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Orginal Article

An innovative approach to a safety format for the estimation of structural robustness

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Abstract: The estimation of structural robustness remains one of the most important stages of the design of structural systems. Recommended design strategies for the robustness assessment are based on the provisions specified in the actual EN 1991-1-7 and ISO 2394:2015. Currently, the EN 1991-1-7 and ISO2394:2015 allows the use of indirect tie-force method, but normally, non-linear pseudo-static analysis is widely used, because it is based on more realistic constitutive relations for basic variables, which enables a simulation of the real structural behaviour. Implementation of the non-linear pseudo-static analysis for the assessment of a structural system in accidental design situations requires to adopt a different approach to safety format.

The paper presents an innovative approach to safety format calibration for non-linear analysis of RC-structures subjected to accidental loads. The proposed method of the robustness estimation is based on the joint energy-saving (conversion) approach and the full probabilistic method for the estimation of a safety format for pseudo-static non-linear response of modified (damaged) structural system. The proposed probabilistic considerations are based on the Order Statistic Theory.

Keywords: robustness, progressive collapse, reliability, safety format, pseudo-static response

1. Introduction

The progressive collapse of a damaged structural system immediately after sudden loss of a column can be prevented or mitigated using the following methods:

1) TF-method (indirect tie-force method);
2) ALP-method (direct Alternate Load Path method);
3) risk-based method;
4) key-element design method.

The indirect TF-method consists of improving the structural integrity of a building by providing redundancy of Load Path and ductile detailing. Currently, the EN 1991-1-7 and ISO2394:2015 allows the use of the indirect method, some regulations concerning this subject are also contained in the EN 1992-1-1.

In this case, criteria are devised to assess the local resistance to withstand a specific assumed accidental load. The direct method, referred in [1] as "Alternate Load Path (ALP) – method" is most widely used in the practical design and based on criteria for evaluating "the capability of a damaged (modified) structural system to bridge over or around the damaged volume of area without progressive collapse developing from the local damage". This direct method requires the designer to prove that a structure is capable to fulfil its performance objectives by bridging over one or more failed (or notionally removed) structural elements, with a potential additional damage level lower than the specified limit (EN2394:2015). The ALP strategy concerns the situation where one or more structural elements (beams, columns, walls) have been damaged, for whatever reason, to such an extent that their normal load bearing capacity has vanished completely. Structural robustness and integrity are commonly defined as the sensitivity of structural system to local failure. The ALP-method consists in the assessment of redistribution of internal forces in a structural system following the sudden loss of a vertical support element on the basis of the non-linear analysis.

According to Starossek [21], the current design methods are inadequate for the assessment of progressive collapse resistance, which can be summarized as follows:

1. Current codes [10], [12], [22], [23] are based on local instead of global failure. Global structural safety against collapse of an entire system or its major part is a function of safety of all the elements against local failure.
2. The second shortcoming of the current design methods is that low-probability events and unforeseeable incidents – i.e. event E for which $P(E)$ is very small – are not taking into account. Starossek [21] argues that for a slender high-rise building, the initial local failure is the simultaneous failure of all vertical elements of a floor, thus the probability of collapse is the sum of probability of a failure of all elements. And if the number of floors is large enough, even very low probabilities of global failure resulting from accidental circumstances can sum up to a probability of the global failure large enough to be seriously considered [21].
3. The third inadequacy of the current design procedures concerns the fact that the probabilistic concept requires the specification of acceptable failure probabilities.

An excellent detailed review of all aspects concerning the regulations and research on progressive collapse and robustness of building structures is presented in [2].

In the general case, the proposed robustness assessment procedure [3] consists of the following steps:

1. determination of the non-linear static response of a considered system;
2. dynamic assessment, using a simplified approach [4] based on energy balance and on obtaining pseudo-static response;
3. determination of the ultimate (pseudo-static) gravity load (response) for the assessment of the robustness of a structural system based on the ultimate value of the static displacement u_{ult} (or ψ_{ult} for punching assessment of sudden column removal based on the [3]);
4. assessment of ductility of connections on the basis of conditions of compatibility between system and subsystem;
5. assessment of safety format for non-linear analysis of a damaged structural system. It should be emphasized that the first five steps and their adaptations are widely considered in numerous international publications, but limited number of works are devoted to safety format assessment at accidental design situation.

