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THE REVERSE TASK OF DISCRETE MECHATRONIC VIBRATION SYSTEMS WITH NEGATIVE STIFFNESS AND CAPACITANCE ELEMENTS

By the use of different distribution methods of dynamical characteristics in the form of slowness function, mechatronic discrete systems have been synthesized. Each model consists of mechanical discrete part and a piezostack actuator connected to $L_x R_x C_x$ external network that has to comply with dynamical requirements in the form of poles and zeros. External network can work within different configurations. In this paper, one investigates the influence of negative parameters of stiffness in mechanical replacement models and capacitance in final mechatronic structures, after dimensionless transformations and retransformations.

1. Introduction

While one of the most important problem in designing of the machines is vibration control, in respect of practical realization and preparation of technical systems, various systems with different damping methods might be used. Possibility of piezoelectric elements application as actuators for vibration isolation has been presented in [1-3]. Other vibration reduction types used in synthesis of mechanical systems, have been compared and presented in [4]. These correspond to passive or active additional damping elements.

The problem of synthesis of mechanical discrete [5] and discrete-continuous systems [6] is formalized and well known. As reverse task to analysis,

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synthesis can be the design method for the systems that have to comply with given requirements in the form of resonant and antiresonant frequencies. The advantage is that numerous repetitions of calculations typical for analysis can be avoided. Researches described in [5, 6] were extended by the authors for solving the reverse task of mechatronic discrete systems, presented on the examples of branched [7] and mixed [8] systems. In this case mechatronic systems were considered as combinations of mechanical discrete models and piezoelectric elements connected to external electric networks $L_x R_x C_x$, where all parameters were with positive values in relation to the assumptions and theorems described in [5, 6].

Distribution of dynamical characteristics in the form of slowness or mobility function by a chosen method determines final the type of mechatronic structure. It means that for the same requirements it is possible to obtain various types of the systems: cascade, branched or mixed. This creates several solutions for given poles and zeros as the criteria. Moreover, by application of additional damping elements proportional to stiffness or inertial elements, the combination of the systems that comply with the given requirements can be significantly increased [7, 8]. The considered mechatronic structures can be obtained by dimensionless transformation and retransformation of mechanical replacement models.

Knowing the possibilities of creating negative capacitance elements and utilizing them for vibration control [3, 9], one formulates the goal of the paper as to check what are the impacts, constrains and limitations if selective parameters take and set out of the required ranges during identified steps of the synthesis process (against the assumptions from [5, 6]). As parameters identification is very important [10], the authors investigated, on the basis of the example of a cascade mechatronic LC system, which stiffness elements are important and have the influence on negative capacitance in the external electric network. Additionally, considering works related to physical realizations of mechanisms and systems that follow the negative stiffness [11, 12], the authors expand hitherto researches from the problem of synthesis of mechanical [5, 6] and mechatronic [7, 8] systems to the investigations focused on negative value parameters: stiffness and capacitance.

2. The reverse task of discrete mechatronic systems

In this section, we present major concerns connected with solving the reverse task of mechatronic systems, structured with discrete mechanical models, piezoelectric elements and external electric networks.

2.1. Synthesis of discrete mechatronic systems

In the case of synthesis of discrete mechatronic systems, the type of the final structure depends on the chosen method of distribution of dynamical characteristics. In this example it was shown in the form of slowness function for a mechanical model with or without additional damping, resp. Fig. 1, Fig. 2:
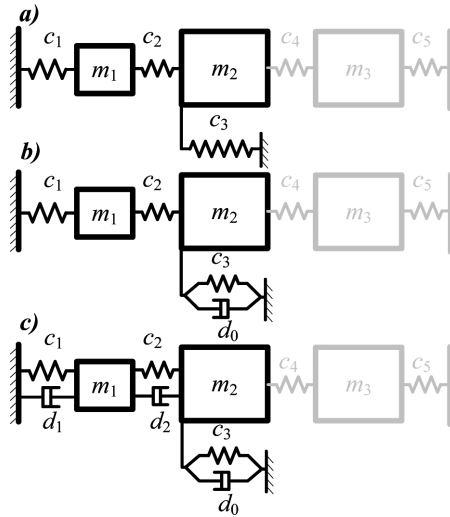

Fig. 1. Mechanical replacement models of final mechatronic cascade systems: a) function $L_x C_x$, b) and c) with additional passive dampers
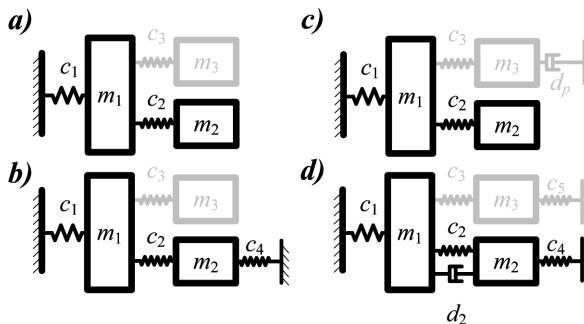

