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# Stress–dilatancy behaviour of remoulded Fujinomori clay

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**Abstract:** The effect of the degree of consolidation and the stress path on the behaviour of remoulded Fujinomori clay for drained triaxial compression and extension was analysed using the Frictional State Concept. It is shown that the stress–dilatancy behaviour can be approximated by a linear general dilatancy equation given by the critical frictional state angle and two soil parameters. The newly formulated dilatant failure state is represented on the stress ratio plastic dilatancy plane by points lying on the friction state line defined by the friction state angle and the Friction State Concept parameters  $\alpha=0$  and  $\beta=1$ . It has been shown that the stress ratio–plastic dilatancy relationship, which is very rarely used in the interpretation of test results, is important for a complete description of the behaviour of soils during shearing.

**Keywords:** clay; stress–dilatancy; frictional state concept; critical state.

## 1 Introduction

The phenomenon of soil dilatancy is the key to the characterising the strength and deformation of soils. The best known are the Rowe [10, 11] and Bolton [2] stress–dilatancy relationships for granular soils. Similarly, the dilatancy equations of Cam-clay and Modified Cam-clay [16] models are known for remoulded and normally consolidated clay. For structured and highly overconsolidated clays, however, the Cam-clay model’s stress–dilatancy relationships should be modified. By

differentiating the plastic potential functions of the classical elasto-plastic model, various stress–plastic dilatancy relationships can be obtained [9]. Szypcio [13] developed a general stress–plastic dilatancy relationship for soils based on the Frictional State Concept (FSC). The linear general stress–plastic dilatancy relationship defined by the critical frictional state angle ( $\phi^o$ ) and two ( $\alpha$ ,  $\beta$ ) parameters of the soil can describe the real behaviour of soil at different stages of shearing. The Critical State Concept assumes that the critical frictional state angle is independent of the deformation mode (triaxial compression (TXC), plane strain and triaxial extension (TXE)), stress level, overconsolidation ratio (OCR) and the initial state of the soil. The critical state angle ( $\phi_{cs}$ ) for some soils depends on the deformation mode [1], and the stress level [5, 12] and structure of soil [3, 4]. The critical frictional state angle and the critical state angle are equal ( $\phi^o=\phi_{cs}$ ) for sands and fully destructured clay [14].

In this paper, the stress–plastic dilatancy behaviour of Fujinomori clay during drained triaxial tests will be analysed. Data from drained TXC and TXE tests conducted by Nakai and Hinokio [8] with various OCRs and different stress paths conducted by Nakai and Matsuoka [6] will be taken for analysis. The influence of OCR and stress path on the stress–plastic dilatancy behaviour will be presented.

## 2 General dilatancy equation

The general stress ratio–plastic dilatancy equation of the FSC [13] is

$$\eta = Q - AD^p \tag{1}$$

where

$$\eta = q/p' \tag{2}$$

$$Q = M^o - \alpha A^o \tag{3}$$

$$A = \beta A^o \tag{4}$$

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$$D^p = \delta \varepsilon_v^p / \delta \varepsilon_q^p \quad (5)$$

where  $\delta \varepsilon_v^p = \delta \varepsilon_v - \delta \varepsilon_v^e$ ,  $\delta \varepsilon_q^p = \delta \varepsilon_q - \delta \varepsilon_q^e$ .

For drained TXC,

$$M^o = M_c^o = (6 \sin \phi^o) / (3 - \sin \phi^o) \quad (6)$$

$$A^o = A_c^o = 1 - M_c^o / 3 \quad (7)$$

$$p' = (\sigma'_1 + 2\sigma'_3) / 3 \quad (8)$$

$$\delta \varepsilon_v = \delta \varepsilon_1 + 2\delta \varepsilon_3 \quad (9)$$

where  $\sigma'_1 = \sigma_a$  is the conventional axial symmetry stress and  $\sigma'_1 = \sigma_r = \sigma_\theta$  are the conventional radial and circumferential stresses.

For drained TXE,

$$M^o = M_e^o = (6 \sin \phi^o) / (3 + \sin \phi^o) \quad (10)$$

$$A^o = A_e^o = 1 - 2M_e^o / 3 \quad (11)$$

$$p' = (2\sigma'_1 + \sigma'_3) / 3 \quad (12)$$

$$\delta \varepsilon_v = 2\delta \varepsilon_1 + \delta \varepsilon_3 \quad (13)$$

where  $\sigma'_1 = \sigma_r = \sigma_\theta$  and  $\sigma'_3 = \sigma_a$  are the conventional stresses for TXE.