This paper briefly presents the main steps of assessing the robustness of a structural system based on classical energy-conservation approach, while focusing on ensuring the target safety format during the use of non-linear analysis for obtaining pseudo-static response in accidental design situation. Taking into account that safety formats for non-linear analysis implemented in currently developed codes have many uncertainties and statistically incorrect and vague formulations, this publication aims to propose a new approach to calibration of the value of the global safety factor related to computational uncertainty for NLFEAs of RC-structures in accidental design situation.

2. Determination of the non-linear response

As discussed in detail in [3], the static response under gravity loading may be established either from detailed non-linear FEM-analysis (non-linear static or non-linear dynamic) or from simplified models, as it is performed, for example, in case of the flat slabs punching [3].

As shown in [2] and [5], non-linear analysis takes into account deformation properties of RC-sections and physical constitutive relations for materials (for example, "σ-ε" for materials based on the mean values of the parameters) and allows a simulation of a real structural behaviour. It reflects an integral response, where all local sections interact and, therefore, it requires an adequate approach for safety assessment.

It should be underlined that non-linear analysis gives a possibility of assessment of global resistance and requires a safety format for global resistance [6]. In accordance with [6], the term global resistance (global safety) is used for "assessment of structural response on higher structural level than cross-section". It should be mentioned that the global resistance format is considered the best practical tool for the safety assessment of RC-elements or structural systems by means of NLFEA within the semi-probabilistic or full-probabilistic approaches.

3. Pseudo-static response of a damaged structural system

According to the approach proposed by [4], sudden loss of a column is considered to be similar in effect to sudden application of the gravity load on a damaged (modified) structure with a removed column. This damaged system can be treated as a single degree of freedom (SDOF) system consisting of vertical deflections at the point of a removed column.

Assuming that the maximum dynamic deflection u_{dyn} at the point of the joint of a removed column is equal to ultimate static displacement u_{st} obtained from the non-linear static response, pseudo-static gravity load $F_{ps,u}$ can be calculated (Fig. 1). The following assumption is formulated on the basis of the proposed approaches [4], [7].

A modified (damaged) structural system with SDOF has the required robustness in accidental design situation, if the gravity load applied immediately after sudden column loss does not exceed ultimate pseudo-static reaction (response) $F_{ps,u}$ which is obtained from the equality balance of the external work over dynamic displacement, and internal energy absorbed by the system (substructure) over the maximum (ultimate) static deflection u_{ult}. Estimation of the Limit State of robustness performed from the following inequality:

$$F_{st} \leq F_{ps,u} \tag{1}$$

where F_{st} is a design value of the generalized gravity load, applied to a structure immediately after sudden column loss.

In general, based on energy-conservation consideration (Fig. 1), the ultimate pseudo-static response is equal:

$$F_{ps,u} = \frac{1}{u_u} \int_0^{u_u} P(u) \cdot du \tag{2}$$

where u_u is the ultimate value of the static deflection (displacement) obtained on the basis of non-linear static response.

