10 demonstrate

that the best accuracy results were obtained for the B component, while the worst ones for the vertical component h.
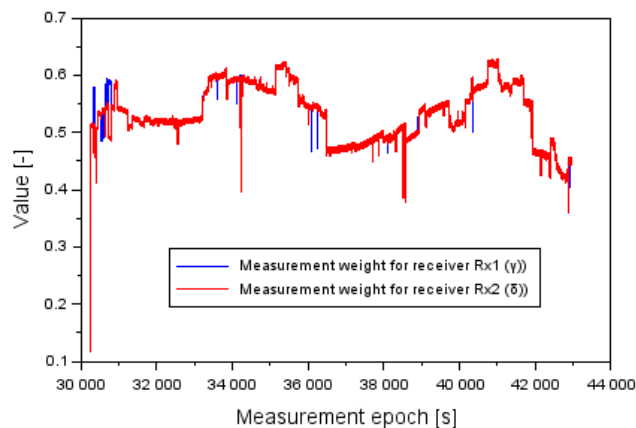

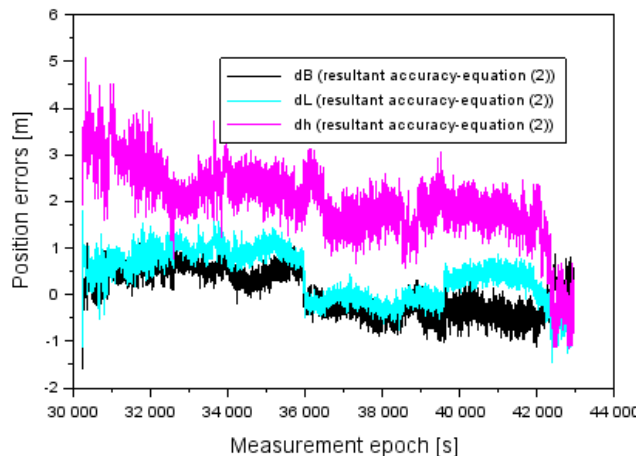**Fig. 9.** The values of measurement weights (γ, δ) (own study)

Fig. 10. The resultant accuracy of EGNOS dual receivers' solution based on Eq. (2) (own study). EGNOS, European Geostationary Navigation Overlay System

6. DISCUSSION

The discussion has been divided into two topics. In the first part, the authors present the influence of the algorithms (1) and (2) on the improvement of the accuracy of EGNOS positioning for a dual set of GNSS receivers. Later, in the second part, the obtained research results are compared with the existing state of knowledge.

Fig. 11 presents the results of the comparison of the obtained average positioning errors dB for receivers $Rx1$ and $Rx2$, based on Eqs (1) and (2). As one may notice, the highest positioning accuracy for the B component was obtained from Eq. (1). On the other hand, the lowest accuracy along the B axis is noticeable for the $Rx1$ receiver. The average accuracy values for the B coordinate are 0.62 m for receiver $Rx1$, 0.40 m for receiver $Rx2$, 0.23 m for Eq. (1) and 0.43 m for Eq. (2). It is worth adding that the mathematical model (1) improved the positioning accuracy along the B axis by 63% compared to the results for receiver $Rx1$, 42% compared to the results for receiver $Rx2$ and 46% compared to the results for the mathematical model (2). On the other hand, the application of Eq. (2) only improved the accuracy of positioning along the B axis by 31% compared to the results obtained for receiver $Rx1$.

Fig. 12 presents the results of the comparison of the obtained average positioning errors dL for receivers $Rx1$ and $Rx2$, based on Eqs (1) and (2). As one may notice, the highest positioning accuracy for the L component was obtained from Eq. (1). On the other hand, the lowest accuracy along the L axis was noticeable for the $Rx2$ receiver. The average accuracy values for the L coordinate are as follows: 0.57 m for receiver $Rx1$, 0.67 m for receiver $Rx2$, 0.30 m for Eq. (1) and 0.55 m for Eq. (2). It is worth adding that the mathematical model (1) improved the positioning accuracy along the L axis by 48% compared to the results for receiver $Rx1$, 55% compared to the results for receiver $Rx2$ and 45% compared to the results for mathematical model (2). On the other hand, the application of Eq. (2) improved the positioning accuracy along the L axis by 5% compared to the results for receiver $Rx1$ and 18% as compared to those for receiver $Rx2$.
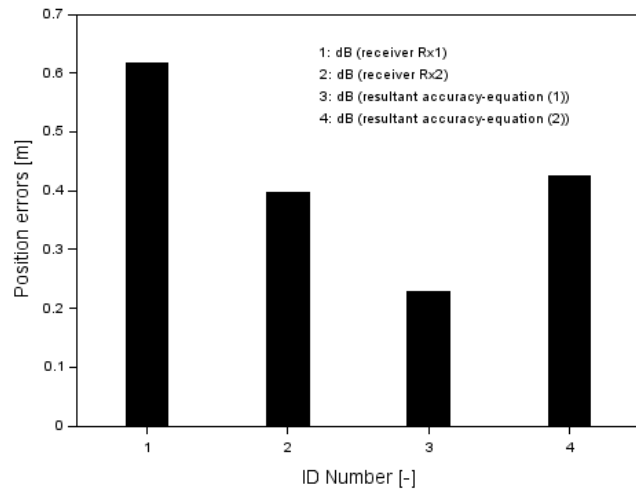

