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Research paper

Sensitivity analysis of fuzzy-analytic hierarchical process (FAHP) decision-making model in selection of underground metal mining method

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ABSTRACT

This study aims to analyse the sensitivity in decision-making which results in the selection of the appropriate underground metal mining method using the fuzzy-analytical hierarchy process (FAHP) model. The proposed model considers sixteen criteria for the selection of the most appropriate mining method out of the seven. The model consists of three-layer viz. the first layer represents the criteria (factors which influence the mining method), the second layer represents the sub-criteria (categorisation of the factors) and the third layer represents the alternatives (mining methods). The priority of the different mining methods was determined based on global weights. The global weights of seven mining method were determined using a different fuzzification factor under different decision-making attitudes (optimistic, pessimistic and unbiased). The sensitivity of the decision-making results was analysed in order to understand the robustness of the model.

1. Introduction

There are many metal mining methods available such as block caving, sub level stoping, sub level caving, room and pillar mining, shrinkage stoping, cut and fill stoping, and square set stoping for excavating ore reserves from underground. The selection of a particular metal mining method depends on multiple factors, and thus the selection can be made using the multi-criteria decision-making (MCDM) technique. It is very important to select the suitable mining method for excavating an ore deposit for economic reasons and for safety. MCDM aims to select the most promising alternative based on the defined criteria and sub-criteria. In the past, many types of MCDM techniques (AHP, FAHP, TOPSIS, etc.) have been developed and used for the selection of mining methods. Namin, Shahriar, Ataee-Pour, and Dehghani (2008) suggested a TOPSIS and Fuzzy TOPSIS kind of integrated model for the selection of coal and metal mining methods. Alpay and Yavuz (2007) proposed a model using AHP and the Yager's decision making techniques for the selection of the underground mining method. Mikaeil, Naghadehi, Ataei, and Khalokakaie (2009) developed the MCDM models using FAHP and TOPSIS for the selection of the mining method for a Bauxite mine in Iran based on thirteen criteria. Naghedehi, Mikaeil and Atei (2009) proposed the FAHP decision-making model for the selection of an appropriate mining method for a Bauxite ore deposit in Iran. The decision-making in the selection of best mining method out of the six was made based on the thirteen influencing parameters. Gupta and Kumar (2012) suggested an AHP-based MCDM model for underground mining method selection. Ataei, Shahsavany, and Mikaeil (2013) suggested a Monte Carlo based AHP (MAHP) approach for the selection of the mining method for a Bauxite ore deposit in Iran. Yavuz (2015) conducted a study of the underground coal mining method selection for five alternatives using AHP and Yager's multi criteria decision-making techniques. Dehghani, Siami, and Haghi (2017) suggested Grey and TODIM approaches for the selection of a mining method.

The selection of a mining method for excavating an ore deposit is a crucial task for the planners at the decision making stage. The selection of a mining method depends on various qualitative and quantitative factors of the ore deposit. These qualitative and quantitative parameters include geometry, geo-mechanical, operational, economical, etc. Mine planners face the difficulty while selecting the mining method due to more number of interdependent parameters. Once the operation for ore extraction has begun with a particular mining method, it is not possible

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or difficult to alter the mining method. Among all the MCDM techniques, the most popular technique that solves decision-making problems is the analytical hierarchy process (AHP). However, AHP is ineffective when applied to ambiguous problems like uncertainty of the criteria parameters (Tsai, Chang, & Lin, 2010). For accommodating the uncertainty of the factors, AHP is integrated with fuzzy logic. Sensitivity analysis is an essential component of fuzzy-AHP decision-making models. The purpose of the sensitivity analysis is to measure the consistency in selecting the best alternative in different conditions. The final priorities of the alternatives are heavily dependent on the weights associated with the main criteria parameters. A small change in the weights of the criteria have a significant impact on the final ranking of the alternatives. Sensitivity analysis provides information about an alteration in the ranking of the alternatives. The sensitivity analysis of different MCDM models has been performed in different fields, but not in the selection of mining method. Chang, Wu, Lin, and Chen (2008) conducted sensitivity analysis of the FAHP model for evaluating and controlling silicon wafer slicing quality. The model outputs were analysed by increasing each criteria weight by 10%, 20%, and 30%. Hsu and Chen (2007) developed FAHP model for the selection of franchisees of a bedding chain retail store. The sensitivity analysis of the model was conducted by changing the values of the uncertainty factors. Tabari, Kaboli, Aryanezhad, Shahanaghi, and Siadat (2008) proposed a FAHP model for site selection and sensitivity analysis was performed for the model they developed by changing the value uncertainty factor from 0 to 1. Tseng and Lin (2008) suggested the selection of competitive advantages in total quality management implementation using FAHP. The sensitivity analysis of the model was analysed for competitive advantage by considering the fuzzification factor (α) to be 0.2, 0.4, 0.6 and 0.8. Tsai et al. (2010) analysed the sensitivity of the FAHP model for evaluating hospital organization performance by changing the fuzification factor, in the range of 0.1–1.

It is clear from the literature that the selection of mining method depends on multiple factors and thus it is important to analyse the degree of sensitivity of decision-making due to either a change in a factor's uncertainty level or a decision maker's attitude. The literature study revealed that sensitivity analysis of decision-making on the type of mining has not been conducted by any other researcher to date. Thus, the present study attempts to analyse the decision-making results in selecting the best mining method using the proposed FAHP model under different fuzzification factors and decision-making attitudes. The model performance was analysed by changing the uncertainty levels of the factors from minimum to maximum in different decision-making attitudes, i.e. optimistic, pessimistic and unbiased.