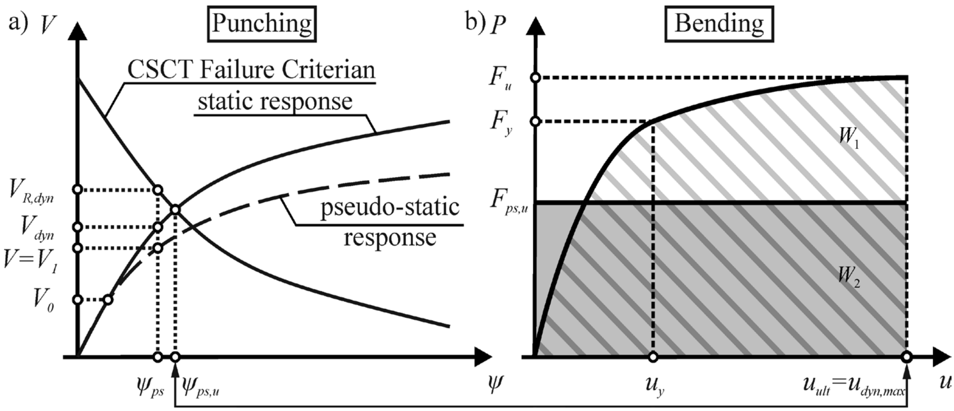

Fig. 1. The principle of assessing the robustness of a structural system with flat slabs based on a combined approach

In case of the flat slab robustness assessment, the following combined procedure is recommended. The maximum dynamic displacement $u_{dyn,max}$, which is used for calculation of the pseudo-static ultimate gravity load $P_{ps,ult}$ in case of the bending failure mode is obtained from the corresponding pseudo-static rotation $\psi_{ps,u}$ calculated on the basis of CSCT-model for punching shear [8] (Fig. 1a).

It should be noted that this approach proposed the system pseudo-static capacity as the single measure of structural robustness, and, therefore, the energy-conservation approach might be criticised. Nevertheless, the results of the detailed analysis given in [3] shows that the implicit error due to these simplifications is relatively small (no more than 5 to 8%) and only slightly affects the final robustness assessment.

4. Safety format for the non-linear analysis

4.1. Reliability index or probability of failure

As shown in [1], the first-generation probability-based Limit Design Criteria (Limit State Design) (such as, for example, EUROCODES) are all based, to varying degrees, on reliability of individual structural members and components.

However, to implement reliability-based design criteria against progressive collapse (for robustness limit state assessment) in the field, the limit state probability (or the reliability index) should be evaluated for a structural system. In contrast to member reliability, this evaluation *"is difficult (complicated) even at the present state of art and with computational resources available"* [1].

As shown in [1], the probability of structural system failure is an order of magnitude which depends less on the redundancy of the system and the degree continuity between members. The recommended value of the acceptable reliability index β_{tag} is based on the design situation, and normally is not specified in design codes.

These threshold values were proposed by [1], assuming that the accepted unconditional probability of failure for extreme (accidental) loads is the same as the one accepted for the failure of structural elements subjected to appropriate load combinations. For example, if the mean rates of occurrence of the accidental event is: $\lambda_i = 10^{-6}\ldots10^{-5}$ (according to [1]), conditional failure probability for a structural system should be an order of $10^{-2}\ldots10^{-1}$, and the target value of reliability index β_{tag} should be an order of 1.5 (for state function *g(x)* in case of the Normal or Lognormal distribution function for resistance). From the other hand, the maximum acceptable target failure probability (reliability index) should be estimated on the basis of LQI-acceptance criterion according to ISO2394:2015.

4.2. Safety factor for global resistance (pseudo-static response)

4.2.1. *Available methods for estimation γ_{global}*

The global resistance approach was initiated by the introduction of non-linear analysis, which is based on the global structural model and offers appropriate tools for the safety assessment [9].

In general case, the design criterion, in line with the global resistance format [10], may be expressed as the following inequality:

$$F_d \leq R_d \tag{3}$$

with $R_d = R_{rep} / (\gamma_R \cdot \gamma_{Rd})$, where: F_d is the design value of actions (effects of actions); R_d – the design value of global resistance; R_{rep} – the representative global resistance of a structure (evaluated according to a selected safety format); γ_R – the global resistance safety factor accounting for the uncertainties related to the inherent randomness of material properties (i.e. aleatory uncertainties); γ_{Rd} – the global safety factor representing the model uncertainties (i.e. epistemic uncertainties).

Various methods based on different levels of implementation of the probabilistic theory can be employed to evaluate the design structural resistance, R_d. According to the general approach of the global resistance format proposed by *fib* MC2010 [10], the design resistance R_d may be estimated by different safety format such as the probabilistic method (PM), identified as an exact approach or by one of the global resistance methods (GRM), identified as simplified approaches [11].