Fig. 11. The comparison of average value of accuracy of latitude (own study)
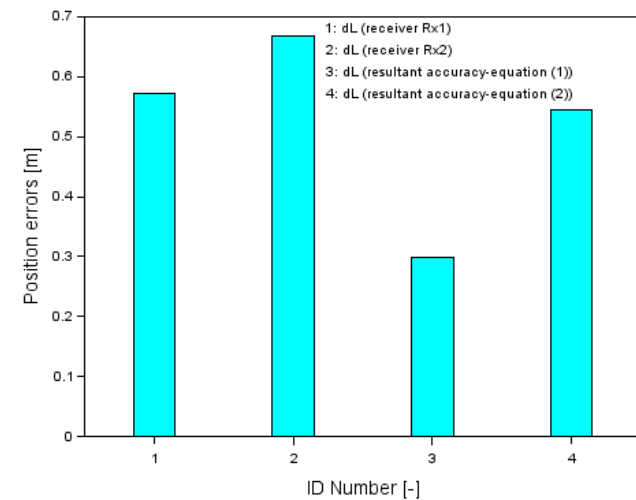

Fig. 12. The comparison of average value of accuracy of longitude (own study)

Fig. 13 presents the results of the comparison of the obtained average positioning errors dh for receivers $Rx1$ and $Rx2$, based on Eqs (1) and (2). As one may notice, the highest positioning accuracy for the h component was obtained from Eq. (1). On the other hand, the lowest accuracy along the h axis was noticeable for the $Rx2$ receiver. The average accuracy values for the h coor-

dinate are as follows: 1.75 m for receiver $Rx1$, 2.06 m for receiver $Rx2$, 1.10 m for Eq. (1) and 2.04 m for Eq. (2). It is worth adding that the mathematical model (1) improved the positioning accuracy along the h axis by 37% compared to the results for receiver $Rx1$, 47% compared to the results for receiver $Rx2$ and 46% compared to the results for the mathematical model (2). On the other hand, the application of Eq. (2) improved the positioning accuracy along the h axis by 16% compared to the results for receiver $Rx1$ and 1% compared to those for receiver $Rx2$.
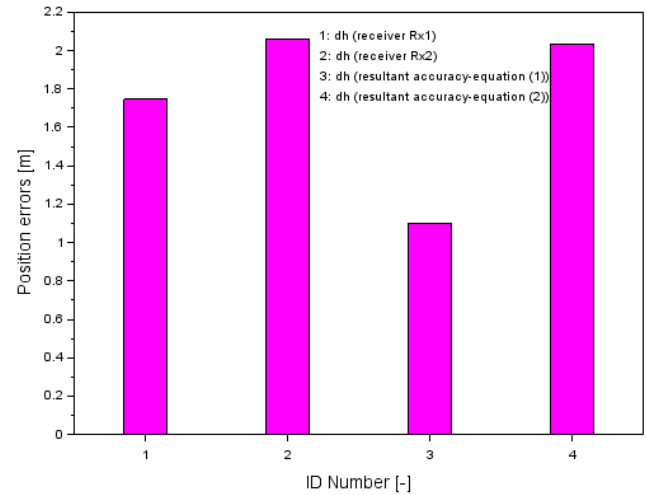

Fig. 13. The comparison of average value of accuracy of ellipsoidal height (own study)

The further part of the discussion shows the effectiveness and efficiency of the research method presented in the paper. Specifically, the mathematical algorithm for Eqs (1) and (2) has been changed with new weighting factor values as described below:

$$\begin{cases} dB = \rho \cdot dB_{Rx1} + \sigma \cdot dB_{Rx2} \\ dL = \rho \cdot dL_{Rx1} + \sigma \cdot dL_{Rx2} \\ dh = \rho \cdot dh_{Rx1} + \sigma \cdot dh_{Rx2} \end{cases} \quad (3)$$

where ρ is measurement weight for receiver $Rx1$, $\rho = \frac{1}{NS_{Rx1}}$, σ is measurement weight for receiver $Rx2$, $\sigma = \frac{1}{NS_{Rx2}}$, NS_{Rx1} is the number of GPS satellite with EGNOS corrections for receiver $Rx1$ [34] and NS_{Rx2} is the number of GPS satellite with EGNOS corrections for receiver $Rx2$.

Eq. (3) uses a different weighting factor calculated as a function of the number of GPS satellites for which EGNOS corrections were determined. The number of GPS satellites represents the satellites tracked by the GNSS receivers during the flight experiment [35]. Fig. 14 shows the test results for the mathematical algorithm used (3). The values of the positioning errors for component B ranged from −2.69 m to +0.26 m while the values of positioning errors for the L component ranged from −1.94 m to +1.64 m; and finally, the values of the positioning error for the h component ranged from −0.36 m to +3.41 m.

