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Models of multimodal networks and transport processes

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Abstract. Models of multimodal cyclic processes, i.e. processes realized with synergic utilization of various local and cyclic acting processes, play a determining role in an evaluation of functioning efficiency inter alia in public transport systems, passengers movement, cargo transport, data and energy transmission etc. We assume that the structure of a system determines repertoire of its behaviors. The paper presents a constraints satisfaction problem, which solving enables an evaluation of potential behaviors of the system of concurrently interacting local cyclic processes. Consequently, it is possible to plan and schedule the multimodal processes realized in that system. The constraints satisfaction problem, enabling the search for the structure of inter-position transport system and guaranteeing realization of assumed schedule of multi-assortment production was formulated for a declarative model of the multimodal transportation processes system. The attached calculation example illustrates the computational efficiency of the proposed approach.

Key words: multimodal processes, cyclic scheduling, constraint satisfaction problems.

1. Introduction

Seeing a company as a set of resources, a set of business (economic) processes implemented there, as well as connections and relationship linking these sets, it is easy to notice that its systemic model consists of available resources infrastructure and a set of portfolios of manufacturing orders. Assuming in a simplified way that the structure of the production enterprise is described by its resources infrastructure, while the behavior is the set of business (production) processes realized in it, the mentioned decision problems come down to two categories of questions: What behaviors are possible in the given structure of the system? What structures enable the realization of a given behavior? In this context, the most commonly formulated enterprise resource management problems, are associated with the search for answers to the questions: Can ordered production be realized in the available resources structure of the enterprise? Will the assumed way of available resources structure of the enterprise expanding allow realization of the ordered production? Can given adjustment of the production plan enable its realization in the available resource structure of the enterprise?

Assuming that each producer has an access to the same amount of the same resources (so called resources infrastructure), it appears that the answers to the above questions are determined by the decisions involving the manner of their utilization, for example decisions oriented to increase in the production flow or to reduce the cost of its service (decisions determining the competitive advantage of the enterprise). For the needs of further discussion, it is assumed that the production flow means a stream of operations occurring in all these processes, which constitute the essence of given products group manufacturing.

Enterprises that produce large quantities of a variety of consumer products typically use cyclic manufacturing strategy. It allows, at regular intervals, to provide the quantified products mixture. Cyclic manufacturing considerably simplifies the control of the flexible production system, i.e. steady schedule is repeated for several time periods. Operational planning related to the appointment of cyclic schedules leads to difficult combinatorial problems. The vast majority of them belong to the class of NP-hard problems, i.e. those for which there are no known solving methods of polynomial computational complexity.

The considered class of concurrently realized discrete cyclic processes is observed in production, communication and timetabling. In the problem of processes scheduling, formulated in the domain of integers, in the conditions of various constraints occurrence, for example imposed on the sequence of operations and the time of resources availability, it is assumed that the processes competing for an access to commonly used resources are synchronized by selected instances of a mutual exclusion mechanism. Diophantine nature of the problem limits the possible behaviors achieved in the given structure of the cyclic processes system. Assuming steady state cyclic behavior of the accepted class of systems, two scheduling problems related respectively to the appointment of production cycles realized in these manufacturing processes systems, and the structure of parameters of systems guaranteeing the steady production cycles values, are considered.

Most research in this area is limited to the technological processes involving the operations typical for processing, plastic forming or assembly of the manufactured elements. The accompanying auxiliary processes associated with the transport and inter-position elements storage, transport and replacement of tools, etc., are either ignored or treated inde-

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pendently. The growing role of transport processes supporting warehouse management, supply chain management, vehicles routing, tracking progress in the implementation of the logistical operations and so on, changes these proportions. Fast growing number of studies which take into account the combined (synergistic) effect of concurrently realized main and auxiliary processes can be observed for several years. Dominant issues concern the field of AGVs fleet planning and scheduling, related to the purpose which is support of the right manufacturing positions at the right time moments, the service guaranteeing realization of the pre-determined production schedules.

In this context, by presenting new proposals for declarative models of multimodal transport networks and processes, this work formulates the problem of the scheduling of AGVs fleet, operating given set of production tasks realized in stream manner. Considered issues fall within the framework of design of dedicated decision support systems, in particular, the design of plug-in extending the selected Enterprise Resources Planning (ERP) systems functionality. The related works are a continuation of previous research assuming Diophantine nature of the problems of scheduling of concurrent cyclic processes systems, in particular these associated with the indeterminacy of the problems related to warranty of the expected system behaviors in terms of imposed structural constraints.

Section 2 presents an review of selected studies conducted in the range of the work. Section 3 and Sec. 4 introduce the problems of modeling of the systems of concurrent cyclic processes and accordingly systems of concurrent multimodal cyclic processes. Declarative variants of the introduced models allowing to formulate the constraint satisfaction problem understood as a problem of the search for the structure of the inter-position transport system guaranteeing realization of a given production schedule is presented in Sec. 5. An illustration of an exemplary course of the experiment and the scope of future work representing a continuation of the studies are presented in the relevant sections – Sec. 6 and Sec. 7.

2. State of the art

Discrete concurrent flowing processes are widely used for modeling communication systems (e.g. public, aviation, rail) [1–3], in manufacturing systems [4–8], information systems (e.g. in multiprocessor solutions, computer networks) [9, 10]. The problems most frequently taken in this type of objects are the problems of scheduling and timetabling [4, 11–13]. These problems, depending on the specificity of the objects and dispatching mechanisms of processes realized in them, are generally formulated in a variety of areas: from Petri nets models implementing [14, 15], through the models of operational research [4, 6], up to the algebraic models [16–18].