If the probabilistic method (PM) [9] is used, [10] the global structural resistance R is represented by an appropriate probabilistic distribution defined with the use of the non-linear FE-analysis. According to the required target level of reliability, the design of global structural resistance R_d can be evaluated directly as:

$$R_d = \frac{1}{\gamma_{Rd}} R\left(\alpha_R \cdot \beta\right) \tag{4}$$

where: $R(\alpha_R \cdot \beta)$ is the desired quantile of the adopted distribution of the global structural resistance R corresponding to the target reliability index β value and α_R is the FORM sensitivity factor [12] ($\alpha_R < 1.0$).

As shown in our studies [13], the probabilistic method (PM) by [12] is not exact, as declared, and has many disadvantages:

1. The global resistance factor used in this safety format is not represented by a single value and must be calculated each time a new analysis is performed;
2. Probabilistic simulation is time-consuming, and therefore its use requires economic justification;
3. The adoption of the LN probability distribution functions may not be correct. The practice of calculations using simulations shows that the distribution of resistance has a so-called *"heavy tails"*. It is calculated with an unknown confidence level γ (for limited groups of results calculations). Every estimated value of the design resistance R_d which is obtained by the probability method based on the limited set of numerical results of non-linear analysis (from $n = 15$ to $n = 35$) represents only an individual value $R_{d,i}$ from probability distribution function of R_d with unknown confidence level γ.

As for the global resistance method (GRM's), different methodologies have been proposed to reduce the computational effort required by the PM, mainly: (I) the partial factor method (PFM) [10], [14], [15], (II) the method of estimating the coefficient of variation of the structural resistance ECOV [9], [10], [14], (III) the global resistance factor (GRF) [10], [15], (IV) the global safety factor (GSF) [16], and (V) the failure mode-based safety factor [11].

The most of the presented methods were analysed and compared in details in the numerous international publications [11], [12], [17]–[20] and in the authors' own studies [13].

Following the "partial factor method (PFM)", the design resistance R_d is obtained by employing a single non-linear analysis performed with the use of the design values of the material strength $f_{d(i)}$.

It can be argued that design values (f_{cd}, f_{yd}) represent extremely low material properties, which does not signify real material behaviour and thus can lead to distorted failure modes.

This method is addressed directly to target design value and thus no extrapolation is involved. Probability of global design resistance R_d is not evaluated and therefore unknown.

According to the "global resistance factor (GRF)" method, the global resistance is defined as follows:

$$R_d = \frac{1}{\gamma_{GL}} R_{rep}\left(f_{cmd}; f_{ym}\right) \tag{5}$$

where: γ_{GL} is the global resistance factor, equal to 1.27 (constant value).

In this method a reduced value f_{cmd} for concrete compressive strength is used as follows: $f_{\text{cmd}} = 0.85 f_{\text{ck}}$. It should be mentioned that the value of concrete compressive strength f_{cmd} does not represent a mean value. The partial factors for both steel and concrete failure recommended by [15] are equalized, and, therefore, "global resistance factor (GRF)" method is approximately consistent with the PFM and has the same uncertainties (mainly only compression type of failure).

Following both the "Estimate of the Coefficient of variation (ECOV)" method and the "Global Safety Format (GSF)" method [16] the design value of the global resistance R_{d} applies:

$$R_d = R_m / (\gamma_R \cdot \gamma_{Rd}) \tag{6}$$

where: R_{d} – the design value of global resistance; R_{m} – the structural resistance predicted by a non-linear resistance model considering the mean values of the material properties ($R_{\text{d}} = R(f_{\text{cm}}, f_{\text{ym}}, \dots, a)$); γ_{R} – the global resistance safety factor according to the uncertainties related to the inherent randomness of material properties.

Assuming a lognormal distribution (LN) of the global resistance of the structure, the global resistance factor γ_{R} can be estimated as:

$$\gamma_R = \exp(\alpha_R \beta \cdot V_R) \tag{7}$$

where: V_{R} is the coefficient of variation of the distribution of the probability of the global structural resistance.

