# **Practical use of RLC-models of transformers' windings for determination of local displacements**

Szymon Banaszak, Konstanty M. Gawrylczyk West Pomeranian University of Technology 70-310 Szczecin, ul. Gen. Sikorskiego 37, e-mail: {szymon.banaszak; konstanty.gawrylczyk}@ zut.edu.pl

The diagnostics of power transformers is a very fast developing branch, due to increasing average age of assets and changes in asset management strategies, nowadays companies introduce asset management based on a real technical condition. One of important methods used for diagnostics of a transformer's active part is Frequency Response Analysis (FRA). It allows determination of mechanical condition of windings, their displacements, deformations and electric faults, as well as some problems with internal leads and connections, core and bushings. For the aim of windings impedance modeling the RLC models are applied. The idea of lumped parameters models was presented in [6]. The new universal model basing on circuit solution is developed in this paper. Lumped parameters used in calculations are obtained with finite element method and Maxwell package. The examples of models created for simple windings were compared to real measurements.

KEYWORDS: transformer's diagnostic, frequency response analysis, lumped parameter models.

### **1. Introduction**

The FRA method is based on the analysis of transfer function of windings. The winding can be described by a set of local capacitances, self and mutual inductances and resistances. Every change in winding geometry leads to change of these parameters, therefore the transfer function's shape is also influenced. The analysis of FRA results is based on comparison of data presented usually as sine signal damping along frequency spectrum in logarithmic scale. This can be compared to results recorded for given transformer in time intervals, between phases, between twin or sister units or with help of computer models. The first method is optimal, but for most of old transformers there is no fingerprint data available.

The next two approaches are usually applied in industrial practice, however they are quite uncertain and may lead to misinterpretations. Each transformer can have differences in FRA curve compared between phases or, if compared to other units, due to constructional differences [2]. Helpful results can be obtained from controlled deformations, but this method cannot be applied in mass scale and generalized [3], however some information gathered from such experiments is useful for further interpretation of test results, also for different constructions.

A nother approach is creation of computer models, which do not need destruction of real objects (as in controlled deformations), however their exactness is limited with disadvantages of chosen methods, simplifications and hardware possibilities.

#### **2. FRA measurements**

At this stage of FRA development results of measurements are given as damping of windings to sine signal in frequency range, according to formula:

$$
FRA[\text{dB}] = 20\log |K_{\text{u}}(f)| = 20\log \frac{|U_{\text{out}}|}{|U_{\text{in}}|}
$$
 (1)

where:  $U_{\text{in}}$  - voltage signal applied to a winding,  $U_{\text{out}}$  - voltage signal measured.

The measurement can be taken either on two ends of the winding (with secondary winding open or shortened and grounded) or between windings of the same phase (capacitive or inductive measurement).

### **3. Lumped parameter model**

To reduce the complexity of network model, several turns, or even one disc were lumped into one electric network element. The proposal of circuit of a single network element was given by Bjerkan in [6] :



Fig. 1. Lumped parameter model of one network element

The parameters used in Fig. 1 have the following meaning:

 $R<sub>i</sub>$  – resistance resulting from ohmic losses and eddy current losses,

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 $L<sub>i</sub>$  – self inductance of the element, with skin effect included,

- $C_{ii}$  self capacitance of the element,
- $G_i$  losses to the ground,

 $C_i$  – capacitance of the element to the ground,

 $M_{ii}$  – mutual inductance to other elements,

 $R_{ii}$  - "mutual" resistance resulting from proximity losses.

These parameters were calculated using approximate analytic formulas. Then, the network elements were interconnected, building the model for a whole transformer. For the network solution, a standard solver could be applied, as Spice or Microcap. A part of network model obtained for 18 turns is shown in Fig. 2.



Fig. 2. Lumped parameters model of 18 turns (presented only part of network)

All above lead to introduction of computer models, which may be useful for interpreting changes in FRA charts. Models are usually based on real physical dimensions and properties of transformers. However, it is still difficult to construct models having frequency response identical to real units and allowing to simulate various defects with similar effect on the FRA curve as real deformation in winding.

Although the obtained characteristics differ from measurement, the direction of the changes and the influenced frequency range may be interpreted correctly.

The reason of large simulation errors in earlier works were  $[3]$ :

- using of approximate formulas for inductances, resistances and capacitances inside the winding,
- too simplistic network model used for solution. Because of utilization of standard network solvers, using of network elements corresponding to single wires in a whole winding was not possible.

The solution of both problems is shown below.