For TXC and TXE,

$$q = \sigma'_1 - \sigma'_3 \quad (14)$$

$$\delta \varepsilon_q = 2(\delta \varepsilon_1 - \delta \varepsilon_3) / 3 \quad (15)$$

$$\delta \varepsilon_v^p = \delta \varepsilon_v - \delta \varepsilon_v^e \quad (16)$$

$$\delta \varepsilon_q^p = \delta \varepsilon_q - \delta \varepsilon_q^e \quad (17)$$

$$\delta \varepsilon_v^e = \delta p' / K \quad (18)$$

$$\delta \varepsilon_q^e = \delta q / 3G \quad (19)$$

$$K = \nu p' / \kappa \quad (20)$$

$$G = \frac{3}{2} \frac{1-2\nu}{1+\nu} \frac{\nu p'}{\kappa} \quad (21)$$

$$\nu = 1 + e \quad (22)$$

where  $\phi^o$  is the critical frictional state angle,  $e$  the void ratio,  $\kappa$  the slope of swelling line in the  $e$ - $\ln p'$  plane (Cam-clay model parameter) and  $\nu$  is the Poisson's ratio.

For drained TXC, the dilatancy equation of the Cam-clay model is [16]

$$D = M_c - \eta \quad (23)$$

and of the Modified Cam-clay model is [16]

$$D = (M_c^2 - \eta^2) / 2\eta \quad (24)$$

$$M_c = 6 \sin \phi_{cs} / (3 - \sin \phi_{cs}) \quad (25)$$

where  $\phi_{cs}$  is the critical state angle.

### 3 Tested soil

The results of drained triaxial tests of Fujinomori clay conducted by Nakai and Hinokio [8] and Nakai and Matsuoka [6] will be analysed. Specimens for tests were prepared by mixing natural Fujinomori clay ( $w_L=44.7\%$ ,  $w_p=24.7\%$ ,  $G_s=2.65$ ) with deaerated water and were one dimensionally consolidated under the pressure of 49 kPa. The initial water content in the clay sample prepared in this way was approximately 40% [6, 8]. This method of sample preparation completely destroys the natural structure of Fujinomori clay.

The samples prepared in this way were consolidated isotropically to the pressure  $p_c$  and unloaded to the pressure  $p_0$ . In the experiments conducted by Nakai and Hinokio [8] for OCR = 1, 2, 4,  $p_0=196$  kPa and for OCR = 8,  $p_0=98$  kPa. In the drained TXC and TXE tests conducted by Nakai and Matsuoka [6],  $p_c=p_0=196$  kPa (OCR = 1) and samples were loaded under different stress paths  $\sigma'_3=\text{constant}$ ,  $p=\text{constant}$ ,  $\sigma'_1=\text{constant}$ . The void ratio of the clay sample at the onset of shear was calculated from the equation

$$e_0 = 1.311 - \lambda \ln(p_c/49) + \kappa \ln(p_c/p_0) \quad (26)$$

For Fujinomori clay,  $\lambda=0.105$  and  $\kappa=0.023$  [7, 8]. In the calculations,  $\nu=0.15$  was adopted.