In order to take into account so many and so different perspectives imposed by modeling and scheduling of discrete, and concurrently, interacting cyclic processes as well as related passengers transportation problem formulation, and constraint programming driven methods aimed at its solution the declarative framework is applied as further considerations platform.

The applied approaches distinguish two methods of the modeling of concurrent discrete processes execution. The first of them is based on a computer simulation of system state model [19]. The results obtained, often presented in the form of Gantt diagram, require further analysis and the simulation of the course of component process is very time-consuming. This group of models include time Petri nets [13], UML, XML languages [7, 9], and Markov chains [20]. The second approach allows to determine the values of parameters of the system functioning using algebraic model, for example algebra (max, +) [20], algebraic equations [10, 14], constraint programming techniques [5, 9, 21] or linear programming [22]. Considered analytical methods allowing to determine the effectiveness of the system functioning and the design of systems with expected quantitative indicators values are presented for example in the study [16]. Prior to [16], there were few papers on analytical methods applied to the simple cases of two cyclic processes sharing the resource [14, 23, 24] or a structurally deadlock-free systems of concurrent cyclic processes [18, 25].

The problem of cyclic scheduling is rarely undertaken by the researchers. This is due to the lack of computational models allowing the construction of efficient algorithms. Presented in the literature studies concerning the cyclic systems are only limited to the flow systems [1, 18]. The examples of typical in this range flow and job-shop cyclic systems models are the following studies [11, 26–28].

The main task of dedicated Decision Support Systems (DSS) is to support of the manager in selected areas of decision-making problems. Heuristic algorithms implemented in them are most often the only way to obtain solutions which are as satisfactory (i.e. obtained within a reasonable time) from the perspective of the size of the dissolving examples, as well as the suitability of the results obtained (i.e. their distance from the optimal solution). In the range of multi-assortment production movement planning, the ESP expectations are generally related to the use of DSS for the two classes of routine questions of decision character:

What are the implications of the premises? (for example: can the backlog of production orders, determined by specified values of operations duration and production batch sizes be implemented in the production system of a specified transport system and storage, in a given time horizon?)

What implies the conclusion? (for example: is there such a solution of the transport and storage system that guarantees realization of the backlog of production orders, determined by the specific values of the durations of activities and the batches size in a given production system, in a given time horizon?

Commercially available tools for decision support enable the support in the range of problems associated with the first class of the above questions [5, 10, 21, 29], while in the area of the cyclic systems they are generally limited to the flow and job-shop systems, for which methods are sought to minimize the duration times of production realized in them.

3. Concurrent cyclic processes

A representative example of the considered hereinafter class of Systems of Concurrently executed Cyclic Processes (SCCP) [5] is the railway system. Single cyclic processes correspond to particular trains circulating in the connection network, stopping according to a specified timetable, on the stations spaced along the route traced in the railway network (e.g. the route consisting the connection: Szczecin – Warszawa – Szczecin).

Assuming that the stations and some fragments of tracks are shared by such kind of processes (trains), it is easy to notice that the various alternative train timetables are sensitive to varying degrees on randomly occurring disturbances related for example to delayed trains. Among other such problems, the following are worth to be mentioned: re-scheduling related to seasonal changes in the timetable, the development of the railway network infrastructure, synchronization of passenger and freight lines timetables, securing the presence of the required safety margin of the critical infrastructure (e.g. capacity at certain network sections and/or directions).

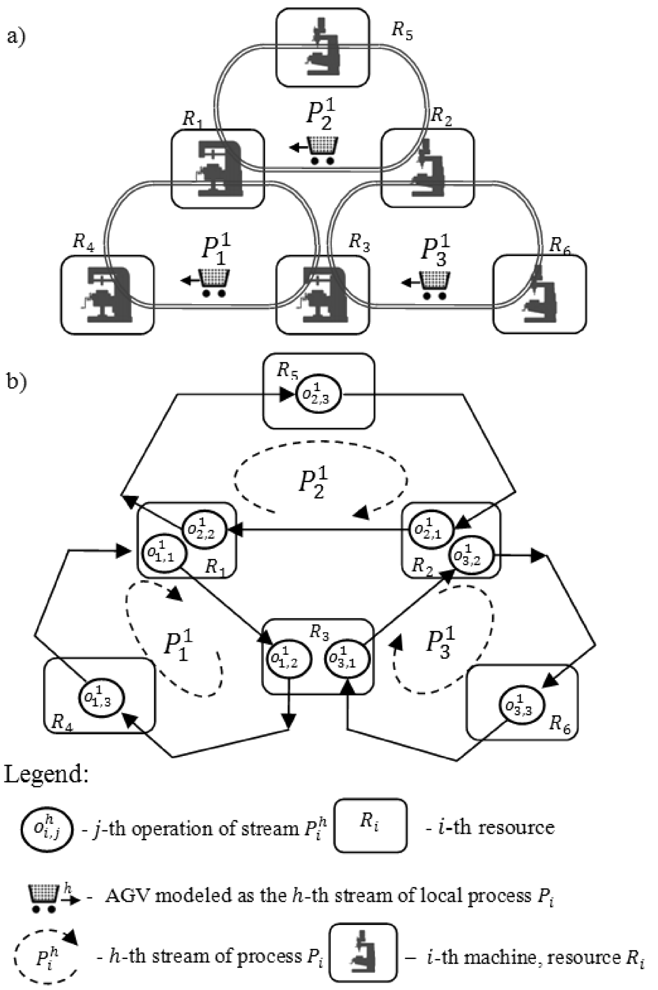

Fig. 1. Robotized flexible manufacturing system (a), its representation in the form of SCCP with three processes (b)

The example presented illustrates the system structure (in the present case – the railway network and trains sharing its resources) influence on its behavior resulting in specified timetables for the lines serving for example local or long-distance connections. It also points to the systemic nature of the problem of timetables planning, emphasizing difficult to predict (non-linear) effect of changes in the structure of the railway line and/or resolving resource conflicts related to prioritization of the trains access to shared line infrastructure resources, on supported by them passenger or freight transport. In particular, this means that the assessment and/or variants of alternative SCCP behaviors fits into the domain of cyclic scheduling problems, problems observed for example in course of timetables determining, telecommunication transmission, production planning [5, 10, 12 24, 29], etc.

