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# ELECTROMAGNETIC TORQUE CALCULATION OF INDUCTION MOTOR WORKING UNDER ONE PHASE FAILURE

## OBLICZENIA ELEKTROMAGNETYCZNE MOMENTU OBROTOWEGO SILNIKA INDUKCYJNEGO PRACUJĄCEGO PRZY ZANIKU JEDNEJ FAZY

**Abstract:** The paper deals with the three phase induction motor (IM) electromagnetic torque waveform calculation for the case of a one phase supply failure. Computation of the motor quantities was made using the spatial phasor theory in the complex plane. On the base of the measured parameters of the IM the trajectories of the stator and rotor current space phasor were calculated. Of these the motor electromagnetic torque waveform was investigated.

**Streszczenie:** W artykule przedstawiono obliczenia momentu elektromagnetycznego silnika indukcyjnego trójfazowego w przypadku awarii zasilania z jednej fazy. Obliczenia mechaniczne wykonano przy użyciu przestrzennej teorii fazorów na płaszczyźnie zespolonej.

**Keywords:** induction motor, torque ripple, space phasor, one phase failure, complex plane **Słowa kluczowe:** silnik indukcyjny, tętnienia momentu, przestrzeń wskazów, awaria jednej fazy, płaszczyzna zespolona

#### 1. Introduction

The cage rotor induction machine (IM) is widely used in industrial applications. About 65-70 % of all electric energy is consumed by electrical drives, and over 90% of them are induction motors. They are essential elements in any power system. Due to its conception, it has quite a low cost compared to the cost of other machines. It has a great electromechanical robustness and there is a good standardization between the diverse motor producers. The relative simplicity of conception of the machine hides quite a great functional complexity, as soon as it is oriented at controlling the performed electromechanical conversion [4].

Reliability is the one of the basic requirements placed on electric drives seeded in various applications. This requirement is joint with the drive system. A frequent fault states is dropout one supply phase, which in most cases result of current overload. In this context several fault tolerant strategies has been investigated. However their implementation requires first and foremost the right information provided by diagnostic step that is used to appropriate reconfiguration of control device. The reconfiguration of control strategy keeps the drive functioning in quasi-normal state until a maintenance schedule. The sequence including fault detection and intervention is necessary and its duration is important.

Simplified diagram of the considered electric drive is illustrated in Fig. 1. The IM is supplied directly from the mains through protective elements. We are interested of electromagnetic torque waveform at failure of one supply phase.



Fig. 1. Drive with induction motor

In the first phase of break-down, the motor will be supplied by phase-to-phase voltage. Motor will operate as a single phase. By the triac switching, motor pass to two-phase operation state.

## 2. Single-phase supply of IM

In normal operation, the IM is supplied with three-phase harmonic voltages. Phase values can be mathematically expressed

$$u_{1} = U_{m} \sin(\omega t) = U_{m} Re(e^{-j\omega t})$$
  

$$u_{2} = U_{m} \sin(\omega t) = U_{m} Re(\mathbf{a} e^{-j\omega t}) \qquad (1)$$
  

$$u_{2} = U_{m} \sin(\omega t) = U_{m} Re(\mathbf{a}^{2} e^{-j\omega t})$$

Where:  $U_m$  is amplitude of supply voltages

and  $\mathbf{a}=e^{-j\frac{2\pi}{3}}$  is phase shifting factor.

By phase values can be defined voltage space phasor for no faulted case as follow

$$\underline{\mathbf{u}} = \frac{2}{3} \left( u_1 + \mathbf{a} u_2 + \mathbf{a}^2 u_3 \right)$$
(2)

In case of failure, the motor coils will be supplied by a single phase-to-phase voltage

$$u_{12} = u_1 - u_2 = U_m \operatorname{Re}\left[(1 - \mathbf{a})e^{-j\omega t}\right] \quad (3)$$

And then voltage space phasor for faulted case can be expressed

$$\underline{\mathbf{u}} = \frac{2}{6} \left( u_{12} + \mathbf{a} u_{12} \right) \tag{4}$$

Fig. 2 shows the voltage space phasor trajectories for faulted and no-faulted supply.



Fig. 2. Voltage space phasor trajectories

For the calculation stator and rotor currents space phasor, was used a equivalent circuit of IM depicted in Fig. 3

For the further calculations are used the following measured parameters of IM:

$$P_{n} = 4,4kW; U_{n} = 3 \times /380V / 50Hz; 2p = 4;$$
  

$$n_{n} = 1380 rev / min.;$$
  

$$R_{1} = 1\Omega; R'_{2} = 2,12\Omega; L_{m} = 0,159H;$$
  

$$L_{1\sigma} = 7,48mH; L'_{2\sigma} = 7,22mH;$$



Fig. 3. Equivalent circuit of IM

Referred to equivalent circuit for the stator current space phasor  $\underline{i}_1$  following equation is valid

$$\underline{\mathbf{i}}_{1} = \frac{\underline{u}}{R_{1} + \left[ j\omega L_{1\sigma} + \frac{j\omega L_{m} \left( \frac{R'_{2}}{s} + j\omega L'_{2\sigma} \right)}{\frac{R'_{2}}{s} + j\omega \left( L_{m} + L'_{2\sigma} \right)} \right]}$$
(5)

There are several ways to solve the circuit in Fig. 3 for the current i2. The easiest one is to determine the "Thevenin" equivalent circuit. Thevenin theorem states that any linear circuit, that can by separated by two terminals from the rest of system can by replaced by a single voltage source in series with the equivalent circuit. The resulting circuit presents a simple series combination of elements, as shown in Fig. 4 [1].



