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Mathematical Modeling of Stress in Circuit Cards Represented by Mechanical Oscillatory Systems

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ABSTRACT

The represented paper is aimed at stress calculation in circuit cards with their representation as a type of mechanical oscillatory systems in purpose of their strength assessment especially in resonance conditions. Three types of oscillatory systems are researched: single-mass; multiple mass and oscillatory system with uniformly distributed mass. In all types the cylindrical bending of circuit cards is considered to be a set of beam-strips with rectangular cross-sections so their stress calculation is performed by conventional methods applied in strength of materials and civil engineering. Mathematical model has been developed for maximal dynamic stress and deflection estimation in circuit card assemblies represented by unique oscillatory system as prismatic beam set on two oscillating supports under inertial resonance excitation generated by constant dynamic force. Comparative analysis of mathematical modeling, MatLab simulation and experimental determination of maximal dynamic stress and deflection accomplished for three types of oscillatory systems verified proximity of obtained results. Single-mass oscillatory system is proposed as equivalent to multiple mass or uniformly distributed oscillatory systems on condition of their equal mass, geometric, elastic and dissipation characteristics in resonance frequency correspondent to the main mode of oscillation, so mathematical model designed for single-mass oscillatory system is recommended for strength and stiffness assessment in engineering calculations where possible difference in determination of stress in equivalent systems can used as safety factor.

Keywords: oscillatory system, dynamic force, stress, deflection, circuit card, resonance.

INTRODUCTION

Variety of modern electronic packages and their parts such as circuit cards (CC) or case walls, etc. are very likely to be exposed to mechanical impacts such as vibration and shocks during their operation. Large number of publications discussing problem of dynamic forces analysis in engineering [1–3] and in electronic equipment [4–7] subjected to vibrations, vibration reduction and suppression design [8, 9] testify of need for improving strength and providing reliability to electronics. Results of the previous research published in [10, 11] emphasized that mechanic impacts, and especially the dynamic forces in CC assemblies are likely to increase manifold so as to damage their bearing parts and electronic components, to which these forces are transmitted especially in resonant oscillations.

The represented research is aimed at stress calculation in CCs with their representation as a type of mechanical oscillatory systems in purpose of their strength assessment. The most attention is drawn to oscillations in resonance conditions. In all representations of oscillatory systems the cylindrical bending of CCs is considered to be a set of beam-strips with rectangular cross-sections so their stress calculation is performed by conventional methods applied in strength of materials and civil engineering [12].

The advantage of the mathematical modeling and of analytical formulas would be in their ability to predict the dependence of the results on the parameters of the tested systems, and it has been successfully applied for the single-mass oscillatory system. The attempts were also made to model multiple mass or even uniformly distributed oscillatory systems, however they did not result in analytical formulas capable to adequately calculate stress and deflection because of complexity of considered systems. In order to replace and for the case of single-mass oscillatory system to verify analytical formulas the MatLab simulation and experimental tests were applied.

Analytical estimation of stress and deflection conducted in the research was based on experimental verification of actual physical and mechanical parameters, which are likely to vary depending on technology, temperature, shape etc. For this purpose analytical and experimental method of sample parameters was applied. This method is based on identification of parameters by solving the reverse strength problems. In this method strain and displacement, which are traditionally estimated by calculations, are, instead, measured experimentally and considered as given values for calculation of physical and mechanical parameters, which are then used in analytical modeling.

Single-mass oscillatory system

The single-mass oscillatory system is represented by the beam with concentrated mass m (Fig. 1). Weight of the beam is assumed negligibly small in comparison with concentrated mass. Such representation is applicable for assembly where mass of electronic components exceeds mass of the substrate and therefore produces imbalance in oscillatory system. The beam is supported by the pinned support O providing with one degree of freedom and roller O' support providing two degrees of freedom. These supports transmit oscillation generated by the shaker.

Mass m, receiving kinematic excitation, undergoes acceleration z_1'' in the inertial frame of reference indicated by 0 (Fig. 1), which represents a support to the shaker generating acceleration z_0''. Then equation of motion for the mass m:

$$m\, z_1'' = F_\mathrm{k} + F_\mathrm{c} \quad (1)$$

where: F_k – elastic force; F_c – damping force.

In inertial frame of reference absolute acceleration z_1'' may be expressed by relative $\Delta z''$ and fictitious z_0'' accelerations as:

$$z_1'' = -\,\Delta z'' + z_0'' \quad (2)$$

Thus, equation of motion in non-inertial frame of reference, represented by O and O' supports, can be written as:

$$m\,\Delta z'' + F_\mathrm{k} + F_\mathrm{c} = m\, z_0'' \quad (3)$$

where: $m\Delta z''$ – relative and mz_0'' – fictitious forces of inertia; $F_\mathrm{k} = k\Delta z$; $F_c = c\cdot\Delta z'$; k – stiffness; c – damping coefficient. Equation (3) written in terms of function $\Delta z(t)$ is:

$$\Delta z'' + \omega_0^2 \Delta z + 2n\Delta z' = z_0'' \quad (4)$$

where: $\omega_0 = \sqrt{\frac{k}{m}}$ – natural frequency; $n = \frac{c}{2m}$ – descent rate.

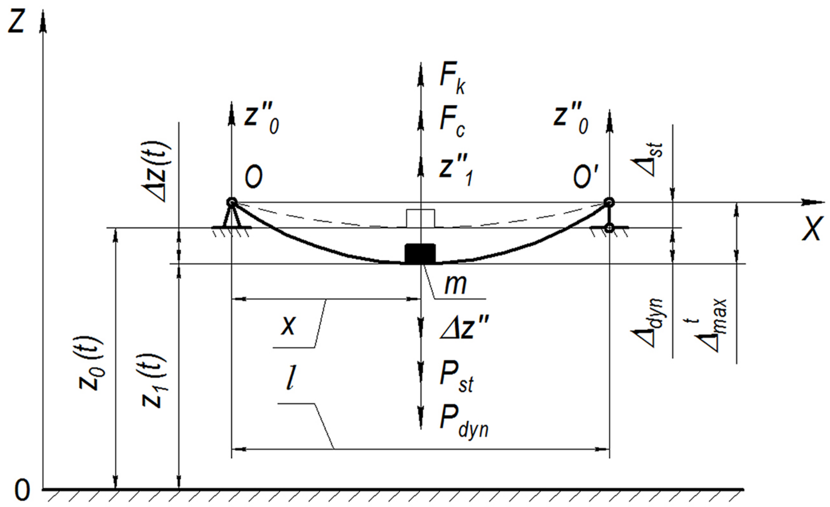


Fig. 1. Dynamic forces acting on the beam with concentrated mass

Shaker generates harmonic oscillations along Z axis described as:

$$z_0(t) = Z_0 \sin(\omega t + \varphi) \tag{5}$$

where: Z_0 – amplitude; ω – angular frequency; t – time; φ – phase of oscillations.