Figure 1b shows an example of SCCP representing robotized flexible manufacturing system (FMS) (Fig. 1a), which takes into account the inter-operations transport, implemented by the devices like robot, conveyor, crane, truck crane, etc.

Integration of technological and inter-operation transport processes observed in the FMS emphasizes the need of periodically recurring processes dispatching. Cyclic scheduling problems considered in the context of such type of systems, are limited to determination of the moments of the beginning of operations $o_{i,j}$ assigned to cyclically performed tasks Z_i ($Z_i \in Z$) and routes of robots movement (mp sequences defining the order in which the robots move between machines/resources $R_i \in R$), so that the production cycle was as short as possible.

In the process aspect, the cyclic execution of Z_i tasks can be interpreted as a stream execution of the corresponding cyclic processes P_i, and therefore represented in the SCCP model. An example of this possibility is the SCCP graphical model presented in Fig. 1a.

Such a system structure, according to the notation introduced in [5], is determined by:

- set of renewable resources: $R = \{R_1, R_2, R_3, R_4, R_5, R_6\}$ of unit volume,
- set of one-stream processes: $P = \{P_1, P_2, P_3\}$, $P_1 = \{P_1^1\}$, $P_2 = \{P_2^1\}$, $P_3 = \{P_3^1\}$, where P_i^h means h-th stream of process P_i. Processes P are characterized by the following values:
  - sequences of operations of streams P_i^h:
  $$O_1^1 = (o_{1,1}^1, o_{1,2}^1, o_{1,3}^1),$$
  $$O_2^1 = (o_{2,1}^1, o_{2,2}^1, o_{2,3}^1),$$
  $$O_3^1 = (o_{3,1}^1, o_{3,2}^1, o_{3,3}^1),$$
  where: $o_{i,j}^h$ – means j-th operation of stream P_i^h
  - routes of processes P_i:
  $$p_1 = (R_1, R_3, R_4),$$
  $$p_2 = (R_1, R_5, R_2),$$
  $$p_3 = (R_2, R_6, R_3).$$

Furthermore, it is assumed that the processes realization is dispatched based on the protocol of mutual exclusion (processes are synchronized by priority dispatching rules assigned to each common shared resource [5]) and the R resources are nonpreemptive.

In the general case, the SCCP structure means further the set of parameters characterizing the resources (their number, capacity, etc.), and realized processes (operations related to them, routes, demand for resources, etc.).

The cyclic schedule X_L of the realization of P set processes operation, presented in Fig. 2, is an example of a potential behavior of the system of considered structure. A set of values determining the manner of operation realization (moments of operation beginning, allocation states, etc.) is further understood as cyclic SCCP behavior [5]. In particular, the SCCP behavior represented in a form of cyclic schedule is defined in a following manner:

$$X_L = (X, \alpha), \tag{1}$$

where X – set of beginning dates $x^h_{i,j}$ of $o^h_{i,j}$ operations of all local processes realized in SCCP, $x^h_{i,j} = x^h_{i,j}(0)$ – date of operation beginning, first execution of h-th stream of P_i process, (for $k = 0$); $x^h_{i,j}(k) = x^h_{i,j}(0) + \alpha \cdot k$, $k \in \mathbb{C}$, $x^h_{i,j}(k) \in \mathbb{C}$, α – period of local processes realization $\alpha \in N^+$.

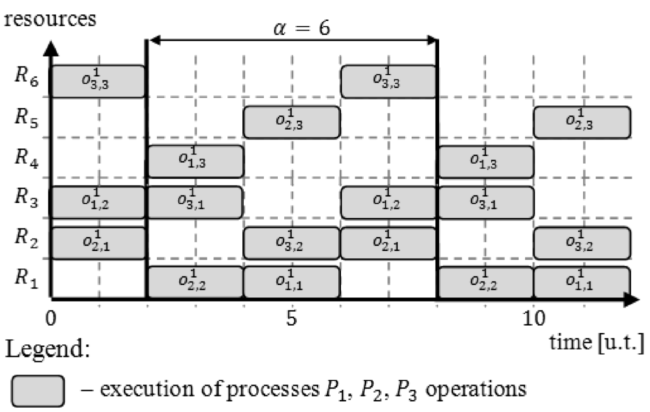

Fig. 2. The cyclic schedule for SCCP from Fig. 1b

SCCP systems lived to see numerous formal models which take into account, inter alia, multi-streaming processes, different rules of dispatching of an access to shared resources (FIFO, LIFO, etc.), variable resources capacity, non-linear/linear processes realization order, mutual exclusion of the processes, etc. [5, 10, 18, 24, 25, 29]. In the literature however, there is a lack of models allowing to take into account the relationships between mutually utilizing processes, i.e. between local processes and using them multimodal processes. The concept of such a model is presented in the next section

4. Concurrent Multimodal Cyclic Processes

SCCP class systems often involve the situations in which the execution of a certain group of processes is conditioned by the simultaneous execution of other processes (e.g. realization of production process operation is conditioned by the realization of appropriate transport processes operations). Such processes, called **multimodal processes** [5] – carried out in "sections" of local cyclic processes are a natural extension of SCCP class systems. Multimodal processes are an extension of the Systems of Concurrent Cyclic Processes SCCP [5]. These systems are used for modeling of the production flow in flexible manufacturing systems, computer processes of operating systems, data transmission, etc.

