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NON-STANDARD TASKS IN MATHEMATICAL EDUCATION

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Abstract

Mathematical knowledge and skills have been playing a more and more important role in our daily lives. At the same time, solving tasks is the essence of mathematics understood as a field of human activity. The subject of this paper are selected issues concerning some atypical tasks which play important role in mathematical education. Presented task is related to the elementary knowledge of probability theory.

1. Main consideration

Solving tasks states the backbone of teaching mathematics at every level of mathematical education. Having in mind the goal of preparing pupils and students, in the course of the educational process, to living in the surrounding reality, one should emphasize the tasks which allow the pursuit of general objectives of mathematical instruction, i.e. those that develop skills and attitudes necessary to a modern person, regardless of his or her field of activity. This goal may be achieved by assigning non-standard tasks to pupils and students.

This paper presents one non-standard task. The task is addressed to students of final grades of secondary schools that are preparing for the final secondary school examination (so called matura) at the advanced level, as well as undergraduate students majoring in mathematics. This task is composed of a problem and two different solutions. The students are asked to verify which of the given solutions is correct.

Solving this type of tasks develops, among others, intellectual attitudes evidenced by logical, creative, and independent thinking as well as by overcoming difficulties, and can improve the ability to analyse the content of

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the task and understanding of the global structure of the task. Moreover, as Polya notes, in mathematics itself, skills are more important than knowledge. What in mathematics does mean skill? It is the ability to solve problems, and not just typical tasks, but also those that require independent judgement, judgement ability, originality, creativity (see [5]).

In the light of the research conducted by the authors, assigning this type of tasks to students is a valuable part of mathematical instruction. The results of the research demonstrate that even able secondary-school and undergraduate students encounter considerable difficulties in solving non-standard tasks (see [3], [4]).

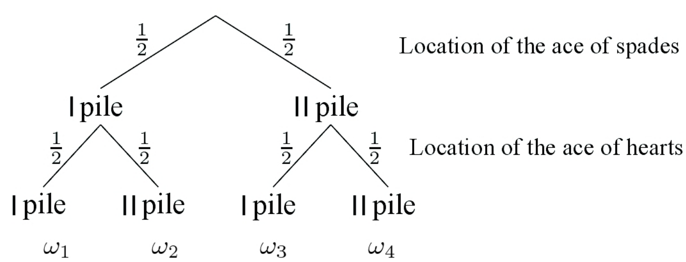
Let us consider the following task (see [1]) that describes a certain real-life situation and refers to the basics of the probability theory.

Task
A deck of 52 cards has been shuffled and divided into two piles of 26 cards each. What is the probability that the ace of hearts and the ace of spades are in the same pile?

Two solutions of the problem are presented below. Verify if they are correct and justify your answer.

Solution 1.

We identify two cards in the deck: the ace of hearts and the ace of spades. Now we analyse in which ways both cards may be distributed in two piles. The following stochastic tree will help us to conduct the analysis.



We have $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$, where:

ω_1 – the ace of spades and the ace of hearts are both located in pile I,

ω_2 – the ace of spades is in pile I and the ace of hearts is in pile II,

ω_3 – the ace of spades is in pile II and the ace of hearts is in pile I,

ω_4 – the ace of spades and the ace of hearts are both located in pile II.

In the model illustrated by the stochastic tree, the event A that both aces are in the same pile consists of two out of four outcomes with equal probability, symbolically $A = \{\omega_1, \omega_4\}$. Hence

$$P(A) = \frac{1}{2}.$$

Solution 2.

Note that dividing a deck of cards into two equal piles may be achieved by drawing 26 cards and considering the remaining 26 cards the other pile. Let us define Ω as the set of 26-element subsets of the set of 52 cards. Then the event

$A = \{$both aces are in the same pile$\}$

may be presented as $B \cup C$, where

$B = \{$both of the ace of hearts and the ace of spades have been drawn$\}$,

$C = \{$neither ace of hearts nor ace of spades has been drawn$\}$.

Events B and C are disjoint; hence

$$P(A) = P(B \cup C) = P(B) + P(C) = \frac{\binom{50}{24}}{\binom{52}{26}} + \frac{\binom{50}{26}}{\binom{52}{26}} \approx 0,45 \neq \frac{1}{2}.$$

As the presented solutions lead to two different outcomes, a conclusion can be drawn that at least one of them is incorrect.

The discussed problem is related to the construction of the mathematical or, more strictly, probabilistic model of the real-life situation. In essence, it must be verified if the probabilistic space assumed as the model of the experiment complies with this experiment, i.e. if it is an appropriate model of the given experiment.

Let us return to the experiment d described in the task, i.e. a random division of the deck of cards into two equal piles. The experiment's outcome consists of a pair of 26-member injective sequences $((a_n), (b_n))$, $n \in \{1, 2, \ldots, 26\}$ whose members belong to the set of 52 cards where $a_i \neq b_j$ whenever $i, j \in \{1, 2, \ldots, 26\}$ (no member of the first sequence is a member of the second sequence and vice versa). Note that every outcome, i.e. every pair of sequences may be identified with a certain permutation of the set of 52 cards, e.g. constituting the sequence $a_1, \ldots a_{26}, b_1, \ldots b_{26}$. Moreover, every permutation of the 52-card set may be interpreted as an outcome of the discussed experiment d (where the permutation is "split in two"). Consequently, there are altogether 52! outcomes of the experiment d; therefore, the probability of each of them is equal to $\frac{1}{52!}$. In the discussed model, the event $A = \{$both aces are in the same pile$\}$ contains the outcomes (pairs of sequences) where both defined elements of the 52-card set (the ace of spades and the ace of hearts) are members either of the sequence (a_n) or of the sequence (b_n).

As far as the discussed event is concerned, the order of putting cards in the piles does not matter. What is significant, is which cards are placed in the first and which in the second pile. Note that knowing the cards in pile one means implies knowing the cards in pile two. Hence, in order to determine the occurrence of event A, it is sufficient to draw 26 cards out of the deck (constituting the first pile, whereas the remaining cards will be in the second pile) and to control if the ace of spades and the ace of hearts are among the cards drawn. If both aces are in the first pile or neither of them has been drawn, the event A has occurred. The above discussion leads to the conclusion that the described experiment d may be substituted by another experiment, d_1, i.e. simultaneous drawing of 26 cards out of the 52-cad deck, that is the experiment described in Solution 2. Therefore, Solution 2 contains the correct result (see [2]).

As already mentioned, the discussed problem is a sample task of the type that should be assigned more often to those learning mathematics. The two contradictory solutions provoke the necessity of a deeper reflection on the problem and its solution. In particular, the task requires reflection on the construction of the described mathematical model and verification of its compliance with the situation in question. As it has been proved by our research, even college students majoring in mathematics are often not prepared for this kind of discussion.

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