Fig. 3. The comparison of FRA charts modeled before and after deformation

### **4. New model based on lumped parameter**

Our model, shown in Fig. 4, seems as simplification of the model from Fig. 1. In fact, the network element shown in Fig. 4 corresponds to the single wire of a winding. It allows for any couplings to all other wires, even if the number of turns is significant. The only thing that has been neglected, are losses to the ground, which in our experience do not influence the response. The solution is carried out by direct algorithm, without using of external solvers.



Fig. 4. New network element modeling a single turn

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Fig. 5. Network model and interconnection of two turns

Model of a single turn, shown in Fig.4, consists of self inductances  $L_i$ , capacitances  $C_i$  and resistances  $R_i$ , as well as mutual inductances  $M_{ij}$  and capacitances  $C_{ii}$ . The adopted model is characterized by the fact, that it includes the mutual inductances and capacitances between all turns of the winding.

Voltage difference on the branch *i* is given by

$$
V_{i} - V_{i+1} = I_{i} \cdot R_{i} + j \omega \sum_{j} M_{ij} I_{j}, \text{ where } M_{ii} = I_{i}.
$$
 (2)

The first voltage  $V_1 = V_m$  is known. So, we have the number of unknown voltages equal number of branches  $N_g$ . Sum of the currents in node  $i+1$  is

$$
I_{i} = I_{i+1} + \frac{j\omega}{2} (C_{i} + C_{i+1}) \cdot V_{i+1} +
$$
  
+ 
$$
\frac{j\omega}{2} \sum_{j\neq i} C_{i,j} \cdot (V_{i+1} - V_{j+1}) + C_{i+1,j} \cdot (V_{i+1} - V_{j}).
$$
 (3)

Number of current equations is  $N_{\rm g}$ , while the number of unknown currents equals  $N_g + 1$ . The current in the last branch  $I_{out} = V_{out}/R_0$  ( $V_{out} = V_{N_g+1}$ ), where  $R_0$ means the resistance of measuring instrument. Considering  $(2)$  and  $(3)$  we obtain the system of  $2N_g$  equations to solve, whose matrix owns the following form:

$$
[A]\begin{Bmatrix} V \\ I \end{Bmatrix} = \begin{bmatrix} A_{\text{vv}} & A_{\text{vu}} \\ A_{\text{uv}} & A_{\text{u}} \end{bmatrix} \begin{Bmatrix} V \\ I \end{Bmatrix} = 0,\tag{4}
$$

where the terms of the matrix  $\vec{A}$  are given by following equations:

$$
A_{\rm{vv}} = \begin{bmatrix} 1 & -1 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & -1 & 0 \\ 0 & 0 & \dots & 1 & -1 \end{bmatrix} N_{\rm{g}}, \tag{5}
$$

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$$
A_{VI} = \begin{bmatrix} -R_1 - j\omega L_{11} & -j\omega M_{12} & \cdots & -j\omega M_{1N_s} & 0 \\ -j\omega M_{21} & -R_2 - j\omega L_{22} & \cdots & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -j\omega M_{N_s-1,1} & -j\omega M_{N_s-1,2} & \cdots & -j\omega M_{N_s-1,N_s} & 0 \\ -j\omega M_{N_s,1} & -j\omega M_{N_s,2} & \cdots & -R_{N_s} - j\omega L_{N_s,N_s} & 0 \\ \hline & \bar{N_s} + 1 & \bar{N_s} & \cdots & \bar{N_s} + 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
A_{IV} = \frac{j\omega}{2} \begin{bmatrix} 0 & C_1 + C_2 + C_{12} & -(C_{12} + C_{23}) & \dots & -(C_{1,N_g} + C_{2,N_g+1}) \\ -C_{31} & -C_{21} & C_2 + C_3 + C_{23} & \dots & -(C_{2,N_g} + C_{3,N_g+1}) \\ -C_{41} & -C_{31} - C_{42} & -C_{32} & \dots & -(C_{3,N_g} + C_{4,N_g+1}) \\ \dots & \dots & \dots & \dots & \dots \\ -C_{N_g+1,1} & -C_{N_g,1} - C_{N_g+1,2} & -C_{N_g,2} - C_{N_g+1,3} & \dots & -C_{N_g,N_g+1} \end{bmatrix} N_g (7)
$$

$$
A_{\rm II} = \begin{bmatrix} -1 & 1 & \dots & 0 & 0 \\ 0 & -1 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & \dots & -1 & 1 \end{bmatrix} N_{\rm g}
$$
 (8)

and the vector

$$
\begin{Bmatrix} V \\ I \end{Bmatrix} = \begin{bmatrix} V_{\text{in}} & V_2 & \dots & V_{N_{\text{s}}} & V_{\text{out}} & I_{\text{in}} & I_2 & \dots & I_{N_{\text{s}}} & I_{\text{out}} \end{bmatrix}^{\text{T}} \tag{9}
$$

The matrix  $A$  is dense populated, so its solution for the whole winding takes considerable time of computation.