### 4 Methodology

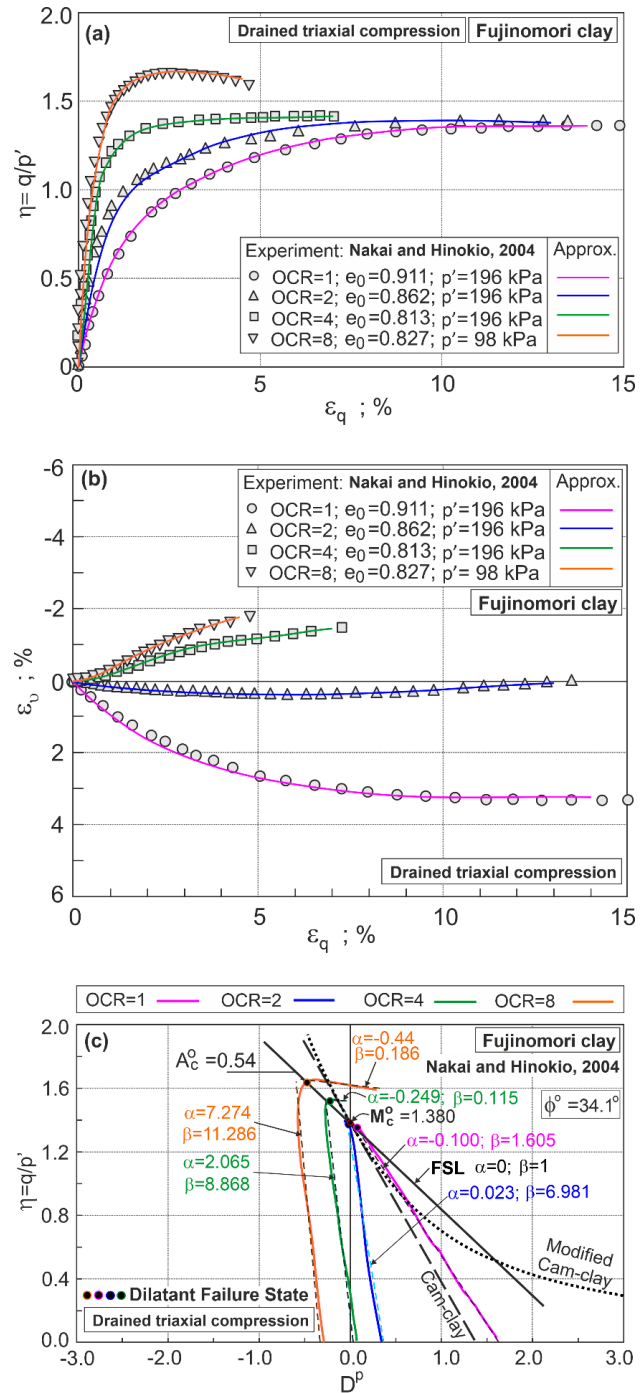
High-degree polynomials were used to segmentally approximate the relationships  $\eta$ - $\varepsilon_q$ ,  $\varepsilon_v$ - $\varepsilon_q$  ( $\sigma'_1/\sigma'_3$ - $\varepsilon_a$ ,  $\varepsilon_v$ - $\varepsilon_a$ ) obtained in the tests. A lot of attention has been given to the connection conditions between the segments. At the connection points, there should not only be the same

function values, but also a very small difference of the first derivatives. Approximation polynomials were used to calculate the stress–plastic dilatancy relationships. For the different stages of elasto-plastic deformation, the stress–plastic dilatancy relations were approximated by linear functions defined by Equation (1) and the parameters  $\alpha$  and  $\beta$  were calculated for each stage. This procedure was described in detail in [15].

## 5 OCR effect of clay behaviour during shearing

The behaviour of the remoulded Fujinomori clay under different OCRs during drained TXC is shown in Figure 1. On the basis of the experimental results [8] of the relationship between the stress ratio and the shear strain (Fig. 1a) and of the volumetric strain and the shear strain (Fig. 1b), the relationship between the stress ratio and plastic dilatancy was calculated (Fig. 1c).