2. Methodology

The analytical hierarchy process (AHP) is a decision-making process used to select the best choice from multiple alternatives. The AHP method was initially proposed by Saaty (1980). The AHP subdivides the problem into many levels in a hierarchical structure. The AHP method requires designing a pair-wise comparison at each level using the decision maker's knowledge. Though AHP solves many decision-making problems; it is inefficient when the influencing parameters in the given problem are uncertain. AHP method is integrated with fuzzy logic to deal with such type of uncertain problems. van Laarhoven and Pedrycz (1983) initially proposed fuzzy logic with AHP. After that, FAHP method was used in many research work problems in the decision-making of uncertain problems.

The proposed study attempts to develop a FAHP model for decision making on the selection of the best mining methods out of seven alternatives based on the criteria. The sensitivity of the model output was analysed by changing the values of the fuzzification factors in different decision-making attitudes. The relative priority of each mining method was determined based on the global weights in different condition. The flowchart of the working procedure is shown in Fig. 1.

2.1. Selection of criteria, sub-criteria, and alternatives

The first step of the proposed model is to select the criteria and sub-criteria for the prioritization of the alternatives (underground metal mining methods). The selection of a suitable mining method depends on various criteria like ore-geometry, the geo-mechanical conditions of the ore, the production capacity of the deposit and various operational parameters (Naghadehi, Mikaeil, & Ataei, 2009). The present study considered 16 criteria (dip, shape, thickness, depth, grade distribution, RMR of the ore, RMR of the hanging wall, RMR of the footwall, RSS of the ore, RSS of the hanging wall, RSS of the footwall, productivity, recovery, dilution, flexibility and safety) (shown in Table 1) for selection of the most suitable mining method for a typical ore deposit. The criteria were further classified into 54 sub-criteria, as shown in Table 1, to develop the hierarchical model. The number of alternatives (mining methods) considered in the model is seven. These are block caving (BC), sublevel stoping (SS), sublevel caving (SC), room and pillar mining (RP), shrinkage stoping (SH), cut and fill stoping (CF), and square set stoping (SQ). The study analyses the sensitivity of the decision-making by considering a different range of fuzzification factors and the decision maker's attitude. The ranges of the sub-criteria were considered based on previous studies (Miller-Tait, Pakalnis, & Poulin, 1995; Tatiya, 2013).

2.2. Design of the hierarchical structure

The hierarchical structure of the proposed FAHP model was designed based on different criteria and sub-criteria. All the identified criteria, sub-criteria and the evaluation alternatives (mining methods) were arranged in different levels of the hierarchy (shown in Fig. 2). The first, second and third level of the hierarchy defines the criteria, sub-criteria and mining methods respectively. The last level defines the goal of the decision-making problem, i.e. the best underground metal mining method (UMMS) for the extraction of the ore deposit.

2.3. Formation of fuzzy-relative importance matrices for each level

The next step is to develop relative importance matrices for each level using the corresponding parameters. The relative importance matrices for each level were built by using the FAHP scale of $\bar{1}$ to $\bar{9}$ (Saaty, 1980). The relative importance scale in normal AHP is 1–9 whereas in FAHP it is $\bar{1}$ to $\bar{9}$. The relative importances and their corresponding definitions are listed in Table 2.

The relative importance values of criteria in selecting the mining method were considered from past studies (Azadeh, Osanloo, & Ataei, 2010; Gupta & Kumar, 2012; Naghadehi et al., 2009). The relative importance matrix of the criteria in the first level of the hierarchy was designed as follows:

$$CP = \begin{array}{cc} & \begin{array}{cccccccccccccccc} DI & SH & TH & DE & GD & ORMR & HRMR & FRMR & PR & RE & DL & ORSS & HRSS & FRSS & FLE & SE \end{array} \\ \begin{array}{c} DI \\ SH \\ TH \\ DE \\ GD \\ ORMR \\ HRMR \\ FRMR \\ PR \\ RE \\ DL \\ ORSS \\ HRSS \\ FRSS \\ FLE \\ SE \end{array} & \begin{bmatrix} \bar{1} & \bar{3} & \bar{1} & \overline{4.5} & \bar{3} & \bar{2} & \bar{2} & \bar{4} & \bar{7} & \bar{7} & \bar{7} & \bar{2} & \bar{2} & \bar{4} & \bar{7} & \bar{7} \\ 1/\bar{3} & \bar{1} & 1/\overline{2.5} & \overline{2.5} & \bar{1} & 1/\bar{3} & 1/\bar{3} & \bar{1} & \bar{5} & \bar{5} & \bar{5} & 1/\bar{3} & 1/\bar{3} & \bar{1} & \bar{5} & \bar{5} \\ \bar{1} & \overline{2.5} & \bar{1} & \bar{4} & \bar{3} & \bar{1} & \bar{1} & \bar{3} & \bar{7} & \bar{7} & \bar{7} & \bar{1} & \bar{1} & \bar{3} & \bar{7} & \bar{7} \\ 1/\overline{4.5} & 1/\overline{2.5} & 1/\bar{4} & \bar{1} & 1/\bar{3} & 1/\bar{5} & 1/\bar{5} & 1/\bar{3} & \bar{3} & \bar{3} & \bar{3} & 1/\bar{5} & 1/\bar{5} & 1/\bar{3} & \bar{3} & \bar{3} \\ 1/\bar{3} & \bar{1} & 1/\bar{3} & \bar{3} & \bar{1} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & \bar{5} & \bar{5} & \bar{5} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & \bar{5} & \bar{5} \\ 1/\bar{2} & \bar{3} & \bar{1} & \bar{5} & \bar{7} & \bar{1} & \bar{1} & \bar{3} & \bar{7} & \bar{7} & \bar{7} & \bar{1} & \bar{1} & \bar{3} & \bar{7} & \bar{7} \\ 1/\bar{2} & \bar{3} & \bar{1} & \bar{5} & \bar{7} & \bar{1} & \bar{1} & \bar{3} & \bar{7} & \bar{7} & \bar{7} & \bar{1} & \bar{1} & \bar{3} & \bar{7} & \bar{7} \\ 1/\bar{4} & \bar{1} & 1/\bar{3} & \bar{3} & \bar{5} & 1/\bar{3} & 1/\bar{3} & \bar{1} & \bar{5} & \bar{5} & \bar{5} & 1/\bar{3} & 1/\bar{3} & \bar{1} & \bar{5} & \bar{5} \\ 1/\bar{7} & 1/\bar{5} & 1/\bar{7} & 1/\bar{3} & 1/\bar{5} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & \bar{1} & \bar{3} & \bar{3} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & \bar{3} & \bar{5} \\ 1/\bar{7} & 1/\bar{5} & 1/\bar{7} & 1/\bar{3} & 1/\bar{5} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & 1/\bar{3} & \bar{1} & \bar{3} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & \bar{3} & \bar{5} \\ 1/\bar{7} & 1/\bar{5} & 1/\bar{7} & 1/\bar{3} & 1/\bar{5} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & 1/\bar{3} & 1/\bar{3} & \bar{1} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & \bar{3} & \bar{3} \\ 1/\bar{2} & \bar{3} & \bar{1} & \bar{5} & \bar{7} & \bar{1} & \bar{1} & \bar{3} & \bar{7} & \bar{7} & \bar{7} & \bar{1} & \bar{1} & \bar{3} & \bar{7} & \bar{7} \\ 1/\bar{2} & \bar{3} & \bar{1} & \bar{5} & \bar{7} & \bar{1} & \bar{1} & \bar{3} & \bar{7} & \bar{7} & \bar{7} & \bar{1} & \bar{1} & \bar{3} & \bar{7} & \bar{7} \\ 1/\bar{4} & \bar{1} & 1/\bar{3} & \bar{3} & \bar{5} & 1/\bar{3} & 1/\bar{3} & \bar{1} & \bar{5} & \bar{5} & \bar{5} & 1/\bar{3} & 1/\bar{3} & \bar{1} & \bar{5} & \bar{5} \\ 1/\bar{7} & 1/\bar{5} & 1/\bar{7} & 1/\bar{3} & 1/\bar{5} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & 1/\bar{3} & 1/\bar{3} & 1/\bar{3} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & \bar{1} & \bar{3} \\ 1/\bar{7} & 1/\bar{5} & 1/\bar{7} & 1/\bar{3} & 1/\bar{5} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & 1/\bar{5} & 1/\bar{5} & 1/\bar{3} & 1/\bar{7} & 1/\bar{7} & 1/\bar{5} & 1/\bar{3} & \bar{1} \end{bmatrix} \end{array}$$

Similarly, the relative importance matrices for sub-criteria in the second level and mining methods in the third level were developed based on the UBC technique of the selection of a mining method (Miller-Tait et al., 1995) and shown in Appendix A. These relative importance matrices were converted into fuzzy matrices by using the following eqn. (1) (Gorai, Kanchan, Upadhyay, Tuluri, Goyal and Tchounwou, 2015).

$$\bar{x}_\alpha = [x - \alpha, x + \alpha]; \ \frac{1}{\bar{x}_\alpha} = \left[\frac{1}{x + \alpha}, \frac{1}{x - \alpha}\right] \tag{1}$$

In general, the α value ranges between 0 and 1, and it may be any

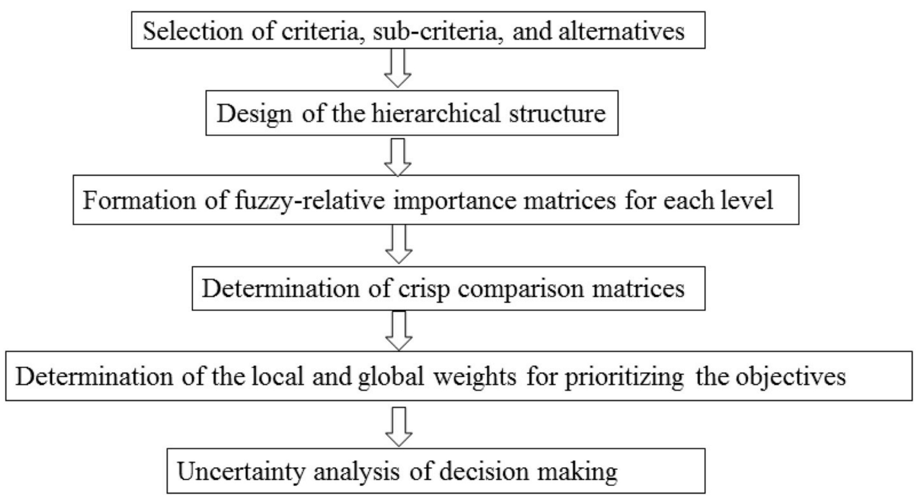

Fig. 1. Flowchart of the working methodology.

Table 1
List of criteria, sub-criteria, and alternatives.