## **5. Obtaining of lumped parameters**

Because of poor exactness of approximate analytical solutions, the lumped parameters were obtained using finite element modeling. As long the transformers' winding owns cylindrical symmetry, the two-dimensional field model may be used. However, the displacements inside the winding may destroy this symmetry, so in *S. Banaszak, K. M. Gawrylczyk / Practical use of RLC-models of transformers'* ...

such case three-dimensional modeling should be introduced. At this stage of research, we use only two-dimensional models. The model consists of single, unconnected wires, excited by known voltage or current.

To provide the own and mutual capacitances the electrostatic model is solved:

$$
\nabla \cdot (\varepsilon_{\rm r} \varepsilon_{\rm 0} \nabla \Phi(r, z)) = -\rho \,. \tag{10}
$$

For own and mutual inductances and resistances the eddy current model described by

$$
\nabla \times \frac{1}{\mu} (\nabla \times A) + j \omega \gamma A = J_s \,. \tag{11}
$$

The solutions were carried out utilizing ANSYS Maxwell package, which offers the possibility to determine the necessary matrices, namely inductance matrix from magnetic energy:

$$
W_{AV} = \frac{1}{4} \int \boldsymbol{B}_i \cdot \boldsymbol{H}_j^* dV, \quad L_{ij} = \frac{4W_{AV}}{I_{Peak}^2} = \int \boldsymbol{B}_i \cdot \boldsymbol{H}_j d\Omega \,, \tag{12}
$$

capacitance matrix from electric energy:

$$
W_{ij} = \frac{1}{2} \int_{\Omega} \mathbf{D}_i \cdot \mathbf{E}_j \, \mathrm{d}\Omega, \ C_{ij} = \frac{2W_{ij}}{V^2} = \int_{\Omega} \mathbf{D}_i \cdot \mathbf{E}_j \, \mathrm{d}\Omega \,, \tag{13}
$$

and the resistances from power dissipation:

$$
P = \frac{1}{2\gamma} \int J \cdot J^* d\Omega, \quad R = \frac{2P}{I_{Peak}^2} = \frac{\int J \cdot J^* d\Omega}{\gamma I_{Peak}^2} = \int J \cdot J^* d\Omega \tag{14}
$$

#### **6. Simulation results and comparison to FRA**

The results provided by the described lumped parameter model were compared to these obtained with transmission line method, described in [7]. The simulation results with both methods yields very similar results until frequency of 1Mhz. For higher frequencies there is visible the signal delay associated with the length of the transmission path, which corresponds to the coil winding.

Next, the simulation results of our new RLC model were compared to measurements done using commercial device Omicron FRAnalyser. The measurement and simulation were done for the winding containing 60 turns. The comparison in Fig. 6 shows very good agreement of simulated results versus measurement.

In the next step, the upper coil of the winding was displaced axially about 4mm, as shown in Fig. 7. Our model enables simulation of such situation, presenting impact of displacement on simulated FRA-charts. The base chart matches this from upper Fig. 6 (RLC).



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Fig. 6. Comparison of simulated and measured FRA-charts



Fig. 7. Axial displacement of upper coil 1





Fig. 8. Impact of upper coil displacement on resulting charts modeled using RLC-model

#### **7. Future work**

The described method can be applied to the analysis of transformers with a large number of turns. The determination of the resistances, capacitances, inductances and mutual inductances can be done using the finite element method, however long computational time should be taken into account. In many cases, the circular-cylindrical symmetry allowing for 2D analysis can be used. Presented models allows for simulation of some deformations of the windings so long, as the cylindrical symmetry will not be lost. For other deformations it will need the full 3D analysis. Further development of presented algorithms is planned and creation of more complex models, corresponding with real complete transformers. This method converted into a practical tool would be very helpful for interpretation of industrial test results.

Presented results are part of research project nr. N N510 698240 entitled "Algorithm for identification of transformers windings deformations on the base of frequency response measurements (FRA)", financed by Polish National Science Center.

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