According to identical differential equation published in [10] the obtained equation (4) has identical solution that gives amplitude of forced oscillation:

$$A = Z_0\omega^2\left(({\omega_0}^2 - \omega^2)^2 + (2n\omega)^2\right)^{-\frac{1}{2}} \tag{6}$$

Function (6) expresses dynamic deflection Δ_{dyn} of the beam with respect to static equilibrium position Δ_{st} in the oscillatory system (Fig. 1), which sum expresses the total deflection Δ^t_{max} produced by the net force P:

$$P = P_{dyn} + P_{st} = \Delta^t_{max}\delta^{-1} \tag{7}$$

where: $P_{dyn} = -F_k$; $P_{st} = mg$; δ – flexibility of the beam in direction of force P.

Flexibility of the beam where a single force is applied is defined by Mohr's method:

$$\delta = \frac{x^2(x-l)^2}{3EJl} \tag{8}$$

where: x – linear coordinate of force application; E – Young's modulus; $J = \frac{bh^3}{12}$ – moment of inertia in cross-sectional area of the beam; l, b and h – length, width and thickness of the beam correspondently.

Formula (8) considers only internal bending moments while internal transverse forces can be neglected.

By using formulas (6–8) the expression for equivalent force is:

$$P = mZ_0\omega^2 \cdot \frac{1}{\sqrt{\left(1-\frac{\omega^2}{{\omega_0}^2}\right)^2 + \frac{4n^2\omega^2}{{\omega_0}^4}}} + mg \tag{9}$$

or:

$$P = P_0 \cdot k_{dyn} + P_{st} \tag{10}$$

where: $P_0 = mZ_0\omega^2$ – amplitude of dynamic force; k_{dyn} – dynamic coefficient; P_{st} – gravity force.

Maximal total normal stress in the critical cross-section of the beam defined by net force (9) and bending formula [12] is expressed as:

$$\sigma^t_{max} = \left[mZ_0\omega^2\left(\left(1-\frac{\omega^2}{{\omega_0}^2}\right)^2 + \frac{4n^2\omega^2}{{\omega_0}^4}\right)^{-\frac{1}{2}} + mg\right]x\left(1-\frac{x}{l}\right)\frac{1}{W} \tag{11}$$

where: $W = \frac{bh^2}{6}$ – section modulus in bending; l, b and h – length, width and thickness of the beam correspondently; x – linear coordinate of m.

Maximal displacement (deflection) is considered as deflection of middle point of the beam:

$$\Delta^t_{max} \approx \frac{P(3xl^2 - 4x^3)}{48EJ} \tag{12}$$

Such assumption results in negligible error in calculations that does not exceed 3%.

Mathematical modeling was conducted for parameters given in Table 1. Average values of physical and mechanical parameters were identified by method of sample parameters, in which physical and mechanical parameters were calculated from experimentally measured values of strain and displacement. Method is described in section of experimental verification, the experimental setup is shown in Figure 11.

The strength condition for CC was specified with respect to lowest ultimate strength in the CC assembly represented by soldered joints [11]:

$$[\sigma] = \frac{\sigma_{sld}}{s} = 16\text{ MPa} \tag{13}$$

where: $s = 2.5$ – safety factor compensating instability in design and technology.

In estimation model the dynamic force is applied with constant dynamic force amplitude P_0 = const provided by constant acceleration $a_0 = Z_0\omega^2 = 10$ m/s^2. Amplitude frequency responses of maximal stress and deflection given in Figure 2 indicate resonance at first critical

Table 1. Parameters of circuit cards

Dimensions, mm	Substrate material	E, GPa	ρ, kg/m^3	Substrate ultimate strength, MPa	Solder joint ultimate strength, MPa	n, 1/s	m, kg	x
l = 180 mm; b = 50 mm; h = 1.5 mm	Fiberglass CAST-V	14	1600	160-300	40	5.48	0.05	l/2

frequency ω_0 = 180.02 rad/s (28.66 Hz) where σ^t_{max} = 20.91 MPa and Δ^t_{max} = 5.38 mm.

With no doubt the most attention is drawn to estimation of maximal stress and deflection, which appear in resonance conditions when $\omega = \omega_0$:

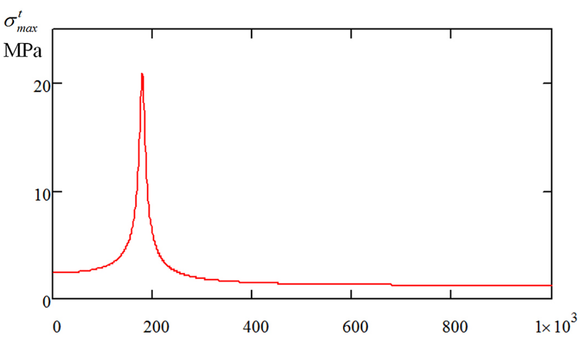
$$\sigma^t_{max} = \left[P_0 \cdot \frac{1}{2n}\sqrt{\frac{3EJl}{mx^2(x-l)^2}} + P_{st}\right]\frac{xl - x^2}{lW} \quad (14)$$

$$\Delta^t_{max} = \left(P_0 \cdot \frac{1}{2n}\sqrt{\frac{48EJ}{l^3 m}} + P_{st}\right)\frac{3xl^2 - 4x^3}{48EJ} \quad (15)$$