Taking into account LN global resistance distribution, the value V_{R} can be estimated as:

$$V_R = \frac{1}{1.65} \ln (R_m / R_k) \tag{8}$$

where: R_{k} is the structural resistance predicted by the non-linear analysis *performed using the characteristic values of the material properties* to define the structural model (i.e. $R_{\text{k}} = R(f_{\text{ck}}, f_{\text{yk}}, \dots, a)$).

It should be noted that the ECOV is the most widely-mentioned and used method. This method is implemented in *fib* MC2010, prEN1992, and other guides. At first glance, the proposed method seems to be very appealing. Following authors [9] and codes [12], the keystone of the method is the determination of the mean (R_{m}) and characteristic (R_{k}) values of the global resistance only (two non-linear analyses). *However, the achieved results and detailed analysis of the proposed ECOV-method show that it is based on one incorrect statistical statement. It is known from the classical probability theory that the sum of the two 5-percentiles in not 5-percentile, and that global resistance R_{k}, calculated with the characteristic values of the basic variables ($f_{\text{ck}}, f_{\text{yk}}$) is not a characteristic value (not 5-quantile) of the global resistance distribution* (see Annex).

For comparison of the presented methods, the numerical study of the global resistance of the statically indeterminate reinforced concrete beam ($b \times h = 300 \times 500$ mm, $L = 6$ m) with the pinned ends was performed.

Based on 121 results of the numerical simulations, it was stated that in the case if characteristic values of materials strengths are used ($f_{\text{ck}}, f_{\text{yk}}$), the resulting global resistance R_{k} is in accordance with near 1-quantile (but, not 5), which, obviously, does not give the right to use the statistic value $t = 1.65$ in Eq. (8). Therefore, Eq. (8) should be rewritten as follows:

$$V_R \cong \frac{1}{c} \ln (R_m / R_k) \tag{9}$$

where: c is the statistic t (for 1-percentile, near $t = 2.15$, as shown in study [17]).

The summarized results of the comparison of safety formats recommended by the codes [10], [12], [14], [15] are given in Tab. 1.

The Tab. 1 does not show the results obtained with the method proposed by Schlune [17] (for beam element mean value of reliability index $\bar{\beta} = 4.14$ with the coefficient of variation of 3.9% was obtained).

Table 1. Comparison of the safety formats

Method	Statistical parameters of resistance, [kN]			Reliability index	Global factors	
	R_m	R_k	R_d	β_i	γ^*_{Rd}	γ_{Rd}
Probabilistic (direct 121 assessment)	280	260	243.47	3.8	1.15 (1)	
prEN1992-2; *fib*MC2010; ECOV	280	252	218.75	5.1	1.21	1.06 (2)
ECOV with $c = 2.15$	280	–	242.40	3.8	1.16	1.06
EN1992-2	266	–	209.45	5.88	1.27	

Notes: 1) the factor was calculated by Eq. (5), with $V_{fc} = 4{,}68\%$, and $\alpha_R\beta = 3.04$;
2) $\gamma_{Rd} = 1.06$ was adopted in accordance with prEN1992, Appendix F.

In calculations with the use of EN1992-2 method (with f_{yk} and $f_{cmd} = 0.85 f_{ck}$), the resulting "mean" resistance $R_{mf} = 266$ kN almost corresponds to the characteristic value $R_k = 260$ kN obtained by Probabilistic method (PM). In this case, the obtained resistance function has a standard deviation that is the same as the standard deviation of the strength of the steel reinforcement (in case of the tension failure mode). However, in the authors' opinion, the more significant problem is that concrete in this method has the same standard deviation as steel reinforcement. Such large value of the reliability index ($\beta = 5.1$) obtained with ECOV-method can be justified by incorrect statistical assessment of the characteristic resistance R_k.

The design values of the global resistance R_d obtained with Probabilistic Method (PM) had an unknown confidence level γ.