Sl. No	Criteria	Sub-criteria
1	Dip (DI)	Flat (FL), Moderate (MO), Steep (ST)
2	Shape (SH)	Massive (MA), Tabular (TA), Irregular (IR)
3	Thickness (TH)	Narrow (NA), Intermediate (IN), Thick (TI)
4	Depth (DE)	Shallow (SA), Moderate (MD), Deep (DP)
5	Grade distribution (GD)	Uniform (UN), Gradational (GR), Erratic (ER)
6	RMR of Ore (ORMR)	Very weak (OVW), Weak (OW), Moderate (OM), Strong (OS), Very strong (OVS)
7	RMR of hanging wall (HRMR)	Very weak (HVW), Weak (HW), Moderate (HM), Strong (HS), Very strong (HVS)
8	RMR of foot wall (FRMR)	Very weak (FVW), Weak (FW), Moderate (FM), Strong (FS), Very strong (FVS)
9	Productivity (PR)	Low (PL), Medium (PM), High (PH)
10	Recovery (RE)	Low (RL), High (RH)
11	Dilution (DL)	Low (DW), Medium (DM), High (DH)
12	RSS of ore (ORSS)	Very weak (ORVW), Weak (ORW), Moderate (ORM), Strong (ORS)
13	RSS of hanging wall (HRSS)	Very weak (HRVW), Weak (HRW), Moderate (HRM), Strong (HRS)
14	RSS of foot wall (FRSS)	Very weak (FRVW), Weak (FRW), Moderate (FRM), Strong (FRS)
15	Flexibility (FLE)	Low (FLL), High (FLH)
16	Safety (SE)	Low (SEL), High (SEH)
Alternatives (Underground Metal Mining Methods)		
Block caving (BC), Sub level stoping (SS), Sub level caving (SC), Room and Pillar (RP), Shrinkage stoping (SH), Cut and fill stoping (CF), Square set stoping (SQ)		

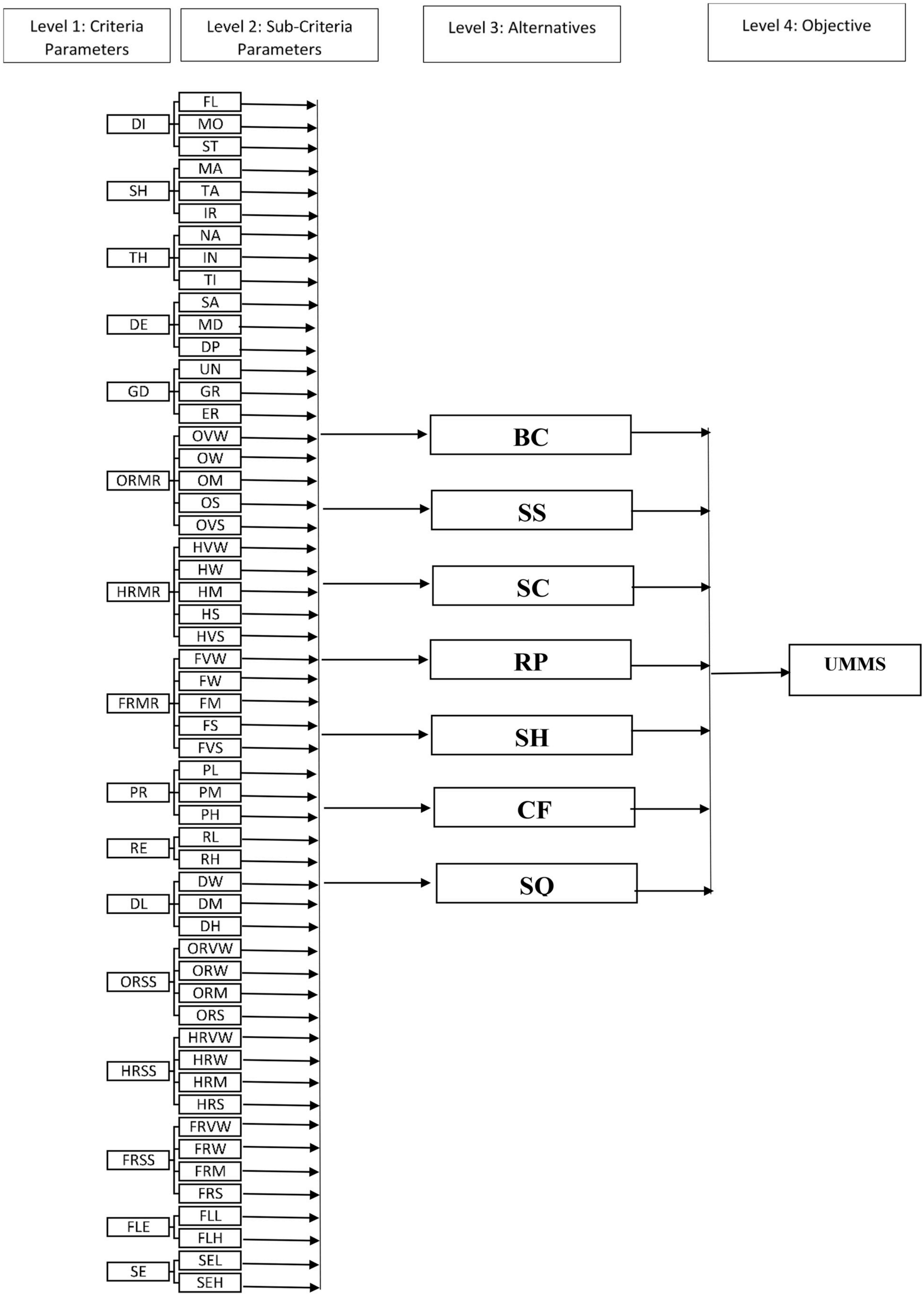

Fig. 2. Hierarchical structure of the FAHP model.

fractional value in between 0 and 1. The higher values of α (i.e., close to 1) represent more uncertainty and the lower values less uncertainty. Using eqn. (1), the relative importance matrices of criteria sub-criteria and underground mining methods were converted into fuzzy matrices. This study used 6 α values (0, 0.2, 0.4, 0.6, 0.8, and 1) to analyse decision making results.