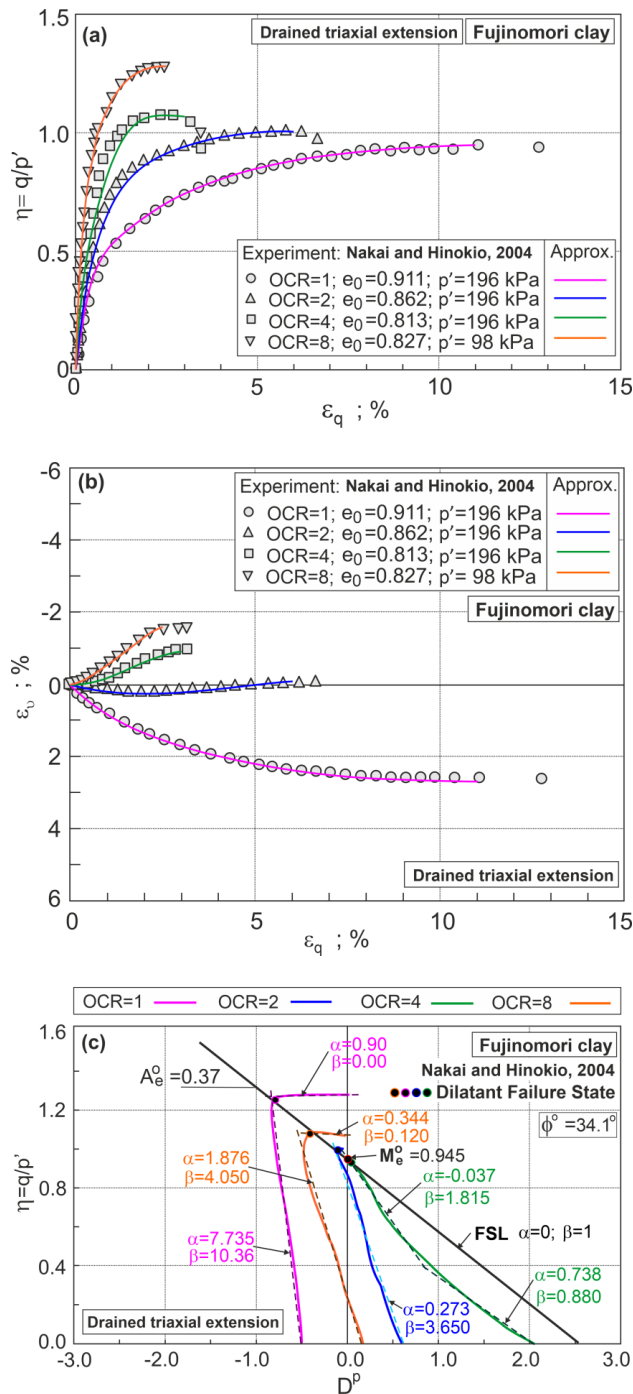
The behaviour of clay during shear is highly dependent on OCR. For OCR = 1 and 2, contractive behaviour and for OCR = 4 and 8, dilative behaviour are observed. In all tests, plastic strains ( $D^p \neq 0$ ) occur from the onset of the shear (Fig. 1c). Dilatant failure states (DFS) are defined as deformation states that are represented by the maximum curvature of the  $\eta$ - $D^p$  curve. A similar definition of DFS was introduced in [15]. These points are marked in Figure 1c. For the tested clay, DFS and failure states are different. The points representing DFS lie on the frictional state line (FSL) defined by Equation (1) with  $\alpha=0$  and  $\beta=1.0$ . According to FSC, FSL intersects the vertical axis at  $M_c^o = 1.38$  ( $\phi^o = 34.1^\circ$ ), representing the slope of the critical state line in the  $q$ - $p'$  plane [13, 14]. The critical frictional state angle  $\phi^o = 34.1^\circ$  is  $0.4^\circ$  higher than critical state angle  $\phi_{cs} = 33.7^\circ$  [6, 8]. In the pre-DFS and post-DFS, the dependence of the stress ratio on plastic dilatancy can be approximated by straight lines defined by Equation (1) with different parameters  $\alpha$  and  $\beta$  (Fig. 1c). The  $\beta$  parameter defining the slope of the approximated stress ratio–plastic dilatancy line in pre-DFS increases with OCR. For post-DFS,  $\beta \approx 0$ . For highly consolidated clay, the shear band is formulated in the sample and the calculated stresses and strains may not be completely correct. The stress–dilatancy relationships calculated from the equations of the Cam-clay model (23) and the Modified Cam-clay model (24) are also shown in Figure 1. The descriptions of the stress–dilatancy behaviour of remoulded clay normally consolidated under a constant mean pressure stress path with the use of FSC and Cam-clay models are very similar and correct.



**Figure 1:** Behaviour of Fujinomori clay under different OCRs in drained triaxial compression tests: a) stress ratio versus shear strain; b) volumetric strain versus shear strain; c) stress ratio versus plastic dilatancy.

The Modified Cam-clay model badly describes the stress–dilatancy behaviour of the tested clay.

The behaviour of Fujinomori clay in the drained TXE tests is shown in Figure 2.



**Figure 2:** Behaviour of Fujinomori clay under different OCRs in drained triaxial extension tests: a) stress ratio versus shear strain; b) volumetric strain versus shear strain; c) stress ratio versus plastic dilatancy.

As in the case of TXC, the relationships  $\eta$ - $D^p$  in different shearing stages were approximated by straight lines defined by Equation (1) with different  $\alpha$  and  $\beta$  parameters. The slope of these lines ( $\beta$ ) increases with OCR. The points representing DFS lie almost exactly on the FSL defined by

the critical frictional state angle  $\phi^o=34.1^\circ$ ,  $\alpha=0$  and  $\beta=1$ . Fully contractive behaviour is observed only for normally consolidated (OCR = 1) clay and dilative behaviour is observed for lightly and heavily overconsolidated (OCR = 2, 4, 8) clay. The plastic dilatancy values for DFS are smaller than for TXC for the same initial conditions.