The obtained results of the EGNOS positioning accuracy for the mathematical model (3) show high efficiency in relation to the results obtained from algorithms (1) and (2). It is worth noting that the average dB accuracy was 0.08 m for algorithm (3), while 0.23 m for Eq. (1) and 0.43 m for Eq. (2). This shows the improvement of the EGNOS positioning accuracy from Eq. (3) by 65% over mathematical model (1) and 81% over mathematical model (2). In

addition, the average dL accuracy was 0.10 m for algorithm (3), while 0.30 m for Eq. (1) and 0.55 m for Eq. (2). This shows the improvement of the EGNOS positioning accuracy from Eq. (3) by 67% over mathematical model (1) and 82% over mathematical model (2). Moreover, the average dh accuracy was 0.38 m for algorithm (3), while 1.10 m for Eq. (1) and 2.04 m for Eq. (2). This shows the improvement of the EGNOS positioning accuracy from Eq. (3) by 65% over mathematical model (1) and 81% over mathematical model (2).
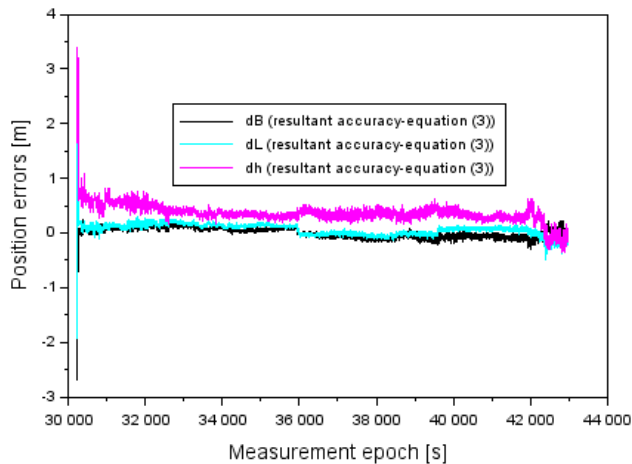

Fig. 14. The resultant accuracy of EGNOS dual receiver solution based on Eq. (3) (own study). EGNOS, European Geostationary Navigation Overlay System

To conclude the discussion, the obtained research results were compared to the analysis of the state of knowledge. The comparison included tests conducted during flight. The obtained positioning accuracy was better than that presented in several studies encountered in the literature [8, 10, 12, 20], all of which involve research works conducted in Poland. On the other hand, the results are comparable to those published in other studies [21, 23, 24, 26].

7. CONCLUSIONS

The paper presents the algorithms that enable an improvement in the accuracy of EGNOS positioning for a dual set of GNSS receivers. In particular, the study presents a computational diagram of a mixed model to improve the accuracy of EGNOS positioning in aerial navigation. In order to achieve it, various measurement weights were applied, which depended on the value of the PDOP geometrical coefficient. The weights were calculated as the inverse square of the PDOP coefficient and the inverse of the PDOP coefficient itself. This enabled performing a linear combination of single position error results from the EGNOS solution for a single GNSS receiver. This was the basis for developing an algorithm to determine the resultant accuracy of EGNOS positioning for two on-board GNSS receivers. The calculations were conducted with the use of actual navigation data from the EGNOS solution obtained from two GNSS receivers installed on board the Diamond DA 20-C aircraft. The final algorithm to improve the accuracy of EGNOS positioning was written in the Scilab v.6.0.0 language environment. The obtained results demonstrated the following:

– The application of Eq. (1) improved the positioning accuracy along the B axis by 63% compared to the results for receiver

$Rx1$, 42% compared to the results for receiver $Rx2$ and 46% compared to the results for the mathematical model (2);

– The application of Eq. (2) only improved the accuracy of positioning along the B axis by 31% compared to the results obtained for receiver $Rx1$;

– Mathematical model (1) improved the positioning accuracy along the L axis by 48% compared to the results for receiver $Rx1$, 55% compared to the results for receiver $Rx2$ and 45% compared to the results for the mathematical model (2);

– The application of Eq. (2) improved the positioning accuracy along the L axis by 5% compared to the results for receiver $Rx1$ and 18% compared to those for receiver $Rx2$;

– The mathematical model (1) improved the positioning accuracy along the h axis by 37% compared to the results for receiver $Rx1$, 47% compared to the results for receiver $Rx2$ and 46% compared to the results for the mathematical model (2);

– The application of Eq. (2) improved the positioning accuracy along the h axis by 16% compared to the results for receiver $Rx1$ and 1% compared to those for receiver $Rx2$.

The results of research demonstrated that the mathematical model developed to improve the accuracy of EGNOS positioning that used measurement weights as a function of the inverse square of the PDOP coefficient proved to be the most effective and yielded the best results in navigation calculations.

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DOI 10.2478/ama-2022-0043

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