2.4. Determination of crisp comparison matrices

The fuzzy pair-wise comparison matrices for each level were converted into crisp comparison matrices using the following eqn. (2) (Lee, 1995).

$$a_{ij}^{\alpha} = \lambda a_{iju}^{\alpha} + (1 - \lambda) a_{ijl}^{\alpha} \tag{2}$$

a_{iju}^{α} and a_{ijl}^{α} in the above eqn. (2) are the upper and lower bound, respectively, of relative importance value a_{ij} in the previously developed matrix. The defuzzified value a_{ij}^{α} returns the crisp value for the relative importance value a_{ij}. In eqn. (2), λ represents the decision making attitude. The value of λ can be any value between 0 and 1. Crisp comparison matrices for the parameters at each level were constructed. After developing the crisp comparison matrices the consistency ratio of all the crisp comparison matrices at each level was examined using eqn. (3) (Saaty, 1980). The *CR* values for all the parameters at each level were found to be less than 0.1. The consistency ratio of the matrix can be determined as

$$CR = \frac{CI}{RI} \tag{3}$$

In the above eqn., the *CI* and *RI* are respectively the consistency index and the random index. The *CI* of a matrix can be determined as

$$CI = \frac{\lambda_{\max} - n}{n - 1}$$

Where $\lambda_{\max}$ is the maximum eigen value, and n is the size of the crisp comparison matrix.

The value of *RI* depends on the size of the matrix. Many researchers determined *RI* values for various sizes of matrices. In this study, the *RI* values suggested by Alonso and Lamata (2006) were considered for the analysis; these RI values for different sizes of matrices are listed in Table 3.

2.5. Determination of the local and global weights for prioritizing the objectives

The local and global weights of the parameters at each level were determined using the geometric mean concept. The geometric mean of the *i*th row (GM_i) of a crisp matrix of a corresponding row parameter can be determined using eqn. (4). Where b_{ij} in eqn. (4) represents the value in the *i*th row and *j*th column of the crisp comparison matrix. *M* is the number of parameters in the crisp comparison matrix.

$$GM_i = \left[\prod_{j=1}^{M} b_{ij} \right]^{1/M} \tag{4}$$

The local weight of the variable can be determined using eqn. (5)

$$w_i = GM_i / \sum_{i=1}^{N} GM_i \tag{5}$$

After determining the local weights, determination of the global weights at the third and fourth levels needs to be performed. The fuzzy global weights (G_k) can be computed from the local weight of the *k*th level and the global weights of the (*k*-1)$^{\text{th}}$ level using eqn. (6).

$$G_k = w_k G_{k-1} \tag{6}$$

The global weights of each mining method were determined using the above equation.

2.6. Sensitivity analysis of decision making

The sensitivity analysis of the proposed decision-making model was conducted by varying the fuzzification factor (α) in eqn. (1) and decision-making attitude (λ) in eqn. (2). The decision-making attitude was considered for three conditions (the optimistic, pessimistic, and neutral). The λ values for optimistic, pessimistic, and neutral conditions were chosen as 1, 0, and 0.5 respectively. The model output was also analysed for six sets (0, 0.2, 0.4, 0.6, 0.8 and 1) of the fuzzification factor (α) in the range of 0–1. The fuzzy pair-wise comparison matrices were formulated using different fuzzification factors (α) for each set of criteria and sub-criteria. The crisp comparison matrices corresponding to each fuzzy pair-wise comparison matrix were derived for three decision-making attitudes. In other words, the crisp comparison matrices were derived for three λ values (λ = 0, 0.5, 1) using eqn. (2). The decision-making model output was analysed for each combination of α and λ.

The sensitivity of decision-making in the ranking of seven mining methods was analysed by considering the fuzzification factor in 16-criteria and 54-sub-criteria. The results indicated that the ranking or priorities of seven mining methods were not altered by either changing of the fuzzification factor from 0 to 1 or changing the decision-making attitude. Therefore, for any value of λ and α, the rank of a particular mining method remains the same. The rank of a particular mining method is decided based on global weights. The higher the global

Table 2
Fuzzy relative importance scale used for making pair-wise comparisons (Saaty, 1980).

Relative importance	Fuzzy Scale	Definition	Explanation
$\tilde{1}$	(1,1,1)	Equal importance	Two activities contribute equally to the objective.
$\tilde{3}$	(3-α), 3, (3+α)	Weak importance	Experience and judgement slightly favour one activity over another.
$\tilde{5}$	(5-α), 5, (5+α)	Essential or strong importance	Experience and judgement strongly favour one activity over another.
$\tilde{7}$	(7-α), 7, (7+α)	Demonstrated importance	Experience and judgement strongly favour one activity over another.
$\tilde{9}$	(9-α), 9, (9+α)	Extreme importance	One activity is strongly favoured and demonstrated in practice.
$\tilde{2}$, $\tilde{4}$, $\tilde{6}$, $\tilde{8}$	(x-α), x, (x+α)	Intermediate values between two adjacent judgements	The evidence favouring one activity over another is of the highest possible order of affirmation.

Note: α is a fuzzification factor.

Table 3
Random index (RI) values for different matrix size.

Size of matrix	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
RI	0	0	0.52	0.88	1.1	1.24	1.34	1.4	1.44	1.48	1.51	1.53	1.55	1.57	1.58	1.59

weight of the mining method, the higher is the rank or priority. To demonstrate the sensitivity of the ranking of various mining methods under different degrees of uncertainty (α) and different decision-makers' attitudes (λ), the results of one criterion (dip) is shown in the text in order to reduce the manuscript length. The sensitivity of the decision-making results for all other parameters are shown in Appendix B. The global weights of different mining methods for a different level of uncertainty or fuzzification factors (α) and the decision-maker's attitude (λ) were determined using eqns. (1), (2) and (6), as explained above in Sections 2.3, 2.4, and 2.5. The global weights of each mining method correspond to flat-dip, moderate-dip, and steep-dip under different fuzzification factors (α) and the decision-maker's attitude (λ), all of which are shown in Tables 4–6 respectively.