One of the examples of multimodal processes application is representation of passenger flows in different underground lines. Figure 3 presents two such processes, representing the transport of passengers between Ríos Rosas and Manuel Becerra underground line stations in Madrid (Spain). The transport possibility in the considered case is conditioned by the availability of suitable transport resources – in this case lines 1 and 6 train (direction Ríos Rosas and Manuel Becerra), and lines 2, 4 and 1, (direction Ríos Rosas and Manuel Becerra). In other words, multimodal processes (streams of passengers) are "transported" by the so-called local processes representing different underground lines trains.

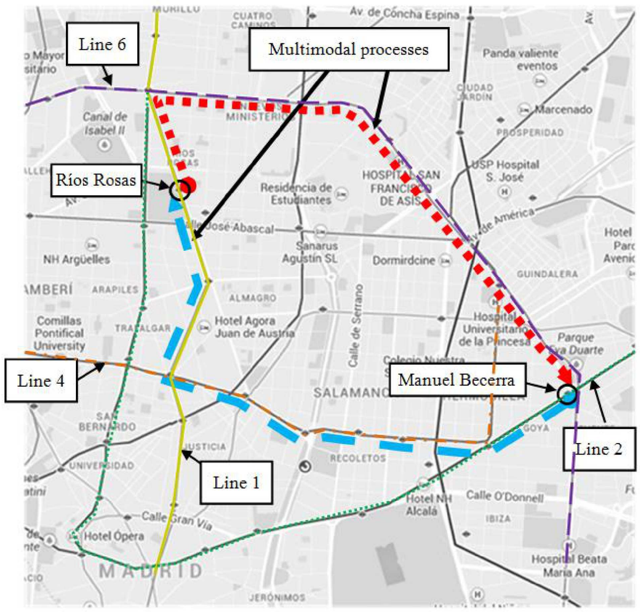

Fig. 3. Madrid – underground lines connections (maps.google.pl)

The presented example is one of many examples of the use of multimodal processes in transport issues modeling. They include inter alia issues related to: daily commuting (bus – suburban railway – underground), courier services (e.g. DHL), streaming data transmission, production flow, etc.

Mentioned examples of multimodal networks allow to consider different problems related to routing and scheduling of multimodal transport processes realized in them. This means that, by adopting some SCCP model of the considered infrastructure of the local cyclic processes system, one can focus on the planning problems of multimodal processes realized in them. In particular, the following problems may be considered taking into account the class of Systems of Concurrent Multimodal Cyclic Processes (SCMCP):

- analysis, e.g. coming down to determination of cyclic multimodal processes reachable in a given SCMCP,
- synthesis, e.g. coming down to determination of system parameters guaranteeing assumed features of cyclic behaviors reachable in given SCMCP (e.g. search for timetable

of lines 2, 1 and 4 trains – Fig. 3 – guaranteeing daily transport of at least 8 000 passengers),
- mutual reachability of cyclic behaviors (so called rescheduling problem), e.g. coming down to an evaluation of possibility of the changes in system behavior between a few reachable cyclic behaviors (e.g. change in the timetable "favoring" direction Ríos Rosas → Manuel Becerra on the timetable "favoring" direction Manuel Becerra → Ríos Rosas – Fig. 3).

As mentioned above, the operations performed in the multimodal processes, except R resources require simultaneous access to other processes realized in the SCCP. Referring to the example from Fig. 3, the multimodal process may represent a passenger traveling with various underground lines, where each line represents an appropriate cyclic process executing their operations related to the movement of trains between successive stations of the line or their stops at the stations. The resources in this case are the stations and sections of underground lines. In this context, the passenger's travel may be presented as a process which operations are related to passenger's transport and the operations of getting on/getting out. The train is required so that the passenger could travel, and it is treated as a moving resource. The "passenger process" requires thus the presence of "train process" to realize the operation. Multimodality in this case means that the passenger during the travel may many times change the underground lines, i.e. to change repeatedly the local processes required for travel plan realization. It is worth noting that the concept of "multimodality" occurs in logistics and is associated with the definition of multimodal transport involving the transport of goods and/or people using various means of transport [1–3].

In the proposed multimodal approach, the distinguishing of certain SCCP elements (such as production routes, transport routes, data streams, etc.) as a separate multimodal processes group allows to conduct a more detailed analysis than is the case of approaches which take into account only local processes. For example, treating the production routings as multimodal processes dependent on transport means, allows inter alia an evaluation of the relationships observed between the cyclic execution of these processes (e.g. means of transport) and cyclic production realized this way.

In general, the **multimodal process** mP_i is the process which require other processes for its realization. In order to distinguish, the concept of **local process** P_i is used to determine the process requiring no other processes for its realization (i.e. the processes discussed in the SCCP model).

Formally, multimodal cyclic process $mP_i \in mP$ is defined as a set of streams: $mP_i = \left\{mP_i^1, mP_i^2, \ldots, mP_i^h, \ldots, mP_i^{lsm(i)}\right\}$ where each stream mP_i^h is characterized by:

