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ANALYSIS OF THE DYNAMICS OF VEHICLE MOTION IN CASE OF SUDDEN BRAKING CONNECTED WITH DAMAGE OF THE TIRE

Summary. One of the urgent problems to be solved in the analysis of road traffic accidents is the calculation of vehicle speed at the time of sudden braking. This makes it possible to determine the dynamics of further movement of the vehicle and to determine the technical ability of the driver to prevent a traffic collision. The article presents a method employed in calculating the speed of the car at the time of sudden braking and determines the dynamics of movement up to its complete standstill. The correct calculation of the speed of movement allows us to obtain the reliability of the restored co-existence, as well as data for an objective analysis of the road traffic accident.

1. INTRODUCTION

When investigating road accidents, one of the focal factors determining the possibility of analyzing the mechanism of the event and its individual elements is determining the speed of the vehicle at the time of the accident. By the magnitude of the speed, the technical ability of the driver to prevent an accident is determined reproduce the relative position of the elements, objects and parties of the event at the moment of occurrence a danger to traffic [1, 4].

The existing methods for determining the vehicle speed are based on the calculation of the traces of braking, and in their absence, based on the observation of eyewitnesses.

However, an estimate of the vehicle speed by the tracks of braking does not always adequately present real events, and eyewitness reports can be rather vague. Appling scientific approaches based on the analysis of physical phenomena occurring in the contact of the wheel with the road surface is a more sound and reliable approach [5].

2. PROBLEM FORMULATION

As an example, let is consider a traffic accident in which a vehicle, as a result of sudden braking due to the destruction of the front left tire, sharply spins at an angle to the left with a sidewise skidding and simultaneous linear movement in the direction of the initial movement and rotation.

It is required to determine the speed of the vehicle at the time of the sidewise skidding, taking into account the linear movement of the center of mass of the vehicle.

3. PROBLEM SOLVING

To answer the question, let us assume as follows: the destruction of the tire occurred instantly; at the time of destruction of the tire of the front left wheel, the driver removed his foot from the accelerator pedal and braked; we will disregard the laws of the forces of rolling resistance, friction in the transmission of the car and air resistance. In addition, we will make allowance that at the time of tire destruction, the vehicle in the process of lateral deceleration simultaneously suffered a turn relative to the point of contact of the front left tire with the road. In this case, the stiffness of the front left wheel suspension during this process is so insignificant that it can be ignored in the calculations.

The vehicle sidewise skid at the point of going into the curve causes the excess of the linear velocity limits acceptable for riding stability. For this road traffic situation, we define the dynamics of the vehicle at the time of the sidewise skidding as follows: the sidewise skidding is followed by a linear movement of the vehicle in the direction of the initial movement and rotation. In this case, the total work of the tire's friction on the road is expressed as follows:

$$\sum A = A_t + A_{fr}, \tag{1}$$

where A_t is the work of the tire's friction on the road during the vehicle rotation, kGm; A_{fr} is the work of the friction on the road at the linear displacement of the mass center of the vehicle, kGm.
We take into account that

$$A_t = G_a \cdot \varphi_s \cdot L \cdot \pi \cdot n_{rev}, \tag{2}$$

where G_a is the weight of the vehicle, kG; φ_s is the coefficient of side grip between the tire and the roadway; L is the vehicle base length (fig. 1; [6]), m; and n_{rev} is the number of revolutions performed by the vehicle (see fig. 1) on the path of its linear motion in the direction of its initial motion.