Table 4 shows the ranks of seven mining methods for six fuzzification factors (α = 0, 0.2, 0.4, 0.6, 0.8 and 1) in three decision-making attitudes, these being pessimistic (λ = 0), unbiased (λ = 0.5), and optimistic (λ = 1) with the flat dip condition of the ore deposit. The trend of global weights for different fuzzification factors indicates that the room and pillar mining method is most appropriate for flat deposit, irrespective of the fuzzification factors and decision-making attitudes. The rank of the room and pillar mining method is always at the top and never alters when changing the values of α and λ. It was also observed that though the global weights of each mining method were altered due to changes in the value of α and λ, the rank of the mining methods never altered.

Similarly, Table 5 shows the ranks of seven mining methods for six fuzzification factors (α = 0, 0.2, 0.4, 0.6, 0.8 and 1) in three decision-making attitudes, these being pessimistic (λ = 0), unbiased (λ = 0.5), and optimistic (λ = 1) with the moderate dip condition of the ore deposit. The trend of global weights for different fuzzification factors indicates that the cut and fill mining method is most suitable for moderate-dip ore deposits irrespective of the fuzzification factors and decision-making attitudes. Here, also, the rank of the cut and fill mining method is always at the top and never alters when changing the values of α and λ. It was also observed that though the global weights of each mining method were altered with changes in the value of α and λ, the rank of the mining methods never altered.

In the same way, Table 6 shows the ranks of seven mining methods for steeply dipping ore deposits. In this case, the square set stoping method exhibits the highest global weight. Here, also, the global weights of each mining method were altered due to changes in the value of α and λ, but the rank of the mining methods never varied. Therefore, the square set stoping method was always at the top and did not alter when changing the values of α and λ.

3. Case study

The validation of the proposed decision-making model was conducted with ore deposit data of the Tummalapalle mine of the Uranium Corporation of India Limited (UCIL). The latitudes of the deposit ranges from 14°18‘36.6″N to 14°20′20″N and the longitude from 78°15′16.57″E to 78°18″3.33″E. The deposit is located in the Cuddapah district of Andhra Pradesh, India as shown in Fig. 3. The direction of the strike of the deposit is WNW-ESE, and the dip varies from 15° to 17°. The ore body is fairly continuous over the entire strike length of 6.6 km and uniformly extending to a depth of 275 m. The width of the hangwall and footwall are, respectively, 3.2 m and 2.5 m. The ore deposit is tabular in shape with little variation in its grade and thickness along the strike and dip direction. The host rock is quite competent. Fig. 3 shows the location of Tummalapalle Uranium Project of UCIL.

In the proposed study, the FAHP model was developed by considering 16-criteria. However, model validation and sensitivity analysis were conducted based on 8-criteria due to the unavailability of the remaining data, these being: dip, shape, thickness, grade distribution, depth of the deposit, the RMR of the ore zone, RMR of hanging wall, and RMR of foot wall. The characteristics of the ore deposit are shown

Table 4
Global weights of seven mining methods for different decision-making attitudes (λ) and fuzzification factors (α) for flat-dip.

	Dip-Flat																	
	λ = 0 (pessimistic)						λ = 0.5 (unbiased)						λ = 1 (optimistic)					
	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1
Block Caving	0.0305	0.0305	0.0302	0.0297	0.0292	0.0267	0.0307	0.0307	0.0307	0.0306	0.0303	0.0299	0.0307	0.0311	0.0314	0.0289	0.0316	0.0309
Sublevel Stoping	0.0080	0.0077	0.0075	0.0073	0.0069	0.0061	0.0079	0.0079	0.0079	0.0079	0.0078	0.0076	0.0080	0.0081	0.0082	0.0075	0.0081	0.0083
Sublevel Caving	0.0049	0.0048	0.0047	0.0046	0.0044	0.0040	0.0049	0.0049	0.005	0.0050	0.0050	0.0051	0.0049	0.0050	0.0051	0.0048	0.0054	0.0056
Room and Pillar	0.0454	0.0454	0.0451	0.0444	0.0434	0.0395	0.0456	0.0457	0.0456	0.0453	0.0447	0.0436	0.0456	0.0462	0.0464	0.0424	0.0461	0.0446
Shrinkage Stoping	0.0025	0.0024	0.0023	0.0023	0.0023	0.0021	0.0025	0.0025	0.0024	0.0025	0.0025	0.0025	0.0025	0.0025	0.0025	0.0023	0.0025	0.0027
Cut and Fill stoping	0.0049	0.0048	0.0047	0.0046	0.0044	0.0040	0.0049	0.0049	0.005	0.0050	0.0050	0.0051	0.0049	0.0050	0.0051	0.0048	0.0054	0.0056
Squareset Stoping	0.0080	0.0077	0.0075	0.0073	0.0069	0.0061	0.0079	0.0079	0.0079	0.0079	0.0078	0.0076	0.0080	0.0081	0.0082	0.0075	0.0081	0.0083

Table 5
Global weights of seven mining methods for different decision-making attitudes (λ) and fuzzification factors (α) for moderate-dip.