## 6 Influence of the stress path on the behaviour of clay during shearing

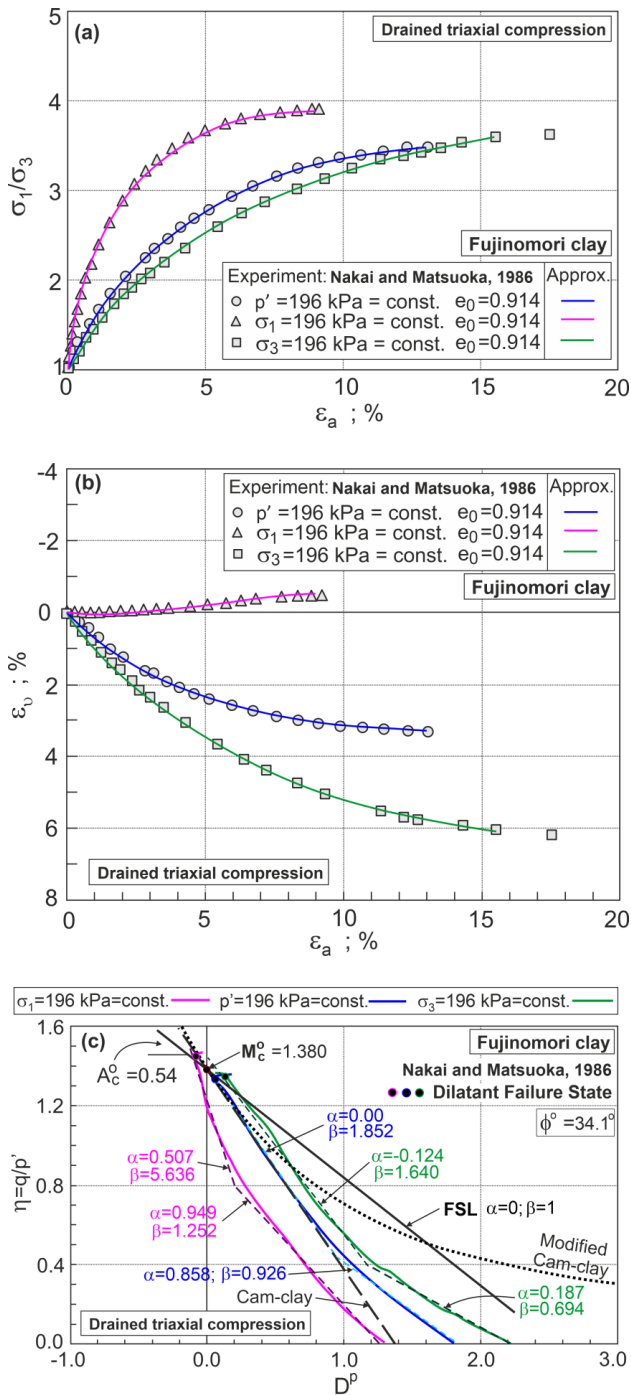
The influence of the stress path on the behaviour of remoulded Fujinomori clay in drained TXC and TXE was studied by Nakai and Matsuoka [6]. The TXC and TXE tests were carried out for stress paths  $\sigma'_3$  - constant,  $p'$  - constant and  $\sigma'_1$  - constant. The behaviour of Fujinomori clay is shown in Figure 3 and Figure 4 for TXC and TXE, respectively.

Linear approximations of the stress to plastic dilatancy ratio in different shearing stages are described by Equation (1) with the critical frictional state angle  $\phi^o=34.1^\circ$ , and the corresponding  $\alpha$  and  $\beta$  parameters are shown in Figures 3c and 4c. In all tests conducted by Nakai and Matsuoka [6], two stages of shear behaviour can be identified in pre-DFS, unlike the tests conducted by Nakai and Hinokio [8] analysed earlier in this paper. DFS can be easily identified for all tests (Figs 3c and 4c). The points representing DFS for TXC lie very close to FSL (Fig. 3c), but not for TXE (Fig. 4c). Based on the presented analysis, it is impossible to finally conclude whether this is an error of the experiment or of the theory. For TXC, the  $\eta$ - $D^p$  relationships of the Cam-clay and Modified Cam-clay models are also shown in Figure 3c. The dilatancy equation of the Cam-clay model (23) correctly describes the behaviour of remoulded, normally consolidated clay sheared under a constant mean pressure stress path. As expected, the stress path affects the shear behaviour of the clay. This effect can be quantified by  $\alpha$  and  $\beta$  parameters of the general FSC dilatancy Equation (1).