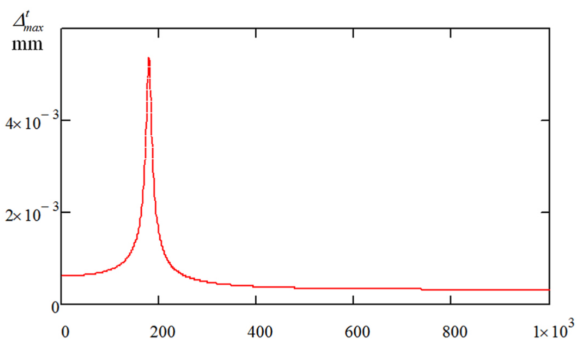
Moreover, analysis of function (14) revealed that neglecting static action of gravity force P_{st}, which is considerably lower in comparison with dynamic force P_{dyn}, gives the expression of dynamic stress:

$$\sigma_{dyn} = P_0 \cdot \frac{1}{2n}\sqrt{\frac{3EJ}{ml}} \cdot \frac{1}{W} \quad (16)$$

which testifies that under given conditions maximal stress does not depend on linear coordinate x of the concentrated mass m. Thus, formula (16) is proposed to use in engineering calculations of stress in CCs represented by oscillatory systems with concentrated mass irrespective to location of its mass.



a)



b)

Fig. 2. Amplitude frequency responses of maximal total normal stress (a) and deflection (b) in CC

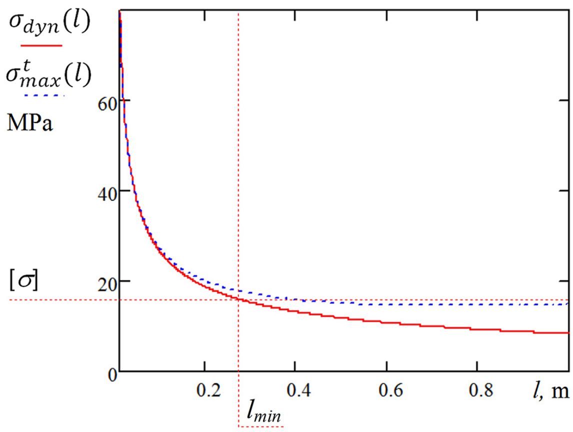


Fig. 3. Total and dynamic stresses depending on length of CC in resonance condition under dynamic force with constant amplitude

Besides, function (16) as demonstrated by (Fig. 3) can be used to specify geometric parameters of CC which would ensure compliance with strength condition. In particular, the minimal acceptable length l_{min} is proposed to calculate by the following formula:

$$l_{min} = \frac{3{P_0}^2 EJ}{4[\sigma]^2 n^2 W^2 m} \quad (17)$$

This formula is validated by relative difference $\Delta\sigma$ between (14) and (16) to comply with threshold tolerance of 15%. For this example (Fig. 3) l_{min} = 273 mm at $\Delta\sigma$ = 10.06%. Formula (14) could be used to increase accuracy if needed.

Mathematical model (16) was initially designed for maximal stress calculation in CC assemblies represented by single-mass oscillatory system with one concentrated mass, however real assemblies consist of multiple electronic components and bearing parts and each of them represents an unique concentrated mass. Thus, such mechanical system subjected to vibration represents a multiple mass oscillatory system.

Multiple mass oscillatory system

Figure 4 illustrates a model of oscillatory system that consists of 3 masses m_1, m_2, m_3 and therefore has three dynamic degrees of freedom since three parameters Δ_1, Δ_2, Δ_3 indicate position of all concentrated masses at their vertical displacements.

According to principle of superposition of forces the displacement in specific direction i correspondent to specific degree of freedom equals

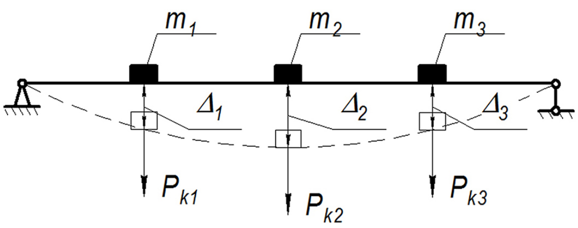


Fig. 4. Circuit card model as multiple mass oscillatory system

to the sum of all displacements in this direction (i = 1..3) produced by all forces acting in directions j (j = 1..3) and is defined by Mohr's method:

$$\Delta_i = \sum_j \delta_{ij} P_{kj} \tag{18}$$

According to equation of motion (3) and expression (18) equation of displacement for the mass m_i:

$$\Delta_i - \sum_j \delta_{ij} \left(m_j \Delta_j'' + 2n\Delta_j' - m_j a_0 \right) = 0 \tag{19}$$

Thus a system of differential equations (i = 1..n) can be written and solution in the following form can be obtained:

$$\Delta_i = A_i \sin(\omega t + \varphi) \tag{20}$$

For representation of only free oscillations where in (19) excitation force (fictitious forces of inertia in this case) and damping force are ignored and so are single displacements (flexibilities) δ_{ij} ($i \neq j$), then system of differential equation is falling apart for equations of single displacements δ_{ii} with their natural frequencies:

$$\omega_{0i} = \sqrt{\frac{1}{\delta_{ii} m_i}} \tag{21}$$

Nevertheless determining basic coordinates Δ_i for the system with more than 2 degrees of freedom encounters certain difficulties.

Symmetric systems with masses symmetrically located may produce either directly or inversely symmetric oscillation forms, where acting forces will be correspondently either directly or inversely symmetrical. In this case single displacements will be calculated as for the group of either directly or inversely symmetric unique forces.