	Dip-Moderate																	
	λ = 0 (pessimistic)						λ = 0.5 (unbiased)						λ = 1 (optimistic)					
	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1
Block Caving	0.0023	0.0022	0.0020	0.0019	0.0017	0.0016	0.0023	0.0023	0.0023	0.0023	0.0023	0.0023	0.0023	0.0024	0.0025	0.0023	0.0025	0.0029
Sublevel Stoping	0.0017	0.0016	0.0015	0.0014	0.0014	0.0013	0.0017	0.0016	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0018	0.0017	0.0019	0.0020
Sublevel Caving	0.0017	0.0016	0.0015	0.0014	0.0014	0.0013	0.0017	0.0016	0.0017	0.0017	0.0017	0.0017	0.0017	0.0017	0.0018	0.0017	0.0019	0.0020
Room and Pillar	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0008	0.0009	0.0008	0.0008	0.0009	0.0008	0.0010	0.0010
Shrinkage Stoping	0.0008	0.0008	0.0008	0.0007	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0008	0.0009	0.0008	0.0008	0.0009	0.0008	0.0010	0.0010
Cut and Fill stoping	0.0092	0.0090	0.0088	0.0085	0.0083	0.0080	0.0092	0.0092	0.0091	0.0091	0.0091	0.0089	0.0091	0.0093	0.0095	0.0088	0.0097	0.0096
Squareset Stoping	0.0092	0.0090	0.0088	0.0085	0.0083	0.0080	0.0092	0.0092	0.0091	0.0091	0.0091	0.0089	0.0091	0.0093	0.0095	0.0088	0.0097	0.0096

Table 6
Global weights of seven mining methods for different decision-making attitudes (λ) and fuzzification factors (α) for steep-dip.

	Dip-Steep																	
	λ = 0 (pessimistic)						λ = 0.5 (unbiased)						λ = 1 (optimistic)					
	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1
Block Caving	0.0020	0.0020	0.0019	0.0018	0.0018	0.0019	0.0020	0.0020	0.0020	0.0020	0.0020	0.0023	0.0020	0.0020	0.0020	0.0018	0.0020	0.0022
Sublevel Stoping	0.0020	0.0020	0.0019	0.0018	0.0018	0.0019	0.0020	0.0020	0.0020	0.0020	0.0020	0.0017	0.0020	0.0020	0.0020	0.0018	0.0020	0.0022
Sublevel Caving	0.0020	0.0020	0.0019	0.0018	0.0018	0.0019	0.0020	0.0020	0.0020	0.0020	0.0020	0.0017	0.0020	0.0020	0.0020	0.0018	0.0020	0.0022
Room and Pillar	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0009	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
Shrinkage Stoping	0.0020	0.0020	0.0019	0.0018	0.0018	0.0019	0.0020	0.0020	0.0020	0.0020	0.0020	0.0009	0.0020	0.0020	0.0020	0.0018	0.0020	0.0022
Cut and Fill stoping	0.0020	0.0020	0.0019	0.0018	0.0018	0.0019	0.0020	0.0020	0.0020	0.0020	0.0020	0.0089	0.0020	0.0020	0.0020	0.0018	0.0020	0.0022
Squareset Stoping	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0089	0.0003	0.0003	0.0003	0.0003	0.0004	0.0004

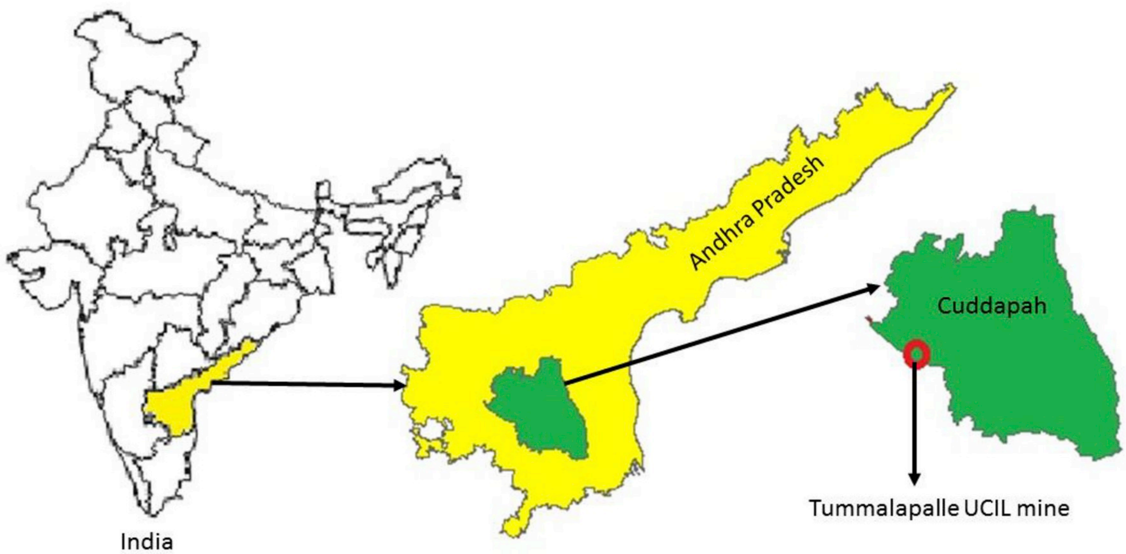

Fig. 3. Location of the tummalapalle uranium ore deposit.

Table 7
Characteristics of the uranium ore deposit of the Tummalapalle mine.

Criteria Parameters	Field data	Characteristics
Dip	15^0	Flat
Shape	tabular	Tabular
Thickness	1.5 m	Narrow
Grade distribution	Uniform	Uniform
Depth	275 m	Moderate
RMR of ore zone	48	Moderate
RMR of hanging wall	34	Weak
RMR of foot wall	48	Moderate

in Table 7.

The global weights of these characteristics were taken from the model developed for the selection of the best mining method for the specified ore deposit. Global weights corresponding to the flat dip, tabular shape, narrow thickness, moderate depth, uniform grade distribution, the moderate RMR of the ore, the weak RMR of the hanging wall, and the moderate RMR of the foot wall, with respect to each mining method, were determined for α value 1 and λ value 0.5 using eqns. (1)–(6). These global weights were determined using a method similar to the one explained in Section 2. These global weights are shown in Table 8.