- route (common for all streams of mP_i process):

$$mp_i = (mp_{i,1}, \ldots, mp_{i,j}, \ldots, mp_{i,lm(i)}), \quad mp_{i,j} \in R \quad (2)$$

determined as combination of selected fragments of local processes routes ($P = \{P_i | i = 1, ..., ln\}$ – processes requiring no other processes for their realization):

$$mp_i = \left(mpr_{i_1}(a_{i_1}, b_{i_1}) \cap mpr_{i_2}(a_{i_2}, b_{i_2}) \cap \ldots \cap mpr_{i_y}\left(a_{i_y}, b_{i_y}\right)\right), \quad (3)$$

where

$$mpr_i(a,b) = \begin{cases} (p_{i,a}, p_{i,a+1}, \ldots, p_{i,b}) & \text{for} \quad a \le b \\ \left(p_{i,a}, p_{i,a+1}, \ldots, p_{i,lr(i)}, \ldots, p_{i,b}\right) & \text{for} \quad a > b \end{cases}$$

is a fragment (i.e., part and/or section) of p_i route of P_i local process containing elements from $p_{i,a}$ to $p_{i,b}$.
$x \cap y$ –means concatenation of sequence x and y, in case when $x = (x_1, \ldots, x_n)$ and $y = (y_1, \ldots, y_m)$ then $x \cap y = (x_1, \ldots, x_n, y_1, \ldots, y_m)$.
Route mp_i is a sequence containing fragments of $mpr_i(a,b)$ routes of P_i local processes, which are used in realization of mP_i process. It is assumed that two subsequent fragments of $mpr_{i_1}(a_{i_1}, b_{i_1}) \cap mpr_{i_2}(a_{i_2}, b_{i_2})$ routes are connected by common resource $p_{j,b_{i1}}$.

- sequence of operation:

$$mO_i^h = \left(mo_{i,1}^h, mo_{i,2}^h, ..., mo_{i,j}^h, ..., mo_{i,lm(i)}^h\right), \quad (4)$$

where $mo_{i,j}^h$ – j-th operation of stream $m^lP_i^h$ – number of operations mP_i.

Time of execution $mt_{i,j}^h \in \mathbb{N}^+$, resource $mp_{i,j} \in R$ and local process $m\mu_{i,j} \in P$ essential for its performance are attributed to each operation $mo_{i,j}^h$.

In order to present the intuition of the introduced definition, an example of the use of multimodal processes for production flow modeling in the FMS considered is presented below.

The system of multi-stream production of three kinds of products, manufactured in accordance with assumed production routes (routes are marked with orange, green and blue lines) is given (see Fig. 4). Manufacturing of one kind of product is represented by multimodal process mP_i $(i = 1, 2, 3)$. Particular products marked as (q) are represented by streams $mP_i^q \in mP_i$. The system involves 13 automated guided vehicles (AGVs) used for products transport between the positions. The operations of products transport are represented by local processes streams P_i^h (P_i^h – means h-th stream of process P_i).

In that context, considered system of multi-stream production (Fig. 4) may be represented by System of Concurrent Multimodal Cyclic Processes (SCMCP) presented in Fig. 5. The system assumes that:

- local/multimodal processes are dispatched based on mutual exclusion protocol (i.e., only one product and/or one vehicle may be on position in a given moment),
- R resources are nonpreemptive (i.e., once started operations of local/multimodal process cannot be disrupted).