Side single displacements which connect directly and inversely symmetric forces become equal zero:

$$\delta_{12} = \delta_{21} = 0; \ \delta_{23} = \delta_{32} = 0 \tag{22}$$

The single displacements defined by Mohr's method are:

$$\delta_{11} = \frac{l^3}{24EJ}; \ \delta_{13} = \delta_{31} = \frac{0{,}0286 l^3}{EJ}; \\ \delta_{22} = \frac{0{,}0052 l^3}{EJ}; \ \delta_{33} = \frac{0{,}0208 l^3}{EJ} \tag{23}$$

Determinant for symmetric vibrations:

$$\begin{vmatrix} \delta_{11}\frac{m}{2} - \frac{1}{\omega^2} & \delta_{13} m \\ \delta_{31}\frac{m}{2} & \delta_{33} m - \frac{1}{\omega^2} \end{vmatrix} = 0 \tag{24}$$

The corresponding frequency equation is the quadratic equation:

$$\lambda^2 a - \lambda b + d = 0 \tag{25}$$

where: $\lambda = \omega^2$; $a = m^2(\delta_{11}\delta_{33} - \delta_{31}\delta_{13})$; $b = m(\delta_{11} + 2\delta_{33})$; $d = 2$, which solution results in natural frequencies of symmetric oscillations:

$$\omega_1 = \sqrt{\lambda_1}; \ \omega_3 = \sqrt{\lambda_2} \tag{26}$$

Frequency equation for inversely symmetric oscillations expressed as:

$$\delta_{22}\frac{m}{2} - \frac{1}{{\omega_2}^2} = 0 \tag{27}$$

has solution:

$$\omega_2 = \sqrt{\frac{2}{\delta_{22} m}} \tag{28}$$

When number of masses is higher than 3, then receiving solution may become impossible especially for the case of considering forced and damped oscillations. Therefore solution to the problem of finding natural frequencies, modes of oscillation and, what matters most, maximal total normal stress and deflection in oscillatory system was offered to obtain by numerical method of simulation by MatLab system.

Simulation conducted by visual programming system *Simscape Multibody* in Matlab represented PCB as a flexible beam performed by multi-body three-dimensional modeling environment of mechanical systems aimed at forming and solving equations of motion. Since stiffness of supports is considered significantly higher than stiffness of the board, supports are considered absolutely rigid.

Modeling flexible plate was performed by using method for approximating flexible body by the set of N solid bodies having concentrated parameters and connected with springs and dampers [13]. Every concentrated mass has one degree of freedom – rotation in ZOX plane (Fig. 5). The mass, spring, and damper elements provide the inertial, restorative, and dissipative forces that collectively account for deformation. Linear spring and damper model was considered.

Stiffness and internal viscous friction are functions of material properties and geometry of flexible elements and are equal for all elements.

The value of the stiffness follows from the equality between the spring torque at the joint and the bending moment on a continuous version of the flexible beam unit. Hooke's law gives the spring torque at the joint:

$$\tau = k_r \gamma \tag{29}$$

where: k_r – rotational spring stiffness; γ – deflection angle.

The bending moment on a continuous beam unit:

$$M = \frac{EJ}{R} \tag{30}$$

where: R – the bending radius of curvature.

In the limit of very small deflections $\gamma \rightarrow l/R$, where l is undeformed length of the elements, spring stiffness is:

$$k_r = \frac{EJ}{l} \tag{31}$$

In the represented model k_r = 12.03 N·m.

Damping (viscous friction) b = 0.0037 for dampers in the joints was specified for experimentally found free oscillations descent rate n = 5.48 1/s (50) of the beam, as described further in the section of experimental verification.

Figure 6 demonstrates simulative approximation of 5-mass oscillatory system. The force application and parameters of oscillatory system were identical to single-mass model with the exception of using 5 symmetrically located masses m_i = 0.01 kg. Modeling flexible beam (substrate) was performed by using method for approximating flexible body by the set of N = 21 solid bodies having concentrated parameters and connected with springs and dampers [13, 14]. Deflections were measured in simulation by Joint Sensor. Maximal stresses were calculated by using bending formula [12], in which maximal bending moments were measured by Body Sensor. 6 pairs of sensors of both types were attached as shown in Figure 6.

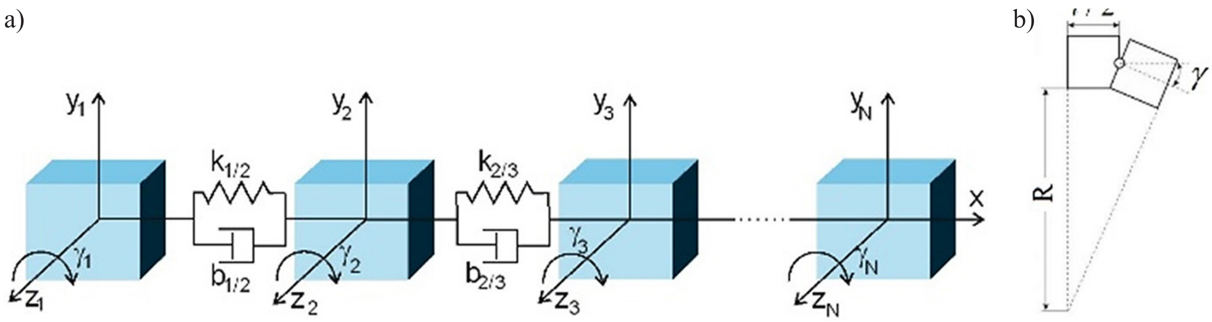


Fig. 5. Flexible beam approximation with distributed mass by: a – set of flexible elements with concentrated masses; b – flexible element