The total score (shown in Table 8) for a specific mining method was determined by summing up the respective weights of all the criteria. All the scores were ranked from the highest to the lowest value. The highest score was obtained for the Room and Pillar mining (0.0582) and thus it was assigned first rank. Hence, the best mining method obtained from the model is the Room and Pillar mining for the excavation of the ore deposit, and the UCIL adopted the same mining method for excavation of the ore deposit.

3.1. Sensitivity analysis of the characteristics (criteria parameters) of the specified mine

Sensitivity analysis for the ranking of the mining methods for the specified ore deposit was carried out for six fuzzification factors (α = 0, 0.2, 0.4, 0.6, 0.8 and 1) in three decision-making attitudes, these being pessimistic (λ = 0), unbiased (λ = 0.5), and optimistic (λ = 1). The global weights of each combination of λ and α were determined using eqns. (1)–(6). At each λ, six fuzzification factors (α) were considered for analyzing the sensitivity in the ranking of the mining methods. The trends of the global weights in each case are shown in Table 9. The trend of global weights for different fuzzification factors clearly indicates that the room and pillar mining method is most suitable for the specified uranium ore deposit, irrespective of the fuzzification factors and decision-making attitudes. The rank of room and pillar mining method was always ranked top and this did not alter when changing the values of α and λ. It was also observed that the global weights of each mining method were altered with changes in the value of α and λ, but the rank of the mining methods rarely changed. The second best mining method was the block caving method.

Table 9 Global weights of different mining methods for the uranium ore deposit (Flat-Dip, Tabular-Shape, Narrow-Thickness, Moderate-Depth, Uniform-Grade distribution, the Moderate-RMR of the ore, the Weak-RMR of the hanging wall, and the Moderate-RMR of the foot-wall).

4. Conclusions

The study aims to analyse the sensitivity in decision-making for the selection of a mining method using the FAHP model. The results indicate that the proposed FAHP decision-making model could be

Table 8
Relative rankings of seven mining methods for the uranium ore deposit of the Tummalapalle mine.

	DI	SH	TH	DE	GD	ORMR	HRMR	FRMR	Score	Rank
BC	0.0299	0.0002	0.0001	0.0007	0.0031	0.0005	0.0006	0.0026	0.0377	2
SS	0.0076	0.0018	0.0002	0.0016	0.0069	0.0054	0.0001	0.0005	0.0241	3
SC	0.0051	0.0018	0.0001	0.0001	0.0031	0.0027	0.0011	0.0026	0.0166	4
RP	0.0437	0.0018	0.0021	0.0007	0.0069	0.0027	0.0001	0.0002	0.0582	1
SH	0.0025	0.0018	0.0021	0.0007	0.0031	0.0027	0.0001	0.0005	0.0135	5
CF	0.0051	0.0018	0.0011	0.0007	0.0007	0.0005	0.0006	0.0005	0.011	7
SQ	0.0076	0.0002	0.0011	0.0001	0.0005	0.0004	0.0011	0.0002	0.0112	6

Table 9
Global weights of different mining methods for the uranium ore deposit (Flat-Dip, Tabular-Shape, Narrow-Thickness, Moderate-Depth, Uniform-Grade distribution, Moderate-RMR of ore, Weak-RMR of hanging wall, and Moderate-RMR of footwall).

	λ = 0 (pessimistic)						λ = 0.5 (unbiased)						λ = 1 (optimistic)					
	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1	α = 0	α = 0.2	α = 0.4	α = 0.6	α = 0.8	α = 1
Block Caving	0.0385	0.0384	0.0381	0.0376	0.0371	0.0344	0.0386	0.0384	0.0386	0.0384	0.0384	0.0381	0.0385	0.0390	0.0393	0.0340	0.0397	0.0391
Sublevel Stoping	0.0247	0.0243	0.0239	0.0235	0.0229	0.0213	0.0247	0.0247	0.0247	0.0247	0.0246	0.0244	0.0247	0.0250	0.0253	0.0205	0.0256	0.0259
Sublevel Caving	0.0165	0.0164	0.0162	0.0162	0.0162	0.0159	0.0165	0.0165	0.0166	0.0166	0.0168	0.0170	0.0165	0.0165	0.0167	0.0138	0.0171	0.0175
Room and Pillar	0.0602	0.0602	0.0599	0.0592	0.0583	0.0539	0.0604	0.0602	0.0601	0.0599	0.0593	0.0585	0.0602	0.0609	0.0611	0.0506	0.0609	0.0594
Shrinkage Stoping	0.0137	0.0136	0.0136	0.0136	0.0137	0.0136	0.0136	0.0137	0.0137	0.0137	0.0137	0.0139	0.0137	0.0135	0.0135	0.0108	0.0136	0.0139
Cut and Fill stoping	0.0111	0.0109	0.0107	0.0106	0.0105	0.0102	0.0110	0.0111	0.0110	0.0111	0.0112	0.0114	0.0111	0.0111	0.0111	0.0120	0.0114	0.0120
Squareset Stoping	0.0117	0.0115	0.0113	0.0111	0.0107	0.0102	0.0117	0.0118	0.0116	0.0117	0.0116	0.0115	0.0117	0.0118	0.0118	0.0109	0.0118	0.0121

robustly used for the selection of a mining method, as the factor's uncertainty levels do not influence the final decision. It was observed that the rank of the highest priority alternative never alters with either changes in the fuzzification factor (α) or the decision-making attitude (λ). It can be inferred from the results that the ranking of the most suitable alternative remains the same irrespective of the fuzzification factors and decision-making attitudes.

Conflict of interest

Authors state that there is not any conflict of interest.

Ethical statement

Authors state that the research was conducted according to ethical standards.

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Appendix A and B. Supplementary data

Supplementary data to this article can be found online at https://doi.org/10.1016/j.jsm.2018.10.003.

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