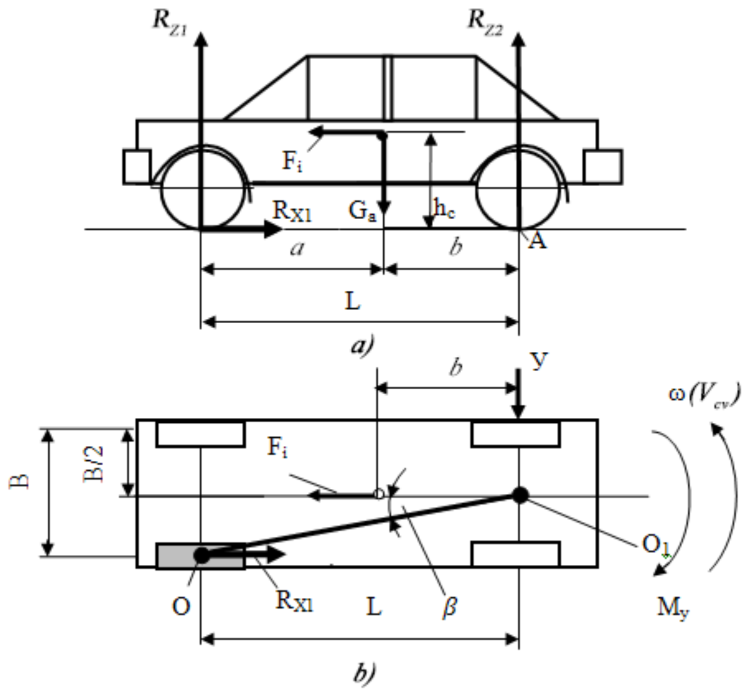

Fig. 1. Diagram of the forces acting on the vehicle when breaking the front left wheel: *a* – side view; *b* – plan view; *ω* – angular spin rate of the center of the rear axle; V_{cv} is the circumference velocity of the rear axle center. The n_{rev} will be calculated as follows

Kinetic energy of rotation of the car:

$$W_t = \frac{J \cdot \omega^2}{2}, \tag{3}$$

where J is the inertial torque toward the O-point, kg·m^2; ω is the angular velocity rate (see fig.1), rad/s.

Where

$$J = \frac{m_a \cdot L^2}{3 \cdot cos^2 \beta}, \tag{4}$$

where m_a is the car mass, kg; L is the car base length, m (see fig.1); and β is the angle between the longitudinal axis of the car and the line connecting points O and O_1 (see fig. 1).

Then, expression (3) will be written as follows:

$$W_t = \frac{m_a \cdot L^2 \cdot \omega^2}{6 \cdot cos^2 \beta}. \tag{5}$$

The angular velocity rate ω will be calculated as follows:

$$\omega = \frac{V_a \cdot cos(90 - \beta) \cdot cos \beta}{L} = \frac{V_a \cdot sin \beta \cdot cos \beta}{L}, \tag{6}$$

where V_a is the linear speed of the car at the start of its rotation, m/s.

Considering expression (6), expression (5) will be written as follows:

$$W_t = \frac{m_a \cdot V_a^2 \cdot sin^2 \beta}{6}. \tag{7}$$

The kinetic energy of rotation goes into the performans of the tire friction while turning during spinning around point O

$$W_t^1 = G_a \cdot \varphi_s \cdot 2 \cdot \pi \cdot \frac{L}{cos \beta} \cdot n_{rev}, \tag{8}$$

where G_a is the car's weight, kG; n_{rev} is the number of revolutions per time of linear displacement of the mass center.

Considering the equality $W_t = W_t,^1$ we will obtain

$$\frac{m_a \cdot V_a^2 \cdot sin^2 \beta}{6} = m_a \cdot g \cdot \varphi_s \cdot 2 \cdot \pi \cdot \frac{L}{cos \beta} \cdot n_{rev}. \tag{9}$$

The linear translation of point O to elementary distance dS^1 over an infinitesimally small period of time dt

$$dS^1 = V^1 dt, \tag{10}$$

where V^1 is the speed of linear movement of the vehicle, m/s.

At the movement of point O to an elementary distance $dS1$, point O1 will strike an infinitesimally small arc dS

$$dS = \frac{L}{cos \beta} \cdot d\gamma, \tag{11}$$

where $d\gamma$ is an infinitesimally small angle of the center of the rear axle (point O_1; see fig. 1), rad.

Where

$$d\gamma = \omega \cdot dt. \tag{12}$$

Considering expression (12), expression (11) will be written as follows:

$$dS = \frac{L}{cos \beta} \cdot \omega \cdot dt. \tag{13}$$

From expression (10)

$$dt = \frac{dS^1}{V^1}. \tag{14}$$

Considering expression (14), expression (13) is written as follows:

$$dS = \frac{L}{\cos\beta} \cdot \omega \frac{dS^1}{V^1}. \tag{15}$$

Upon integration of expression (15), we will obtain:

$$\int dS = \frac{L}{\cos\beta} \cdot \omega \frac{1}{V^1} \int dS^1. \tag{16}$$

Also, we will obtain the following formula:

$$S = \frac{L}{\cos\beta} \cdot \omega \frac{S^1}{V^1}. \tag{17}$$

By substituting value (17) ω from expression (6), we obtain

$$S = \frac{S^1 \cdot L}{V^1 \cdot \cos\beta} \cdot \frac{V_a \cdot \sin\beta \cdot \cos\beta}{L}. \tag{18}$$