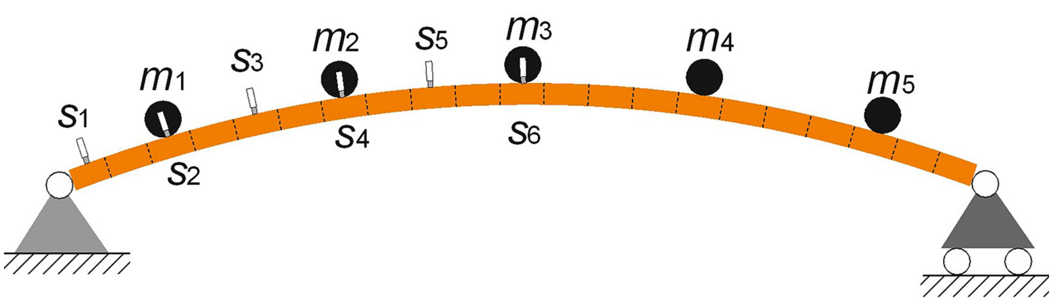


Fig. 6. Simulative approximation of 5 mass oscillatory system

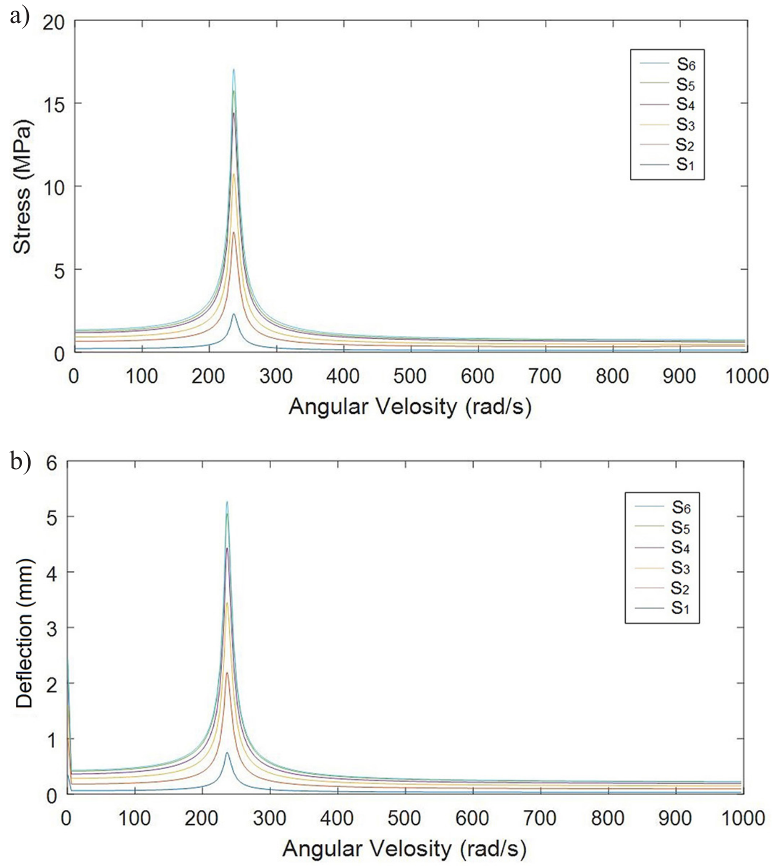


Fig. 7. Amplitude frequency responses of maximal total normal stress (a) and deflection (b) for multiple mass oscillatory system

The simulation of oscillatory system has resulted in amplitude frequency response graphs read by 6 sensors for maximal total normal stress (Fig. 7a) and deflection (Fig. 7b). Noteworthy is that highest levels of measured parameters are correspondent to the first critical frequency ω_{01} = 237 rad/s for the main mode of oscillation.

Obviously the more detailed model is the more accurate calculation results will be. Moreover consideration of all masses in real CC assembly representing numerous electronic components and a substrate laminated with conductive tracks, pads and other features and therefore having no-zero mass introduces complex and cumbersome problem. Such multiple mass mechanical system with actually infinite number of degrees of freedom is desirable to study as a system with mass distributed over its volume and in case of a beam representation – over its length and consider this distribution as uniform.

Oscillatory system with uniformly distributed mass

Equation of motion for the beam with uniformly distributed mass is represented for infinitesimal element with length *dx* (Fig. 8) arbitrarily selected along the length *l* of the beam. Element *dx* undergoes vertical displacement described by the function $z(x,t)$ forced by uniformly distributed dynamic inertial force $f(x,t)$. In this case force $f(x,t)$ is represented by fictitious forces of inertia (analogically to single mass oscillatory system described above in non-inertial frame of reference) which is spent to overcome uniformly distributed relative force of inertia $dm \cdot z(t)''$, damping force F_c, and concentrated elastic force F_k in cross-sections of the beam, which isolate element *dx* by the left and right faces where forces $F_k(x,t)$, $F_k(x+dx,t$, and moments of internal resistance $M(x,t)$, $M(x+dx,t)$ appear as shown in Figure 8.

The equation of motion for *dx* forced by all forces indicated in Figure 8:

$$\rho s dx \frac{\partial^2 z}{\partial t^2} + \frac{\partial F_k}{\partial x} dx + F_c dx = f dx \tag{32}$$

where: ρ – density; s – cross-sectional area of the beam.

Division by *dx*, substitution of $F_k(x,t) = \frac{\partial M}{\partial x}(x,t)$ and application of

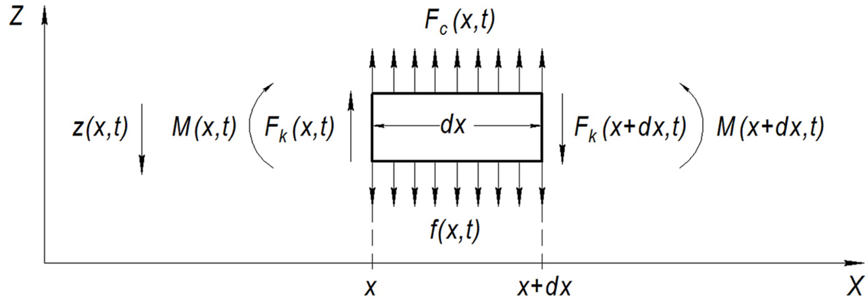


Fig. 8. Dynamic force application to infinitesimal element of the beam with uniformly distribute weight

differential equation of bent axis of the beam $M = EJ\dfrac{\partial^2 z}{\partial x^2}$ [12] gives the following expression:

$$\rho s\frac{\partial^2 z}{\partial t^2} + EJ\frac{\partial^4 z}{\partial x^4} = f - F_c \tag{33}$$

Solution to the equation of free transverse periodic oscillations represented by equation (33), in which the right side of it equals zero, is considered to be the harmonic function of beam deflection:

$$z(x,t) = A_x(x)\,sin(\omega t + \varphi) \tag{34}$$

where: $A_x(x)$ – amplitude function of beam axis, which represents the main mode of oscillations.