Fig. 2. Mechanical replacement models of final mechatronic branched structures: a) and b) branched system L_x, c) branched system $L_x R_x$, d) branched system $L_x C_x$

$$U(s) = H \frac{d_l s^l + d_{l-2} s^{l-2} + ... + d_0}{c_k s^k + c_{k-2} s^{k-2} + ... c_1 s},$$

(1)

where:

l – even degree of the numerator,
k – degree of the denominator,
$l - k = 1$,
H – any real number.

Basing on [7, 8], by the use of algorithms for dimensionless transformations and retransformations, one transforms mechanical replacement models into final mechatronic structures.

2.2. Piezoelectric element and external electric network

In order to investigate the influence of negative parameters on dynamical characteristics of the system, the mechanical cascade model was connected with a piezo element and an external electric network consisting of L_xC_x, Fig. 3.
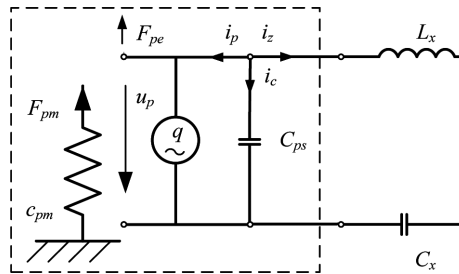

Fig. 3. The model of piezostack actuator connected with external electric network L_xC_x

To combine mechanical and electrical properties of the piezoelectric effect, one writes the commonly known piezo equations:

$$\sigma = K_E s - eE \tag{2}$$

$$D = es + \varepsilon_s E \tag{3}$$

where:
σ – mechanical stress,
K_E – Young's Modulus,
s – mechanical strain,
D – electrical displacement,
ε_s – electrical permittivity,
E – electrical field,
e – piezoelectric constant,
The voltage on the electrodes of piezostack actuator [7] is described as:

$$u_p = f(e, F_{pe}), \tag{4}$$

where:

F_{pe} – electric part of piezoelectric force [5] written as:

$$F_{pe} = -e \frac{A_p}{l_p} u_p, \tag{5}$$

where:

A_p – surface area of electrodes,
l_p – distance between piezoelectric electrodes.
Current intensity can be written as a function of:

$$i_p = f(e, \dot{x}). \tag{6}$$

The number of degrees of freedom of the mechatronic structures depends on the number of used piezoelectric elements and is always lower than the number of degrees of freedom of the mechanical replacement model.

3. Negative parameters

In this section, we present the influence of negative stiffness and capacitance on dynamical characteristics of synthesized system.

3.1. Mechatronic cascade system *LC*

In order to investigate negative stiffness and capacitance, the following requirements have been assumed:
– resonant frequencies:

$$\omega_1 = 100 \left[\frac{\text{rad}}{\text{s}} \right], \quad \omega_3 = 300 \left[\frac{\text{rad}}{\text{s}} \right], \quad \omega_5 = 500 \left[\frac{\text{rad}}{\text{s}} \right],$$

– antiresonant frequencies:

$$\omega_0 = 0 \left[\frac{\text{rad}}{\text{s}} \right], \quad \omega_2 = 200 \left[\frac{\text{rad}}{\text{s}} \right], \quad \omega_4 = 400 \left[\frac{\text{rad}}{\text{s}} \right].$$

The examples of mechanical replacement models, which realize the given resonant and antiresonant frequencies, and that will be transformed into the final mechatronic structures are shown in Fig. 1. For further analysis, one selects a mechatronic cascade system. This system in the mechanical replacement model has three DOF (Fig. 1a) and is additionally excited by the force $F(t)$ applied to the mass m_1.

The matrix equation for the system, after dimensionless transformations and retransformations in non-dimensional time τ, can be written as:

$$
\begin{bmatrix} \lambda_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda_2 \end{bmatrix} \begin{bmatrix} x_1'' \\ x_2'' \\ x_3'' \end{bmatrix} + \begin{bmatrix} 1+\gamma & -1 & 0 \\ -\beta_1 & \beta_1+\beta_2+1 & -\beta_2 \\ 0 & -1 & \alpha+1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} F(\tau) \\ 0 \\ 0 \end{bmatrix} \quad (7)
$$

where:

τ – non-dimensional time,

λ_i – dimensionless inertial parameters,

γ, β_i, α – dimensionless stiffness parameters,

$i = \{1, 2\}$.

The parameter α, which depends on selection of stiffness c_3, may have a negative value and affects the capacitance in the external electric network.