Transforming expression (18), we obtain

$$S = \frac{S^1 \cdot V_a \cdot \sin\beta}{V^1}. \tag{19}$$

where

$$S = \frac{2\pi \cdot L}{\cos\beta} \cdot n_{rev}. \tag{20}$$

Considering expressions (19) and (20), we obtain:

$$\frac{S^1 \cdot V_a \cdot \sin\beta}{V^1} = \frac{2\pi \cdot L}{\cos\beta} \cdot n_{rev}. \tag{21}$$

From this, it follows that

$$V_a = \frac{2 \cdot \pi \cdot L \cdot V^1}{S^1 \cdot \sin\beta \cdot \cos\beta} \cdot n_{rev}. \tag{22}$$

On substituting value V_a into expression (9), we will obtain

$$\frac{4 \cdot \pi^2 \cdot L^2 \cdot (V^1)^2 \cdot n_{rev}{}^2 \cdot \sin^2\beta}{\cos^2\beta \cdot (S^1)^2 \cdot \sin^2\beta \cdot 3} = g \cdot \varphi_s \cdot \pi \frac{L}{\cos\beta} \cdot n_{rev}. \tag{23}$$

By transformation of expression (23) and considering

$$(V^1)^2 = 2 \cdot S^1 \cdot j_a, \tag{24}$$

we obtain

$$\frac{4 \cdot \pi \cdot L \cdot n_{rev} \cdot 2 \cdot S^1 \cdot j_a}{\cos\beta \cdot (S^1)^2 \cdot 3} = g \cdot \varphi_s. \tag{25}$$

where j_a is slowing down of the vehicle drifting at its further linear motion, $j_a = \varphi_1 \cdot g$; φ_1 is the coefficient of longitudinal grip between the tire and the roadway.

By transformation of expression (25) considering j_a, we obtain

$$\frac{8 \cdot \pi \cdot L \cdot n_{rev} \cdot g \cdot \varphi_1}{\cos\beta \cdot S^1 \cdot 3} = g \cdot \varphi_s. \tag{26}$$

Considering expression (26), we obtain

$$n_{rev} = \frac{3 \cdot \varphi_s \cdot \cos\beta \cdot S^1}{8 \cdot \pi \cdot L \cdot \varphi_1}, \tag{27}$$

where S^1 is the linear movement of the center of mass of the vehicle from the moment of its sidewise skidding down to complete standstill, m.

By transformation of expression (2) and considering expression (27), we obtain

$$A_t = G_a \cdot \varphi_s \cdot S^1 \cdot 0{,}375 \cdot \varphi_s / \varphi_1 \cdot \cos\beta. \tag{28}$$

Tire friction work at a linear car traverse is expressed as follows:

$$A_{fr} = G_a \cdot \varphi_s \cdot S^1. \tag{29}$$

Considering expressions (28) and (29), expression (2) will be written as follows:

$$\sum A = G_a \cdot \varphi_s \cdot S^1 \cdot (0{,}375 \cdot \frac{\varphi_s}{\varphi_1} \cdot \cos\beta + 1). \tag{30}$$

From this, the dynamics of the car motion at the sidewise skid will be written as follows:

$$V_a = \sqrt{\frac{2\left[G_a \cdot \varphi_s \cdot S^1 \cdot (0{,}375 \cdot \frac{\varphi_s}{\varphi_1} \cdot \cos\beta + 1)\right]}{m_a}}. \tag{31}$$

The vehicle's braking distance from the sudden braking down to a complete standstill is calculated from the following formula:

$$\sum t = t_t + t_l + t_{tl}, \tag{32}$$

where t_t is the turn time of the car with respect to point O (see fig. 1) and for angle α (fig. 2) by braking force with the left front wheel, s; t_l is the time of linear movement of the center of mass of the car from the moment of its sidewise skid down to complete standstill от, s; and t_{tl} is the time spent on rotation of the car at its forward-rotation movement, s.