Substituting (33) into (34) gives:

$$\frac{EJ}{\rho s}\frac{\partial^4 A_x}{\partial x^4} - \omega^2 A_x = 0 \tag{35}$$

Characteristic equation for (35) is:

$$\frac{EJ}{\rho s}\beta^4 - \omega^2 = 0 \tag{36}$$

Whence:

$$\beta^4 = \frac{\omega^2 \rho s}{EJ} \tag{37}$$

Solutions to (37) are:

$$\beta_{1,2} = \pm\sqrt[4]{\frac{\omega^2\rho s}{EJ}};\ \beta_{3,4} = \pm i\sqrt[4]{\frac{\omega^2\rho s}{EJ}}.$$

Solution to (35) is:

$$A_x(x) = C_1\,cos(\beta x) + C_2\,sin(\beta x) + \\ + C_3\,cosh(\beta x) + C_4\,sinh(\beta x) \tag{38}$$

where: $\beta = \beta_1$; $C_1 - C_4$ – constants, which depend on boundary conditions:

$A_X(0) = 0$; $A_X''(0) = 0$; $A_X(l) = 0$; $A_X''(l) = 0$.

Then $C_1 = C_3 = C_4 = 0$; $C_2 \sin(\beta l) = 0$. When $C_2 = 0$ solution to (38) is trivial, therefore when $C_2 \neq 0$ it is considered that:

$$\sin(\beta l) = 0 \tag{39}$$

Whence:

$$\beta_j l = j\pi$$

where: $j = 1,2\ldots$

Considering (37) the natural frequencies of transverse periodic oscillations will be expressed as:

$$\omega_{0j} = \frac{j^2\pi^2}{l^2}\sqrt{\frac{EJ}{\rho s}} \tag{40}$$

Since strength and stiffness assessment is performed in engineering calculations by maximal magnitudes of stress and deflection then consideration of lower critical frequencies or even of the first one ω_{01}, when $j = 1$, representing the main mode of oscillation is required.

This statement was approved by MatLab simulation, in which flexible body of the beam (circuit card assembly) was approximated by the set of 21 elements (solid bodies) joined in between them by springs and dampers. The total mass of the beam $m = 0.05$ kg was uniformly distributed over all elements. 6 sensors attached as shown in Figure 6 measured total normal stresses and deflections.

Simulation has resulted in amplitude frequency response graphs read by 6 sensors for maximal total normal stress (Fig. 9a) and deflection (Fig. 9b). Noteworthy is that highest levels of measured parameters are correspondent to the first critical frequency $\omega_{01} = 255.3$ rad/s for the main mode of oscillation.

Comparative analysis of stress and deflection diagrams obtained by the research of oscillatory systems of all mentioned types (Fig. 10) demonstrated that the main mode of oscillatory system with uniformly distributed mass is similar to multiple mass

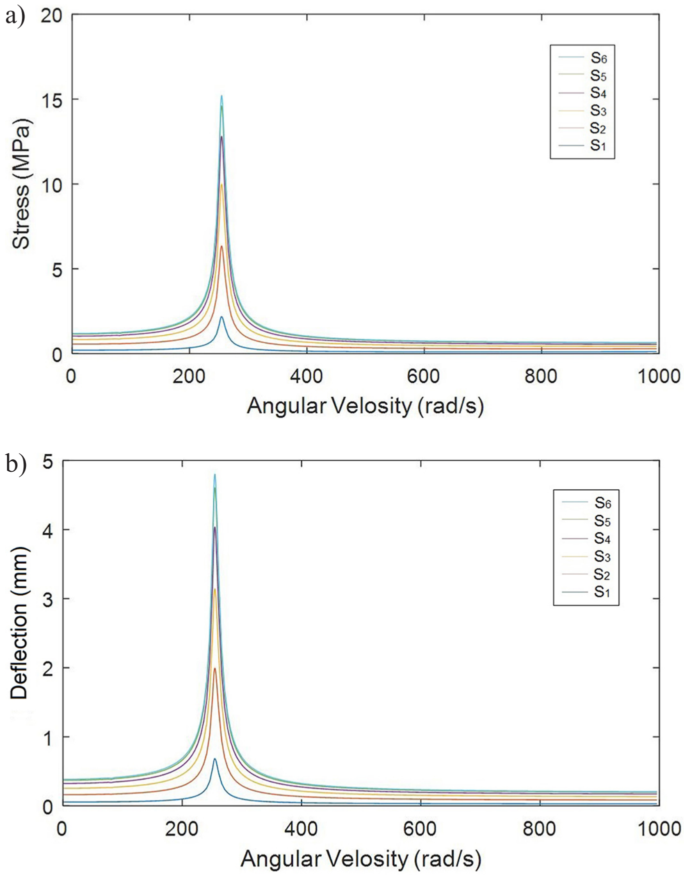


Fig. 9. Amplitude frequency responses of maximal total normal stress (a) and deflection (b) for oscillatory system with uniformly distributed weight

oscillatory system and moreover to the single-mass oscillatory system with concentrated mass, what allows conjecture of their equivalence.

Analysis of equivalence for strength and stiffness assessment in oscillatory systems

Similarity of main modes of oscillation indicated by proximity of maximal total normal stresses and deflections obtained for three mentioned types of oscillatory systems, on condition of equal dynamic force applied and mass, geometric, elastic and dissipation characteristics, allowed conjecture of representing these systems as equivalent for strength and stiffness assessment.

There is known [15], that an oscillatory system with uniformly distributed mass m can be