After transformations, the matrix equation for the system can be written as:

$$
\begin{bmatrix} c_1+c_2 & -c_2 & 0 \\ -c_2 & c_2+c_3+c_4 & -\dfrac{eA_p}{l_p} \\ 0 & \dfrac{C_{ps}}{C_x} & \dfrac{\varepsilon_s}{e}\left(1+\dfrac{C_{ps}}{C_x}\right) \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ u_p \end{bmatrix} +
$$

$$
+ \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & L_x C_{ps} & \dfrac{\varepsilon_s}{e}L_x C_{ps} \end{bmatrix} \begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \\ \ddot{u}_p \end{bmatrix} = \begin{bmatrix} F(t) \\ 0 \\ 0 \end{bmatrix} \quad (8)
$$

where:

C_{ps} – piezoelement capacitance,

L_x – inductance element in external electric network,

C_x – capacitance in external electric network,

A_p – surface area of electrodes,

l_p – distance between piezoelectric electrodes.

3.2. Negative stiffness

During synthesis, the selection of stiffness c_3 has an impact on the value of stiffness c_5, which can have positive or negative value. In regards to requirements given in section 3.1, in accordance with [5, 6], the value of the stiffness c_3 should be selected from the interval $(0; 156.46)$. Over this range the stiffness c_5 has a negative value.

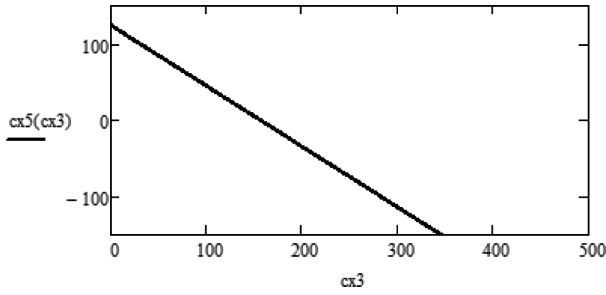The dependence between stiffness c_3 and c_5 is linear and is shown in Fig. 4.

Fig. 4. Impact of selection of stiffness c_3 for c_5

3.3. Dynamical characteristics of the system

The amplitudes for the considered system, in the case of all stiffness elements having positive value, are shown in Fig. 5.
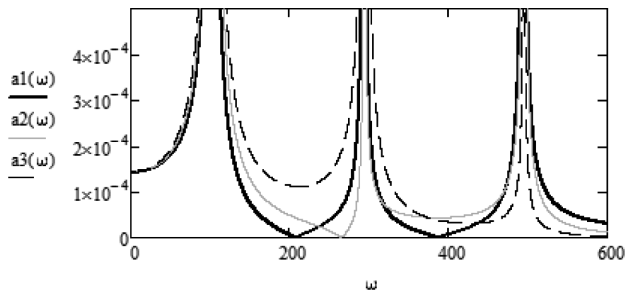

Fig. 5. Amplitudes for the cascade system with positive values of stiffness elements

For further consideration, one assumes the following cases:
1. c3 $\ll$ (0; 156.46),
2. c3 $>$(0; 156.46),
3. c3 $\gg$ (0; 156.46).
The dynamical flexibility $V1_x(\eta)$ has been written as:

$$V1_x(\eta) = \left| \frac{1 + \alpha - \lambda_2\eta^2}{\begin{array}{c}\left(1 + \beta_1 - \lambda_1\eta^2\right)\left(1 + \gamma + \beta_2 - \eta^2\right)\left(1 + \alpha - \lambda_2\eta^2\right) - \\ -\gamma\left(1 + \beta_1 - \lambda_1\eta^2\right) - \beta_2\left(1 + \alpha - \lambda_2\eta^2\right)\end{array}} \right| \quad (9)$$

With regard to each case, dynamical characteristics have been compared with one another and shown in Fig. 6: $V1_p(\eta)$ – case 1, $V1_n(\eta)$ – 2 and $V1_m(\eta)$ – 3.
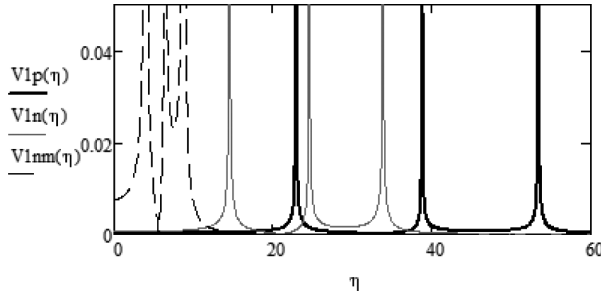
Fig. 6. Dynamical flexibility $V1_x(\eta)$ for different value of stiffness c_3

3.4. Negative capacitance

Negative capacitance in external electric network depends on dimensionless parameter α. This parameter in mechanical displacement model is connected with stiffness c_5:

$$\alpha = \frac{c_5}{c_4}. \tag{10}$$

Referring to each case defined in section 3.3, important parameters of the system have been collated in Table 1. The source for the parameters is related with distribution method of dynamical characteristics in synthesis (stiffness elements c_3 and c_5, based on equation (1)) and dimensionless transformations and retransformation (C_x, L_x, α). Whereas the value of capacitance C_{ps} has been assumed from physically realizable range. Calculations of the parameters has been done in Mathcad environment.