## 7 Conclusions

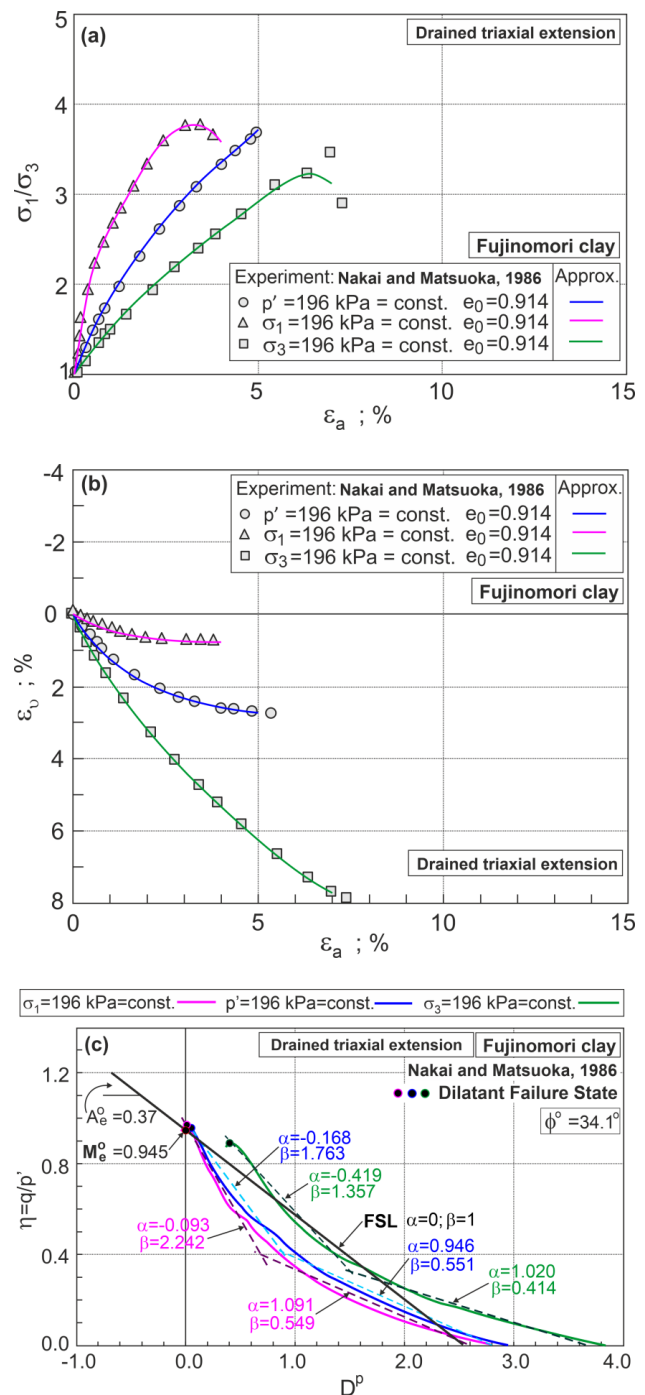
OCR and stress path significantly influence the shearing behaviour of the remoulded clay.

The general dilatancy equation of FSC can be used to describe the behaviour of the stress-plastic dilatancy relationships of soils during shearing.



**Figure 3:** Behaviour of Fujinomori clay in different stress paths for drained triaxial compression: a) principal stresses ratio versus axial strains; b) volumetric strains versus axial strains; c) stress ratio versus plastic dilatancy.

The Cam-clay model dilatancy equation describes well the stress–dilatancy behaviour only for remoulded, normally consolidated clay loaded under a constant mean pressure stress path.



**Figure 4:** Behaviour of Fujinomori clay in different stress paths for drained triaxial extension: a) principal stresses ratio versus axial strains; b) volumetric strains versus axial strains; c) stress ratio versus plastic dilatancy.

DFS independent of OCR and stress paths are very characteristic states of soil behaviour during shear.

The points representing DFS lie on the friction state line defined in the  $\eta$ - $D^p$  plane by the critical frictional state

angle with parameters  $\alpha=0$  and  $\beta=1$  for drained TXC and TXE.