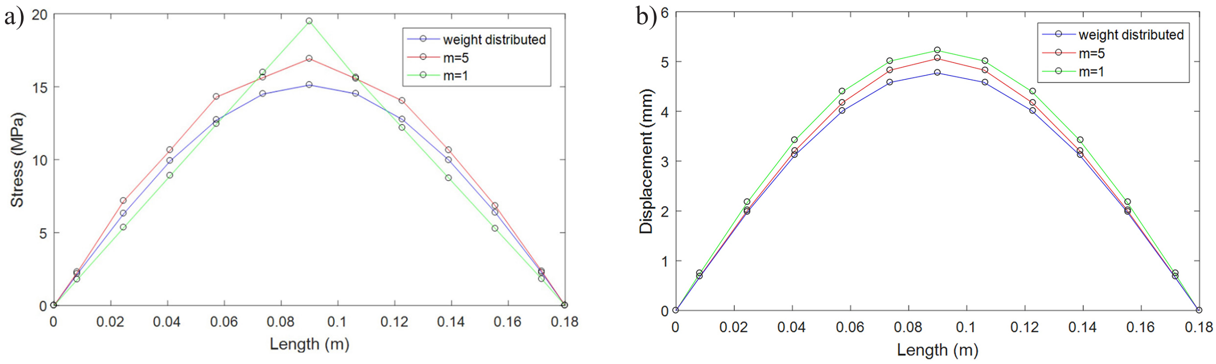


Fig. 10. Diagrams of maximal total normal stress (a) and deflection (b) for main mode of oscillation correspondent to first oscillation mode for multiple mass ($m = 5$), uniformly distributed (weight distributed) and single-mass ($m = 1$) oscillatory systems

represented as oscillatory system with concentrated mass converted into reduced mass β·*m* and applied in a specific place on the beam, in this case in its center, to perform strength and deflection assessment by using mathematic model designed for single-mass oscillatory system.

Equivalence of two systems can be reached by equality of their kinetic energies therefore the value of reduced mass β·*m* is assumed such that its kinetic energy:

$$T' = \beta\left(\frac{\rho s l (z')^2}{2}\right) \tag{41}$$

is equal to kinetic energy of mass distributed along the length *l* of the beam:

$$T = \int_l \left(\frac{\rho s (z_x')^2}{2}\right) dx \tag{42}$$

where: z' – displacement velocity of concentrated mass centered to the beam; z_x' – displacement velocity of infinitesimal element, determined by its position on the axis *x* along the beam.

Whence:

$$\beta = \frac{\int_l \frac{(z_x')^2}{z^2} dx}{l} \tag{43}$$

Hypothesis about similarity of displacement diagrams produced by the static action of weight of distributed mass *m* and concentrated mass β·*m* gives the relationship:

$$\frac{z_x'}{z'} = \frac{\Delta_{xst}}{\Delta_{st}} \tag{44}$$

where: Δ_{xst} – displacement of infinitesimal element *dx* produced by static action of weight of distributed mass in oscillatory system; Δ_{st} – displacement of the beam center produced by static action of weight of concentrated mass (Fig. 11).

Displacement of the section *x*:

$$\Delta_{xst} = \frac{mg}{EJ} x \left(\frac{l^2}{16} - \frac{x^2}{12}\right) \tag{45}$$

and displacement of the beam center:

$$\Delta_{st} = \frac{mgl^3}{48EJ} \tag{46}$$

Considering (44) and substituting (45) and (46) into (43) results in β = *17/35*.

Nevertheless using equivalent model of oscillatory system with reduced mass is only valid when mass of CC assembly is uniformly distributed and in case of multiple mass or single-mass oscillatory systems such approach would bring about lowered result in calculation of stress and deflection.

Analysis of simulation results given as graphs in Figure 10 verified that value of maximal total normal stress in oscillatory systems is that less than more distributed their mass is and, on the contrary, stress is highest for single-mass oscillatory system. Thus, calculation of highest values of maximal total normal stress through representation of single-mass oscillatory system as equivalent to any other systems even to oscillatory system with uniformly distributed mass allows strength assessment by using mathematical models designed for single-mass oscillatory system (16) or (14), whereby possible difference in calculation related to using equivalent system instead of original one can be used as safety factor.

Experimental verification

Research objectives were circuit cards representing three types of oscillatory systems: 1) single-mass oscillatory system with concentrated mass centered to the beam; 2) multiple mass oscillatory system consisting of 5 masses placed uniformly along the beam (Fig. 6); 3)

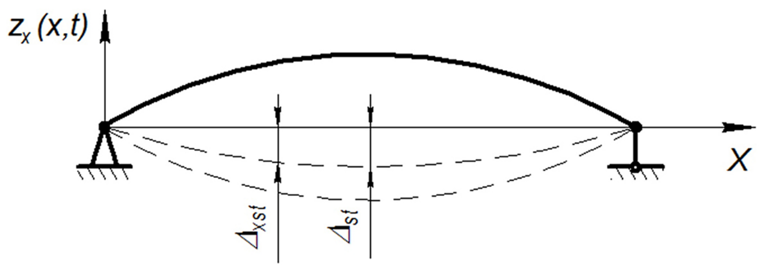


Fig. 11. Main mode of oscillation of the beam with uniformly distributed mass

oscillatory system with uniformly distributed mass. Circuit cards were selected on condition of equal mass, geometric, elastic and dissipation characteristics and dynamic force application and identical to those in mathematical modeling and MatLab simulation.

Actual elastic and dissipation parameters were determined by the method of sample parameters based on measuring strain and displacement in the experimental setup shown in Figure 12.

Young's module E and descent rate n where found by static middle-point bend test of CC samples under force threshold limit:

$$[P] = \frac{2bh^2}{3l}[\sigma] \tag{47}$$

where: l = 180 mm; b = 50 mm; h = 1,5 mm; $[P]$ = 6.67 N.

During the test the strain was measured by strain gauges along with reading force P and deflection Δ. Two strain gauges with 10 mm base were attached both in the longitudinal (direction of maximal normal stress) and in the transverse directions (indicated no strain). For higher accuracy the Young's module E was found in two ways:

$$E_1 = \frac{Pl}{4\varepsilon W} \text{ and } E_2 = \frac{Pl^3}{48\Delta J} \tag{48}$$

The mean value of E = 14.5 GPa was calculated over 5 experiments. Tests were conducted by using method of acoustic emission reading from the sensor attached to PCB surface. Detecting no acoustic emission verified of only elastic strain produced during the test and approved Hook's law application. Descent rate n of oscillatory system was found by free oscillation graph read from vibration sensor attached to the center of PCB by the formula:

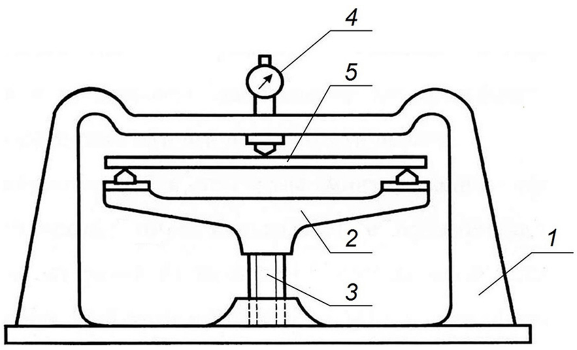


Fig. 12. Experimental setup for three-point bend test: 1 – base; 2 – movable bar; 3 – lifting rod; 4 – clock indicator; 5 – circuit card

$$n = \frac{1}{T} \cdot ln\frac{A(t)}{A(t+T)} \tag{49}$$

where: $A(t)$ and $A(t+T)$ – amplitudes measured in time t and $t + T$ correspondently, T – oscillation period.