In the picture (see fig.2), the angle of the deviation of the line, connecting the contact point (point O; see fig1b) between the front left wheel and the roadway and the center of the rear axle (point O_1; see fig.1 b) at the moment of turn with the front wheel, is shown. From the picture, it follows:

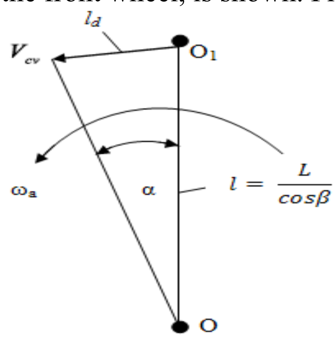

Fig. 2. Scheme of deviation of the rear axle of the car in relation to the point of the tire contact between the front left wheel and the roadway

$$\frac{F_i \cdot B}{2} = \frac{J \cdot \omega_a^{\ 2}}{2}, \tag{33}$$

where F_i is the inertia force at the moment of braking with the left front wheel (see fig.1), kG; J is the moment of inertia with respect to point O (see fig.1), kgm^2; and ω_a is the angular velocity of the car (see fig. 2), rad/s.

Let us transform the right-hand side of expression (33); we will obtain

$$\frac{J \cdot \omega_a^{\ 2}}{2} = \frac{m_a \cdot l^2 \cdot \omega_a^{\ 2}}{6}, \tag{34}$$

where

$$\omega_a = \frac{\alpha}{t_t}. \tag{35}$$

Then, expression (33) will be written as follows:

$$\frac{F_i \cdot B}{2} = \frac{m_a \cdot L^2 \cdot \alpha^2}{6 \cdot cos^2 \beta \cdot t_t^{\,2}}, \tag{36}$$

The arc length l_d (see fig. 2) can be written as follows:

$$V_{cv} \cdot t_t = l \cdot \alpha = \frac{L}{cos\,\beta} \cdot \alpha, \tag{37}$$

where V_{cv} is the circular velocity of the center of the car's rear axle, m/s.

Where

$$t_t = \frac{L}{cos\,\beta \cdot V_{cv}} \cdot \alpha. \tag{38}$$

Considering expression (38), expression (36) can be written as follows:

$$\frac{F_i \cdot B}{2} = \frac{m_a \cdot V_{cv}^{\,2}}{6}. \tag{39}$$

where

$$V_{cv} = \sqrt{\frac{3 \cdot F_i \cdot B}{m_a}}. \tag{40}$$

where

$$F_i = m_a \cdot j_T \cdot \delta, \tag{41}$$

where j_T is the deceleration of the car when braking with the front left wheel [6], m/s^2; δ is the coefficient considering the impact of rotation mass inertia of the car,

$$\delta = 1 + \sigma_1 \cdot u_{\kappa n}^{\,2} + \sigma_2, \tag{42}$$

where $u_{\kappa n}$ is the reduction rate of the hydromechanical transmission.

For the hydromechanical transmission of the car:

$$\sigma_1 = \frac{J_{rp} \cdot \eta_{tr} \cdot k_{ct}}{m_a \cdot r_k^{\,2}} \cdot \frac{d\omega_н}{d\omega_m}, \tag{43}$$

where J_{rp} is the inertia of the rotating parts of the engine and related transmission parts, kg·m^2; η_{tr} is the transmission efficiency; k_{ct} is the transformation coefficient; $\omega_н$, ω_m are the angular velocity of shaft and turbine rotations, respectively, c^{-1}; r_k is the wheel rolling radius, m; and m_a is the car mass, kg;

$$\sigma_2 = \frac{\sum J_k}{m_a \cdot r_k^{\,2}}, \tag{44}$$

where $\sum J_k$ is the total inertia moment of the car wheels, kg·m^2.

$$j_T = \frac{b \cdot g \cdot \varphi_1}{(2 \cdot L - h_c \cdot \varphi_1)}. \tag{45}$$

Considering expression (41), expression (40) will be written as follows:

$$V_{cv} = \sqrt{3 \cdot j_T \cdot \delta \cdot B}, \tag{46}$$

By substituting expression (46) into expression (38), we will obtain

$$t_t = \frac{L \cdot \alpha}{cos\,\beta} \cdot \sqrt{\frac{1}{3 \cdot j_T \cdot \delta \cdot B}}, \tag{47}$$

where angle α [6] equals

$$\alpha = \frac{0{,}5 \cdot B \cdot \delta \cdot cos\,\beta}{\varphi_s \cdot L} \cdot \frac{b \cdot \varphi_1}{(2L - h_c \cdot \varphi_1)}. \tag{48}$$