4.2.2. Assessment of resistance non-linear FEM-model uncertainties

As shown in [7], the result of estimation depends on assumptions and criteria for the model used in the non-linear analysis. It should be noted that the different FEM-software, which was applied for non-linear structural analysis (obtaining of the static non-linear response), will have its own different level of FEM-model uncertainties in addition to local cross-section resistance model that covers all relevant failure mechanisms. So, the effects of computer model uncertainties should be treated separately. The coefficient variation V_{vR} and mean values of the computer model uncertainties are estimated on the basis of theoretical background described in [7]. It is suggested that these features are derived from the comparison of the experimental test data and numerical calculation results but through probabilistic consideration.

The set of the test results obtained with the experimental investigations of the different types of statically indeterminate structures [7] demonstrates that different failure mechanism was collected from some references and used for assessment of the coefficient variation V_{vR} and model uncertainty factor γ_{Rd} (Tab. 2). The model uncertainty factor γ_{Rd} takes into account the difference between the real behaviour of structure and the results of the numerical modelling suitable for a specific structure.

It should be noted that values of γ_{Rd} are different for different FEM software. These values for FEM-program should be estimated on the basis of full probabilistic approach, taking into account statistical parameters of the FEM-model uncertainties and included in Program Manual.

Table 2. Statistical parameters of the NL FE-model uncertainties according to [7] and values of γ_{Rd}

Type of structure	β_{tag}	α_R	μ_{Rd}	V_{Rd}	σ_{Rd}	distr.	γ_{Rd}
beams, frames	1.5	0.32	1.00	0.157	0.157	LN	1.08
slabs			1.03	0.066	0.066		1.03

Notes: 1) β_{tag} = 1.5 for accidental design situation;
2) α_R – sensitivity factor, assumed equal 0.32 in case of a non-dominant resistance variable, and 0.8 – in case of a dominant variable; 3) factor γ_{Rd} calculated as follows:

$$\gamma_{Rd} = \frac{1}{\mu_{Rd}} \exp(\alpha_R \cdot \beta_{tag} \cdot V_{Rd})$$

4.2.3. Probabilistic analysis with the use of non-parametric (order) statistics

As shown in [6], [7] the full probabilistic analysis is the general tool for safety assessment of RC-structures, and thus it can be applied in case of non-linear analysis.

Generally, a probabilistic analysis based on numerical simulations includes the following steps:

1. numerical model formulation based on non-linear finite elements. This model describes the resistance function *r(r)* and can perform deterministic analysis of resistance for a given set of input variables;
2. randomization of input variables (random properties are defined by random distribution type and its statistical parameters);
3. probabilistic analysis of resistance (this can be performed, for example, by numerical method of Monte-Carlo-type of sampling, such as LHS sampling). Results of this analysis provide set of random parameters of resistance (and actions);
4. evaluation of safety level using reliability index β or probability of failure. A disadvantage of this approach is the fact that the target value of design resistance is located in the tail of probability distribution function (PDF), determined best fit for the sampling. The design value of the resistance is obtained by extrapolation and strongly depends on the choice of PDF of resistance.

According to the proposed approach [5], the global resistance factor γ_{glob} should be determined with the following equation:

$$\gamma_{global} = \frac{R_{m(0.5)}}{R_{d(0.01)}} \tag{10}$$

where $R_{d(0.01)}$ is design resistance (0.01-percentile of the probabilistic distribution function (pdf) of resistance); $R_{m(0.5)}$ is mean (median) value of resistance (as 0.5-percentile). Based on the Order Statistic (nonparametric) the theory of original procedure for estimation of the desired *p*-th percentile assuming arbitrary confidence level (γ) was developed and presented in detail in [5]. The main advantage of the order nonparametric statistics consists in its independence from the type of probability density function (PDF) as well as from the main statistical parameters of the continuous population.