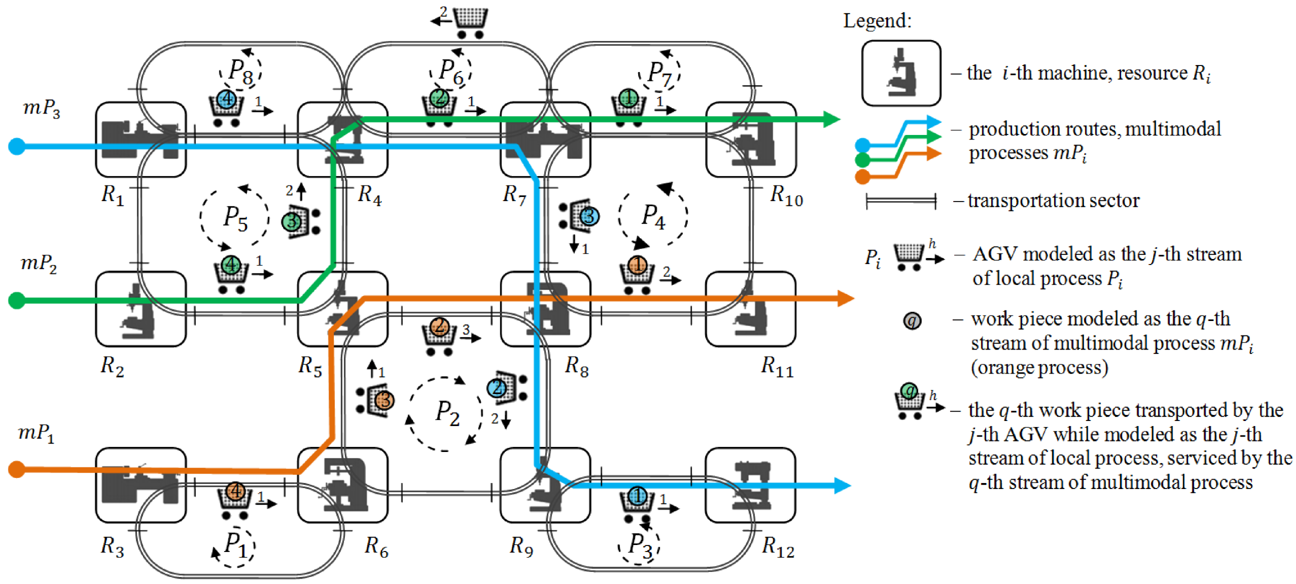

Fig. 4. System of stream production with AGV transport sub-system

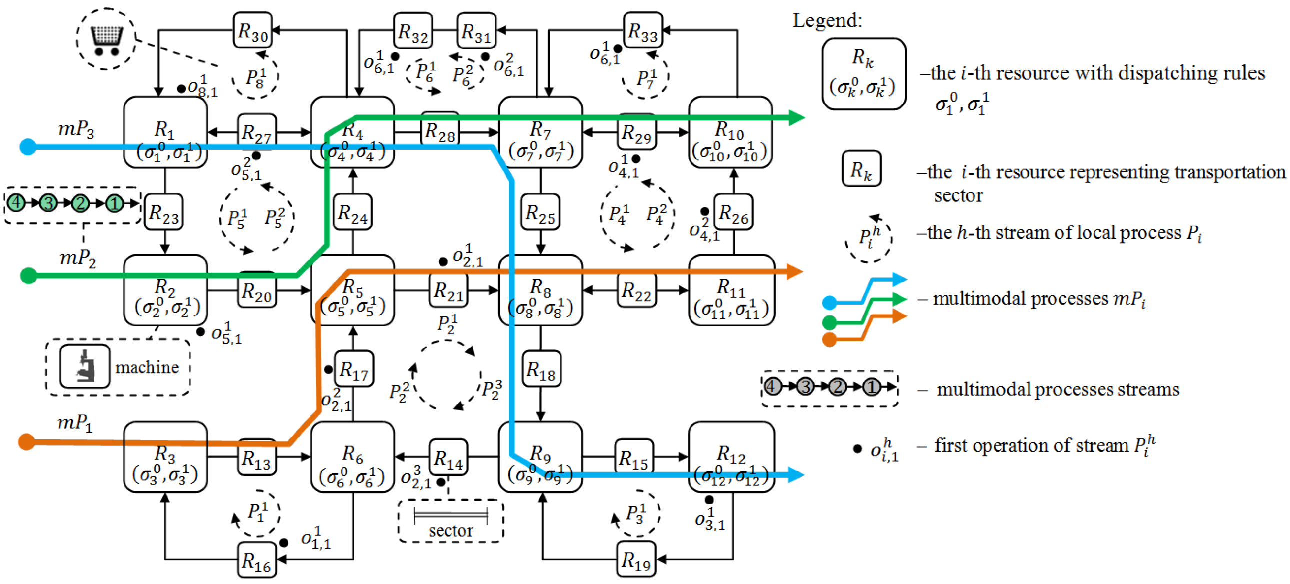

Fig. 5. SCMCP modeling system from Fig. 4

In other words, the considered SCMCP can be treated as an extension of SCCP, where SCCP (e.g. AGV transport sub-system) provides a framework for multimodal processes execution (e.g. production flows). The structure of SCMCP may be described in a form of the following sequence:

$$SC = ((R, SL), SM), \tag{5}$$

where $R = \{R_1, \ldots, R_{33}\}$ – is a set of 33 resources. Resources $R_1 - R_{12}$ represent work positions, ($R_1 - R_3$ are input resources, and $R_{10} - R_{12}$ are output resources), resources $R_{13} - R_{33}$ represent transport sectors the vehicles move along.

SC structure contains two levels of behaviors: the level of local processes SL (inter-position transport level) and level of multimodal processes SM (multi-version production level).

The level of local processes SL is characterized by a set of 8 processes:

$$P = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\},$$

$$P_1 = \{P_1^1\}, \quad P_2 = \{P_2^1, P_2^2, P_2^3\},$$

$$P_3 = \{P_3^1\}, \quad P_4 = \{P_4^1, P_4^2\},$$

$$P_5 = \{P_5^1, P_5^2\}, \quad P_6 = \{P_6^1, P_6^2\},$$

$$P_7 = \{P_7^1\}, \quad P_8 = \{P_8^1\},$$

from which processes: P_4, P_5, P_6 are two-stream processes (i.e. it is assumed that two vehicles move along one route), and process P_2 is three-stream process (i.e. three vehicles move along one route). Following routes correspond to the processes:

$$p_1 = (R_{16}, R_3, R_{13}, R_6),$$
$$p_2 = (R_{21}, R_8, R_{18}, R_9, R_{14}, R_6, R_{17}, R_5),$$
$$p_3 = (R_{12}, R_{19}, R_9, R_{15}),$$
$$p_4 = (R_7, R_{25}, R_8, R_{22}, R_{11}, R_{26}, R_{10}, R_{29}),$$
$$p_5 = (R_2, R_{20}, R_5, R_{24}, R_4, R_{27}, R_1, R_{23}),$$
$$p_6 = (R_{32}, R_4, R_{28}, R_7, R_{31}).$$

The operations of the streams conducted along specified routes are determined by: $O_i^h = \left(o_{i,1}^h, ..., o_{i,lr(i)}^h\right)$, $i = 1 \ldots 8$, $h = 1 \ldots ls(i)$, where each operation $o_{i,j}^h$ has attributed the moment of its beginning $x_{i,j}^h$ and duration time $t_{i,j}^h$ (it is acceptable that operations from various streams may have different duration times).

Initial operations $o_{i,1}^h$ of streams P_i^h were marked in Fig. 5 as $\bullet$. It was accepted that initial operations $o_{i,1}^h$ of the same process streams may be realized on various resources. For example, initial operations of the streams of process P_2 are executed on resources R_{21}, R_{17}, R_{14}.

Concurrently executed local cyclic processes are synchronized by priority dispatching rules assigned to common shared resources Streams (vehicles) access to the resources is determined using priority dispatching rules (dispatching rules for short) $\Theta^0 = \left\{\sigma_1^0, \ldots, \sigma_{33}^0\right\}$ presented in Table 1.