Table 1.
Values of the most important parameters for the considered system with regard to given cases

Units	Parameters	c3 ≪ (0; 156.46)	c3 >(0; 156.46)	c3 ≫ (0; 156.46)
[N/m]	c_3	80	200	3000
[N/m]	c_5	60	−34.57	−2257
–	α	$1.6 \cdot 10^{-3}$	$-9 \cdot 10^{-4}$	−0.06
[F]	C_{ps}	$1.5 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$	$1.5 \cdot 10^{-4}$
[F]	C_x	0.09	−0,17	$-2.41 \cdot 10^{-3}$
[mH]	L_x	0.21	0.54	8.16

The influence of selection of the parameter α on the capacitance in the external electric network is shown in Fig. 7.

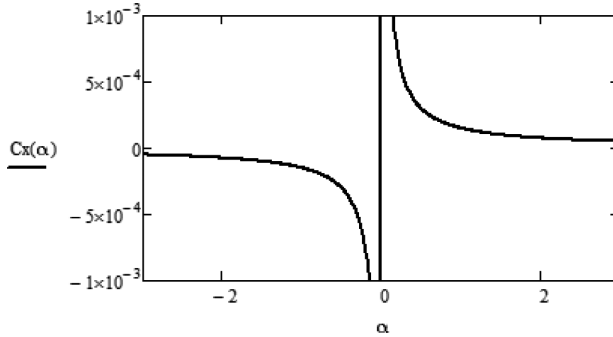The considered mechatronic structure is shown in Fig. 8.

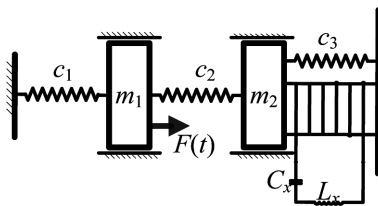Fig. 7. Capacitance in external electric network in function of parameter α

Fig. 8. Mechatronic cascade system with one piezoelectric actuator in the $L_x C_x$ configuration

4. Conclusions

In the synthesis of mechatronic systems, the selection of proper parameters leads to negative values of the chosen stiffness elements. By dimensionless transformations and retransformations, these systems affect dynamical flexibility of the system and the values of capacitance in the external electric network which is connected to the piezostack actuator. Negative value of the external capacitor can be realized by the synthetic impedance with the use of operational amplifiers as mentioned and proposed in [3, 9].

Considering the results of the investigations, it is possible to obtain various types of the structures for the same criteria. It depends on the designer and the required structure type which method of synthesis is needed and will be used. The paper shows that for the same criteria it is possible to synthesize systems not only with all elements with positive values of parameters, belonging to the required and defined range, but also with negative values of parameters intentionally taken out of the required and assumed range. The impact of selection of proper stiffness on dimensionless parameters and dynamical flexibilities was shown in Figs. 4, 6, 7 and in Table 1. Taking into account physical possibilities of realization of negative capacitance, one can widen possible applications of the considered systems.

The paper is an introduction to a new problem, which is planned to be developed and verified in detail in the next researches. The goal for the

further works will be determination of values of negative parameters optimal for dynamical characteristics, and investigations on regularization, stability conditions of cascade and branched systems.

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**Zadanie odwrotne dyskretnych drgających układów mechatronicznych
ze sztywnością i pojemnością ujemną**

S t r e s z c z e n i e

Korzystając z wybranych metod rozkładu funkcji charakterystycznej w postaci powolności zsyntezowano mechatroniczne dyskretne układy drgające. W każdym przypadku układ zbudowano z mechanicznego modelu dyskretnego oraz piezo aktuatora typu „stack" połączonego z zewnętrznym obwodem elektrycznym $L_x R_x C_x$. Układy zaprojektowano ze względu na wymagania dynamiczne w postaci biegunów i zer. Zewnętrzny układ elektryczny może występować w różnych konfiguracjach. W pracy, po bezwymiarowych transformacjach i retransformacjach, zbadano wpływ ujemnych parametrów: sztywności w mechanicznym modelu zastępczym oraz pojemności w finalnym układzie mechatronicznym $L_x C_x$, na charakterystyki rozważanych układów.