Fig. 4. Thevenin equivalent circuit of IM

To find the Thevenin voltage, we must recalculate  $U_m$  as follow

$$U_{mth} = U_m \frac{\omega L_m}{\sqrt{R_1^2 + (\omega L_{1\sigma} + \omega L_m)^2}}$$
(6)

The Thevenin impedance is given by

$$Z_{th} = R_{th} + j\omega L_{th} = \frac{j\omega L_m \left(R_1 + j\omega L_{1\sigma}\right)}{R_1 + j\omega \left(L_{1\sigma} + L_m\right)} \quad (7)$$

Because  $L_m = L_{1\sigma}$  and  $\omega(L_m + L_{1\sigma}) = R_1$ , the Thevenin resistance and inductance are approximately given by

$$R_{th} \approx R_1 \frac{L_m}{L_{1\sigma} + L_m}$$

$$L_{th} \approx L_{1\sigma}$$
(8)

From the Thevenin circuit, the rotor current space phasor is calculated as

$$\underline{\mathbf{i}}'_{2} = -\frac{\underline{u}_{th}}{R'_{2}/s + R_{th} + j\omega(L_{1\sigma} + L'_{2\sigma})}$$
(9)

For amplitude of supply voltage  $U_m = 316V$ and frequency f = 50Hz, the Thevenin voltage is  $U_{th} = 301,75V$ . Thevenin resistance has a value  $R_{th} = 0.955\Omega$ .



Fig. 5. Current space phasor trajectories

Fig. 5 shows the stator and rotor current space phasor trajectories calculated on the base of equations (5) and (9). Compute was made for rating motor speed and slip (s = 0, 08).

Electromagnetic torque can be calculated on the base of stator and rotor currents space phasors as follow

$$M_{em} = \frac{3}{2} p L_m Im \left( \underline{\mathbf{i}}_1 \, \underline{\mathbf{i}}_2^* \right) \tag{10}$$

Fig. 6 shows the calculated time plot of the electromagnetic torque waveform of IM working in case of one phase failure. The motor torque is pulsating, as a result of a fault rotating electromagnetic field changes to pulsating.

In case when one wants to express electromagnetic waveform in a space phasor form, equation (10) takes form

$$\underline{\mathbf{m}}_{em} = \frac{3}{2} p L_m Im \left( \underline{\mathbf{i}}_l \ \underline{\mathbf{i}}^*_2 \right) e^{\frac{-j\omega t}{p}} \qquad (11)$$



Fig. 6. Electromagnetic torque waveform

Fig. 7 shows the plot of electromagnetic torque space phasor trajectory.



Fig. 7. Electromagnetic torque trajectory

## 3. Two-phase supply of IM

At the moment of indication of the supply failure is advantageous to connect the motor coil node through a triac or other switch to neutral point, as depicted in Fig. 1. The IM will be supplied by a two-phase voltage.

The supply voltage space phasor takes a form

$$\underline{\mathbf{u}} = \frac{2}{3} \left( u_1 + \mathbf{a} \, u_2 \right) \tag{13}$$

Fig. 8 shows the supply voltage space phasor trajectory, when IM operates with motor coil node connected to neutral point.

In Fig. 9 are shown IM stator and rotor currents space phasor trajectories. The space phasors were calculated on the base of equation (5) and (9).



Fig. 8. Two-phase voltage space phasor trajectory

Trajectories were changed from line segments for the ellipses. Motor magnetic field was changed from pulsating for rotating, but elliptical.



Fig. 9. Two-phase current space phasor trajectories



Fig. 10. Electromagnetic torque waveform



Fig. 11. Electromagnetic torque trajectory

Fig. 10 shows the time plot of IM electromagnetic torque calculated on the base of equation (10). Torque is pulsating again, however, with reduced amplitude. The avarage-value of the electromagnetic torque is increased.

In Fig. 11 is shown the electromagnetic torque trajectory based on equation (11).

#### 4. Conclusion

The presented contribution shows the possibility of the electromagnetic torque waveform computation of a three-phase induction motor using a space phasors. Proposed mathematical method is simple and gives sufficiently accurate values. The method is applicable to multiphase motors too.

Calculation shows that course of the electromagnetic torque in case of one phase failure is pulsated, with the double frequency of supply voltage. Connection of motor coil node with neutral point improves torque conditions.

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