Table 1
Dispatching rules of SL level processes for system from Fig. 5

Dispatching rules Θ^0		
$\sigma_1^0 = (P_5^2, P_8^1, P_5^1)$	$\sigma_{12}^0 = (P_3^1)$	$\sigma_{23}^0 = (P_5^2, P_5^1)$
$\sigma_2^0 = (P_5^1, P_5^2)$	$\sigma_{13}^0 = (P_1^1)$	$\sigma_{24}^0 = (P_5^1, P_5^2)$
$\sigma_3^0 = (P_1^1)$	$\sigma_{14}^0 = (P_2^3, P_2^1, P_2^2)$	$\sigma_{25}^0 = (P_4^1, P_4^2)$
$\sigma_4^0 = (P_6^1, P_8^1, P_6^2, P_5^1, P_5^2)$	$\sigma_{15}^0 = (P_3^1)$	$\sigma_{26}^0 = (P_4^2, P_4^1)$
$\sigma_5^0 = (P_2^2, P_2^3, P_5^1, P_5^2, P_2^1)$	$\sigma_{16}^0 = (P_1^1)$	$\sigma_{27}^0 = (P_5^2, P_8^1, P_5^1)$
$\sigma_6^0 = (P_2^3, P_2^1, P_1^1, P_2^2)$	$\sigma_{17}^0 = (P_2^2, P_2^3, P_2^1)$	$\sigma_{28}^0 = (P_6^1, P_6^2)$
$\sigma_7^0 = (P_4^1, P_4^2, P_6^1, P_7^1, P_6^2)$	$\sigma_{18}^0 = (P_2^1, P_2^2, P_2^3)$	$\sigma_{29}^0 = (P_4^1, P_4^2, P_7^1)$
$\sigma_8^0 = (P_2^1, P_4^1, P_2^2, P_2^3, P_4^2)$	$\sigma_{19}^0 = (P_3^1)$	$\sigma_{30}^0 = (P_8^1)$
$\sigma_9^0 = (P_2^1, P_2^2, P_3^1, P_2^3)$	$\sigma_{20}^0 = (P_5^1, P_5^2)$	$\sigma_{31}^0 = (P_6^2, P_6^1)$
$\sigma_{10}^0 = (P_4^2, P_7^1, P_4^1)$	$\sigma_{21}^0 = (P_2^1, P_2^2, P_2^3)$	$\sigma_{28}^0 = (P_6^1, P_6^2)$
$\sigma_{11}^0 = (P_4^1, P_4^2)$	$\sigma_{22}^0 = (P_4^1, P_4^2)$	$\sigma_{30}^0 = (P_7^1)$

In general, dispatching rules σ_k^l is defined as a sequence, which elements determine the order of an access of processes streams to shared resource R_k: $\sigma_k^l = (s_{k,1}^l, \ldots, s_{k,j}^l, \ldots, s_{k,lh(k,l)}^l)$, where: $s_{k,d}^l$ determines the stream executed on resource R_k in j-th sequence. For example, the rule $\sigma_1^0 = \left(P_5^2, P_8^1, P_5^1\right)$ means that an access of processes streams to resource R_1 has the following sequence $\ldots, P_5^2, P_8^1, P_5^1, \ldots$

SM level (multimodal processes) is characterized in turn by a set of three four-stream multimodal processes:

$$mP = \{mP_1, mP_2, mP_3\},$$

where $mP_i = \left\{mP_i^1, mP_i^2, mP_i^3, mP_i^4\right\}$, $i = 1, 2, 3$.

Assumption of multi-stream character in case of multimodal processes results from the fact that concurrent production of numerous products of one kind may be realized in the system simultaneously. Each stream mP_i^h of multimodal processes is performed along one of the following routes:

$$mp_1 = ((R_3, R_{13}, R_6) \cap (R_{17}, R_5) \cap (R_{21}, R_8) \cap (R_{22}, R_{11})) = (R_3, R_{13}, R_6, R_{17}, R_5, R_{21}, R_8, R_{22}, R_{11}),$$

$$mp_2 = ((R_2, R_{20}, R_5) \cap (R_{24}, R_4) \cap (R_{28}, R_7) \cap (R_{29}, R_{10})) = (R_2, R_{20}, R_5, R_{24}, R_4, R_{28}, R_7, R_{29}, R_{10}),$$

$$mp_3 = ((R_1, R_{27}, R_4) \cap (R_{28}, R_7) \cap (R_{25}, R_8) \cap (R_{18}, R_9), (R_{15}, R_{12})) = (R_1, R_{27}, R_4, R_{28}, R_7, R_{25}, R_8, R_{18}, R_9, R_{15}, R_{12})$$

where $(R_3, R_{13}, R_6)\,(R_{17}, R_5)\,(R_{21}, R_8)\,(R_{22}, R_{11})$ – fragments of routes of local processes streams P_1^1, P_2^1, P_2^3, and P_4^2 (i.e., arbitrarily selected vehicles) subsequently used in streams realization (products transport) of process mP_1 (marked with orange line in Fig. 5), $(R_2, R_{20}, R_5)\,(R_{24}, R_4)\,(R_{28}, R_7)\,(R_{29}, R_{10})$ – fragments of routes of local processes streams P_5^1, P_5^2, P_6^1 and P_7^1 subsequently used in streams realization of process mP_2 (marked with green line in Fig. 5), $(R_1, R_{27}, R_4)\,(R_{28}, R_7)\,(R_{25}, R_8)\,(R_{18}, R_9)\,(R_{15}, R_{12})$ – fragments of routes of local processes streams P_8^1, P_6^2, P_4^1, P_2^2 subsequently used in streams realization of process mP_3 (marked with blue line in Fig. 5).

Operations of streams executed along such specified routes are determined by: $mO_i^h = \left(mo_{i,1}^h, ..., mo_{i,lm(i)}^h\right)$, $i = 1 \ldots 3$, $h = 1 \ldots 4$, where each operation $mo_{i,j}^h$ has attributed the moment of its beginning $mx_{i,j}^h$.

The set of dispatching rules Θ^1 for considered multimodal processes (level SM) is presented in Table 2.