According to the proposed approach [5], the estimator of resistance $\hat{R}_{p,\gamma}$ (in case of accidental design situation in terms of the ultimate pseudo-static response $F_{ps,u}$) of p-th percentile with desired confidence level γ can be represented as a normalized linear combination of the first three order statistics:

$$\hat{R}_{p,\gamma} = R_{lowest} - \lambda_{(1),\gamma}\Delta_{2-1} - \lambda_{(2),\gamma}\Delta_{3-2} \tag{11}$$

where $R_{lowest} = R_{(1)}$ is the lowest value of resistance in the ordered sample (set of numerical results);

$\Delta_{2-1} = R_{(2)} - R_{(1)}$ and $\Delta_{3-2} = R_{(3)} - R_{(2)}$ are non-negative differences;

$R_{(1)}, R_{(2)}, R_{(3)}$ – the first, second and third order statistics, respectively;

$\lambda_1 = \lambda(\gamma, n)$; $\lambda_2 = \lambda(\gamma, n)$ – a dimensionless coefficient, which depends on sample size n and specified confidence level γ.

Calibration of the coefficients λ_1, λ_2 for the wide range of confidence level γ performed with the use of a set of n-size random samples obtained by numerical Monte-Carlo simulations is shown in detail in [5].

Values of dimensionless coefficients λ_1, λ_2 (rounded to the hundredth place) for the assessment of the 0.01-percentile with different confidence levels γ are listed in Tab. 3.

Substituting (11) to (10) gives:

$$\gamma_{global(\gamma)} = \frac{1 - \lambda_{1(0.5;\gamma)}\delta_1 - \lambda_{2(0.5;\gamma)}\delta_2}{1 - \lambda_{1(0.01;\gamma)}\delta_1 - \lambda_{2(0.01;\gamma)}\delta_2} \tag{12}$$

with $\delta_1 = \dfrac{\Delta_{2-1}}{R_{lowest}}$, $\delta_2 = \dfrac{\Delta_{3-2}}{R_{lowest}}$.

Table 3. Values of the coefficient λ_1, λ_2 for different confidence levels γ for p-th percentile estimation ($N = 35$)

γ	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.75	0.8	0.9
	$p = 0.01$									
$\lambda_{1(\gamma)}$	-0.46	-0.28	-0.11	0.09	0.32	0.63	1.05	1.35	1.75	4.32
$\lambda_{2(\gamma)}$	-0.14	+0.03	0.19	0.37	0.58	0.86	1.26	1.53	1.9	4.29

Using the proposed approach for the assessment of robustness of a damaged structural system, a non-linear analysis of two-span frame (2 × 6 m) with beams with a cross-section of 300 × 500 mm and reinforcement ratio of $\rho_l = 0.33\%$ ($\rho_l' = 0.66\%$) was performed. The following input data were adopted in accordance with EN 1992-1-1: concrete compressive strength class C20/25, reinforcement steel B500, and constitutive relationship "σ-ε" for materials.

At the *first stage*, the series of separate deterministic non-linear analyses of the modified structural system were performed with the use of NL FEM-software for a given set of input variables. The probabilistic models of basic variables included in a non-linear state function are listed in Tab. 4.

For each of the deterministic non-linear static response, a pseudo-static response was obtained on the basis of the provisions [5] given above, and then the ultimate value of the pseudo-static gravity load $F_{ps,u}$ corresponding to the maximum dynamic displacement $u_{dyn,max}$ equal to the ultimate static displacement $u_{st,ult}$ was calculated.

As a result, a set of random values (in proposed example, $N = 35$) of ultimate pseudo-static loads (resistance) was obtained. This set of ultimate pseudo-static loads for the global safety factor γ_{global} obtained with the proposed procedure (12) was used.

Table 4. Probabilistic models of basic variables

Variable		Unit	Parameters of probability distribution function (*pdf*)			
			dist	Characteristic value	mean	COV
f_c	concrete compressive strength	MPa	N	20	28	0.173
f_y	yield strength of reinforcement steel	MPa	LN	500	560	0.054
b	width	m	N	0.3	0.3	0.033
h	height	m	N	0.5	0.5	0.033
k_R	model uncertainty for resistance: for beams for slabs	-	LN	-	 1.00 1.03	 0.167 0.066

Note: N – normal distribution; LN – lognormal distribution.