The relationships between stress and plastic dilatancy, although rarely presented in the literature, are important for a full understanding of the behaviour of clay during shearing.

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## References

- [1] Balasubramaniam, A. S., Zue-Ming, H., Uddin, W., Chaudhry, A. R., Li, Y. G. (2007). Critical state parameters and peak stress envelopes for Bangkok clays. *Q. J. Eng. Geol. Hydrogeol.* 11(3), 219-232. <https://doi.org/10.1144/GSL.QJEG.1978.011.03.02>
- [2] Bolton, M. D. (1986). The strength and dilatancy of sands. *Géotechnique* 36(1), 65-78. <https://doi.org/10.1680/geot.1986.36.1.65>
- [3] Fearon, R. E. (1998). The behaviour of a structurally complex clay from an Italian landslide. PhD Dissertation, City University London, UK. <https://openaccess.city.ac.uk/id/eprint/7575/>
- [4] Fearon, R. E., Coop, M. R. (2000). Reconstitution: what makes an appropriate reference material? *Géotechnique*, 50(4), 471-477. <https://doi.org/10.1680/geot.2000.50.4.471>
- [5] Indraratna, B., Sun, Q. D., Nimbalkar, S. (2015). Observed and predicted behaviour of rail ballast under monotonic loading capturing particle breakage. *Canadian Geotechnical Journal*, 52(1), 73-86. <https://doi.org/10.1139/cgj-2013-0361>
- [6] Nakai, T., Matsuoka, H. (1986). A generalized elastoplastic constitutive model for clay in three-dimensional stresses. *Soils and Foundations*, 26(3), 81-98. [https://doi.org/10.3208/sandf1972.26.3\\_81](https://doi.org/10.3208/sandf1972.26.3_81)
- [7] Nakai, T., Matsuoka, H., Okuno, N., Tsuzuki, K. (1986). True triaxial tests on normally consolidated clay and analysis of the observed shear behaviour using elastoplastic constitutive models. *Soils and Foundations*, 26(4), 67-78. [https://doi.org/10.3208/sandf1972.26.4\\_67](https://doi.org/10.3208/sandf1972.26.4_67)
- [8] Nakai, T., Hinokio, M. A. (2004). A simple elastoplastic model for normally consolidated soils with unified material parameters. *Soils and Foundations*, 44(2), 53-70. [https://doi.org/10.3208/sandf.44.2\\_53](https://doi.org/10.3208/sandf.44.2_53)
- [9] Rahimi, M. (2019). Review of Proposed Stress-Dilatancy Relationships and Plastic Potential Functions for Uncemented and Cemented Sands. *J. Geol. Res.* 1, 19-34. <https://doi.org/10.30564/jgr.v1i2.864>
- [10] Rowe, P. W. (1962). The stress-dilatancy relation for static equilibrium of an assembly of particles in contacts. In *Proceedings of the royal Society of London. Series A, Mathematical and Physical Sciences.* 269(1339), 500-527. <https://doi.org/10.1098/rspa.1962.0193>
- [11] Rowe, P. W. (1969). The relation between shear strength of sands in triaxial compression, plane strain and direct shear. *Géotechnique*, 19(1), 75-86.
- [12] Shu, R., Kong, L., Liu, B., Wang, J. (2021). Stress-Strain Strength Characteristics of Undisturbed Granite Residual Soil Considering Different Patterns of Variation of Mean Effective Stress. *Applied Sciences* 11(4), 1874. <https://doi.org/10.3390/app11041874>
- [13] Szypcio, Z. (2016). Stress-dilatancy for soils. Part I: The frictional state theory. *Studia Geotechnica et Mechanica*, 38(4), 51-57. <https://doi.org/10.1515/sgem-2016-0030>
- [14] Szypcio, Z. (2016). Stress-dilatancy for soils. Part II: Experimental validation for triaxial tests. *Studia Geotechnica et Mechanica*, 38(4), 59-65. <https://doi.org/10.1515/sgem-2016-0031>
- [15] Szypcio, Z., Dołżyk-Szypcio, K. (2022). The Stress-Dilatancy Behaviour of Artificially Bonded Soils. *Materials*, 15(20), 7068. <https://doi.org/10.3390/ma15207068>
- [16] Wood, D. M. (1990). Shear behaviour and critical state soil mechanics. Cambridge University Press, New York, USA.