Then, the slew time considering expressions (45) and (48) will be written as follows:

$$t_t = \frac{L}{\cos\beta} \cdot \sqrt{\frac{1}{3 \cdot \frac{b \cdot g \cdot \varphi_1}{(2 \cdot L - h_c \cdot \varphi_1)} \cdot \delta \cdot B}} \cdot \frac{0{,}5 \cdot B \cdot \delta \cdot \cos\beta}{\varphi_s \cdot L} \cdot \frac{b \cdot \varphi_1}{(2 \cdot L - h_c \cdot \varphi_1)}. \tag{49}$$

The time of linear movement of the center of mass of the car from the sidewise skid start moment down to complete standstill of the car will be obtained considering the linear velocity using the following formula:

$$V_l = \sqrt{\frac{2 \cdot G_a \cdot \varphi_s \cdot S^1}{m_a}}. \tag{50}$$

From this, the linear movement of the center of mass of the car

$$t_l = \frac{S^1}{\sqrt{\frac{2 \cdot G_a \cdot \varphi_s \cdot S^1}{m_a}}}. \tag{51}$$

The time, spent on rotation at its translation movement,

$$t_{tl} = \frac{2 \cdot \pi \cdot L}{V_a \cdot \sin\beta \cdot \cos\beta} \cdot n_{rev}. \tag{52}$$

Considering expression (27) by converting expression (52), we obtain

$$t_{tl} = 0{,}75 \cdot \frac{\varphi_s \cdot S^1}{\varphi_1 \cdot V_a \cdot \sin\beta}. \tag{53}$$

Then, the time duration of the vehicle drift from the start of the spin till its complete stop at the end of its motion will be expressed as follows:

$$\sum t = \frac{L}{\cos\beta} \cdot \sqrt{\frac{1}{3 \cdot \frac{b \cdot g \cdot \varphi_1}{(2 \cdot L - h_c \cdot \varphi_1)} \cdot \delta \cdot B}} \cdot \frac{0{,}5 \cdot B \cdot \delta \cdot \cos\beta}{\varphi_s \cdot L} \cdot \frac{b \cdot \varphi_1}{(2 \cdot L - h_c \cdot \varphi_1)} + \\ + \frac{S^1}{\sqrt{\frac{2 \cdot G_a \cdot \varphi_s \cdot S^1}{m_a}}} + 0{,}75 \cdot \frac{\varphi_s \cdot S^1}{\varphi_1 \cdot V_a \cdot \sin\beta}. \tag{54}$$

To assess the reliability of the suggested method of speed calculation, the vehicle speeds were determined using the suggested method and computer simulations using the software «*PC-CRASH*». The initial data are as follows: drift distance from the moment of braking to a full stop – S^1 = 10, 30, 50, 70, 90 m; vehicle weight – 1545 kg; base – 2,78 m; track – 1,5 m; epy tire grip coefficient in the longitudinal direction – φ_1= 0,7; and tire-to-surface friction coefficient in the transverse direction φ_s = 0,35. The results of the calculations are presented in the table.

Table 1

Vehicle speed values on sudden braking of a vehicle plotted against the drift distance along the initial direction of the vehicle

The speeds calculated with the suggested method						Computer-simulated speeds					
S^1, m	10	30	50	70	90	S^1, m	10	30	50	70	90
V_a, m/s	9,0	15,7	20,3	24,0	27,3	V_a, m/s	9,0	13,6	21,4	24,2	29,3

The table shows that the vehicle speeds, calculated with the suggested method, are in agreement with the speeds as determined by computer simulation.

4. CONCLUSIONS

The suggested method for calculating the vehicle speed at the time of sudden braking provides a supporting rationale for scientifically based restoration of the course of events of a road accident and to identify the data that can serve as probative evidence for a traffic accident.

The currently available methods for calculating the speed of vehicles in the case of sudden breaking are based either on the visual perception of an event by an accident witness [7, 8] or on the data of the accident investigative experiment [9, 10]. In the first case, the reliability of the event coverage is hinged on human psychophysical abilities [11, 12], whereas in case of the simulation, it is complicated by different technical and operational properties of vehicles being used in the accident investigative experiment [13, 14]. For this reason, accident modeling solely based on the laws of physics [15, 16] is considered adequate and overriding.

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