4.2.4. Assessment of the global safety factor

The following two approaches of determining the global safety (resistance) factor γ_{glob} were examined:

- Approach 1 – the values of the factors γ_{Rd} and γ_R were determined separately (according to [6] and proposed procedure (12), respectively), and then the value of the global safety factor γ_{glob} was calculated as the product $\gamma_{Rd}\,\gamma_R$;
- Approach 2 – the value of the global safety factor was determined in accordance with (12). In this case, the model uncertainty is considered as the basic variable of the non-linear resistance model (see k_R in Tab. 4). Tab. 5 shows influence of the confidence level of estimation γ on the global safety factor γ_{glob}, obtained with the Approach 1 and Approach 2.

For the purpose of comparison, the values of the global safety factor (global resistance factor) obtained by other methods were listed in Tab. 5.

Table 5. Influence of the confidence level γ on the global safety factor γ_{glob}

Approach	Confidence level, γ						Notes
	0.5	0.6	0.7	0.75	0.9	unknown	
Approach 1	1.22	1.23	1.26	1.31	1.49	-	$\gamma_{glob} = \gamma_{Rd}\,\gamma_R$
Approach 2	1.41	1.46	1.49	1.73	3.00	-	from simulation with k_R as basic variable
ECOV [6]	-	-	-	-	-	1.225	$\exp(\alpha_R\,\beta V)$
EN 1992-2 *fib* MC 2010	-	-	-	-	-	1.26	constant value with $f_{cm} = 0.85 f_{ck}$

The primary analysis of obtained results shows that the procedure of calibrations according to the Approach 2 gives sufficiently larger values of the global safety factor γ_{glob} than according to the Approach 1, especially with increased confidence level of estimation. This is obviously due to the fact that the statistical parameters of a model uncertainty (μ_{Rd}, σ_{Rd}) in "varying degrees" affect the final value of the global safety factor when it is estimated based on the Approach 1 or Approach 2.

When the Approach 2 is applied, the model uncertainty becomes the dominant basic variable (k_R in Tab. 4), whereas according to Approach 1, when the coefficient of variation V_{Rd} changes from 6.6% to 16.7%, calculation exp($\alpha_R\ \beta V_{Rd}$) leads to change of the value of factor γ_{Rd} from 1.03 to 1.08 (only). It was found that the global safety factor γ_{glob} values according to EN 1992-2, *fib* MC 2010, and ECOV-method are very close to value γ_{glob}, obtained by the Approach 1 for different confidence level (γ_{glob} from 1.22 to 1.29, see Tab. 5), but differ from γ_{glob} – the values obtained by the Approach 2.

The use of the values of the global safety factors in accordance with the Approach 1 to mean values of resistance $\hat{R}_m$ obtained from non-linear analysis can increase the risk of over-estimation the design value of resistance $\hat{R}_d$. The application of the calibration procedure, based on the proposed approach of interval estimation of percentile by the method of order (non-parametrical) statistics, creates the foundation for more objective assessment of the value of γ_{glob} . It allows one to perform p-th percentile estimation with a desired confidence level γ without resorting to the selection of the resistance distribution function type.

5. Conclusions

The simplified pseudo-static column removal scenario with appropriate gravity load combination may be used for checking of the structural systems robustness and progressive collapse prevention in accidental design situation. When performing a nonlinear analysis (NLFEA) of a modified structural system, one of the main problems remains to ensure the required safety format. It should be noted that for different FEM-software, the values of factor γ_{Rd} will be different and should be included in a Software Manual. Based on the performed studies it could be concluded that all proposed and implemented in recently developed codes safety formats for NLFEA of RC-structures are contained. An innovative calibration procedure of the global safety factor γ_{glob} was proposed on the basis of the Order Statistics (non-parametric) estimation method. The main advantage of the proposed approach is that the result of the percentile estimation does not depend on the choice of the probability distribution function (PDF). There are significant differences (up to 230% depending on the confidence level) in the γ_{glob} value for the approach when the model uncertainty k_R is considered as a basic variable in the non-linear resistance model and for the approach when the value of the global safety coefficient is defined as the product $\gamma_{Rd}\gamma_R$.

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