The mean value of n = 5.48 s^{-1}. The further vibration tests were conducted for experimental verification of mathematical model. Vibration tests were conducted by constant dynamic load with amplitude P_0 provided by constant vibration acceleration a_0 = 10 m/s^2. CCs were installed in clamps designed as regular fixtures (Fig. 13) mounted on the shaker. Maximal normal stress

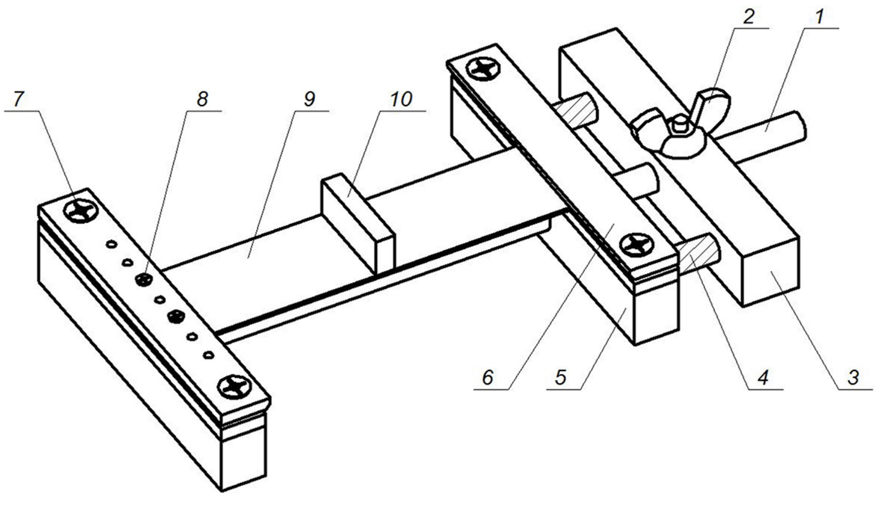


Fig. 13. Clamp for circuit cards on vibration tests: 1 – rod; 2 – fastening nut; 3 – adjustable bar; 4 – springs; 5 – support bars; 6 – top plates; 7 – fixture screws; 8 – stop screws holding CCs in their lose holes; 9 – circuit card; 10 – concentrated mass

was performed by reading strain gauge attached to the middle point of CC surface and calculation by the Hook's law. Maximal dynamic stress σ_{dyn} was calculated by the following formula:

$$\sigma_{dyn} = \sigma_{max}^{t} - \sigma_{st} \tag{50}$$

where: σ_{st} – static stress produced by gravity force and measured by the strain gauge attached to unloaded CC before it was set on the shaker.

Maximal deflection Δ_{dyn} was measured by vibration sensor set in the center of CC. Second vibration sensor attached to fixtures was reading oscillations generated by the shaker. The resonance frequencies were detected by method of floating frequency [10]. The first critical frequency was detected by the sensor in the middle of CC. Stress measured on this frequency is considered as maximal total normal stress in the critical section of the circuit card and calculated by (50) – as maximal dynamic stress.

Results of vibration experimental tests, simulation and mathematical modeling of maximal dynamic stress and deflection in circuit cards are given in Table 2.

Comparing maximal magnitudes of dynamic stress and deflection obtained by experimental tests with results of simulation and mathematical modeling indicates of their similarity and insignificant relative difference from 0.53 to 8.81 %.

Thus, the conducted research approved the idea of estimating maximal dynamic normal stress by representing single-mass oscillatory system as equivalent to multiple mass or even uniformly distributed oscillatory systems on condition of their equal mass, geometric, elastic and dissipation characteristics in resonance frequency correspondent to main mode of oscillation and use mathematical model designed for single-mass oscillatory system for engineering strength and stiffness assessment whereby possible difference in determination of stress in equivalent systems can used as safety factor.

Table 2. Maximal dynamic stress and deflection in circuit cards

Parameters / Oscillatory system	ω_0, rad/s	σ_{dyn} , MPa	Δ_{dyn}, mm
Experimental data			
Multiple mass	242.0	16.25	4.63
Distributed mass	261.3	14.19	3.91
Single-mass	176.9	18.96	4.75
Simulation data			
Multiple mass	242.0	16.55	4.83
Distributed mass	261.3	15.44	4.00
Single-mass	176.9	19.06	5.02
Mathematical modeling data			
Single-mass	180	19.71	5.07

CONCLUSIONS

Estimation of maximal dynamic normal stress is proposed by representing single-mass oscillatory system as equivalent to multiple mass or even uniformly distributed oscillatory systems on condition of their equal mass, geometric, elastic and dissipation characteristics in resonance frequency correspondent to the main mode of oscillation and use mathematical model designed for single-mass oscillatory system for engineering strength and stiffness assessment whereby possible difference in determination of stress in equivalent systems can used as safety factor.

Comparative analysis of mathematical modeling, MatLab simulation and experimental determination of maximal dynamic stress and deflection accomplished for three types of oscillatory systems verified proximity of obtained results.

Mathematical model has been developed for maximal dynamic stress and deflection estimation in circuit card assemblies represented by unique single-mass oscillatory system as prismatic beam with concentrated mass set on two oscillating supports under inertial resonance excitation generated by constant dynamic force. Since dynamic stress is caused by displacement about static equilibrium position and is independent on linear coordinate of the concentrated mass it provides strength assessment with no respect to linear coordinate of the concentrated mass and can be recommended for engineering calculations.

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