Table 2
Dispatching rules of SM level processes for system from Fig. 5

Dispatching rules Θ^1	
$\sigma_1^1 = (mP_3^1, mP_3^2, mP_3^3, mP_3^4)$	$\sigma_{13}^1 = (mP_1^1, mP_1^2, mP_1^3, mP_1^4)$
$\sigma_2^1 = (mP_2^1, mP_2^2, mP_2^3, mP_2^4)$	$\sigma_{15}^1 = (mP_3^3, mP_3^4, mP_3^1, mP_3^2)$
$\sigma_3^1 = (mP_1^1, mP_1^2, mP_1^3, mP_1^4)$	$\sigma_{17}^1 = (mP_1^1, mP_1^2, mP_1^3, mP_1^4)$
$\sigma_4^1 = (mP_3^4, mP_2^4, mP_3^1, mP_2^1$ $mP_3^2, mP_2^2, mP_3^1, mP_2^1)$	$\sigma_{18}^1 = (mP_3^3, mP_3^4, mP_3^1, mP_3^2)$
$\sigma_5^1 = (mP_1^3, mP_2^4, mP_1^4, mP_2^1$ $mP_1^1, mP_2^2, mP_1^2, mP_2^3)$	$\sigma_{20}^1 = (mP_2^1, mP_2^2, mP_2^3, mP_2^4)$
$\sigma_6^1 = (mP_1^1, mP_1^2, mP_1^3, mP_1^4)$	$\sigma_{21}^1 = (mP_1^3, mP_1^4, mP_1^1, mP_1^2)$
$\sigma_7^1 = (mP_2^3, mP_3^4, mP_2^4, mP_3^1$ $mP_2^1, mP_3^2, mP_2^2, mP_3^3)$	$\sigma_{22}^1 = (mP_1^3, mP_1^4, mP_1^1, mP_1^2)$
$\sigma_8^1 = (mP_3^3, mP_1^3, mP_3^4, mP_1^4$ $mP_3^1, mP_1^1, mP_3^2, mP_1^2)$	$\sigma_{24}^1 = (mP_2^1, mP_2^2, mP_2^3, mP_2^4)$
$\sigma_9^1 = (mP_3^3, mP_3^4, mP_3^1, mP_3^2)$	$\sigma_{25}^1 = (mP_3^3, mP_3^4, mP_3^1, mP_3^2)$
$\sigma_{10}^1 = (mP_2^3, mP_2^4, mP_2^1, mP_2^2)$	$\sigma_{27}^1 = (mP_3^1, mP_3^2, mP_3^3, mP_3^4)$
$\sigma_{11}^1 = (mP_1^3, mP_1^4, mP_1^1, mP_1^2)$	$\sigma_{28}^1 = (mP_3^4, mP_2^4, mP_3^1, mP_2^1$ $mP_3^2, mP_2^2, mP_3^1, mP_2^1)$
$\sigma_{12}^1 = (mP_3^3, mP_3^4, mP_3^1, mP_3^2)$	$\sigma_{29}^1 = (mP_2^3, mP_2^4, mP_2^1, mP_2^2)$

It is assumed in the example considered, that some of the elements of its structure are known, i.e. local (corresponding to vehicles) and multimodal processes routes (corresponding to manufactured products) (Fig. 5), dispatching rules (Tables 1 and 2) and the execution times of multimodal processes operation (Table 3). In turn, the execution times of the local processes operation $t^h_{i,j}$. are unknown. In other words, it is not known what is the time of vehicles staying at certain positions, and the duration of their passage along the transport sectors.

Except the values characterizing the structure, also the manner of production routes realization (i.e., multimodal processes) is known. The realization of processes following the routes is determined in advance and results from the assumptions included in the adopted production plan of given cyclic schedule shown in Fig. 6.

The schedule presents realization of the operation (products processing on resources $R_1 - R_{12}$) of four streams mP^1_i, mP^2_i, mP^3_i, mP^4_i, each of the multimodal processes ($i = 1, 2, 3$).

Realization of the operation is executed according to accepted rules (Table 2), the processes operate with period: $m\alpha = 320$ time units, where each product release the system in intervals of 80 time units (operation execution times are presented in Table 3).

The answer for the following question is sought in the context of such assumed, expected system behavior:

Is it possible in the system from Fig. 5 to organize the work of transport vehicles in the manner which guarantees products manufacturing in accordance to the approved production plan?

By the organization of vehicles work we understood here such selection of particular operations execution time (set of times $t^h_{i,j}$) of local processes, which guarantees transport of elements between the positions assuring specified, punctual realization of the production plan.

The accepted schedule takes into account the tasks related to the transport and loading/unloading of the elements. It was assumed that element transport between the positions requires 1 time unit ($\Delta t = 1$), and tasks related to load/unload involve the first and last unit of operation execution time interval, respectively (Fig. 6). It means in the practice, that the presence of suitable vehicle on a position where element processing is realized on, is required at the moment of operations beginning (unload on the position) and at its termination (load on the vehicle). The exceptions are input positions (R_1, R_2, R_3) as well as output ones (R_{10}, R_{11}, R_{12}), in which download/collection of the elements is performed in an automatic manner (without transport vehicles contribution).

The problem presented is an example of a widened **synthesis problem** [5], which searches for the manner of local processes realization guaranteeing specified schedule of multimodal processes in a given SCMCP. In general, the synthesis problem is defined in the following manner.

Table 3
Times of execution of multimodal processes operations from Fig. 5

$k = 1 \ldots 4$	R_1	R_2	R_3	R_4	R_5	R_6	R_7	R_8	R_9	R_{10}	R_{11}	R_{12}	$R_{13} - R_{12}$
mt^k_1	–	–	77	–	64	77	–	20	–	–	20	–	1
mt^k_2	–	77	–	20	10	–	30	–	–	30	–	–	1
mt^k_3	77	–	–	49	–	–	20	50	20	–	–	10	1

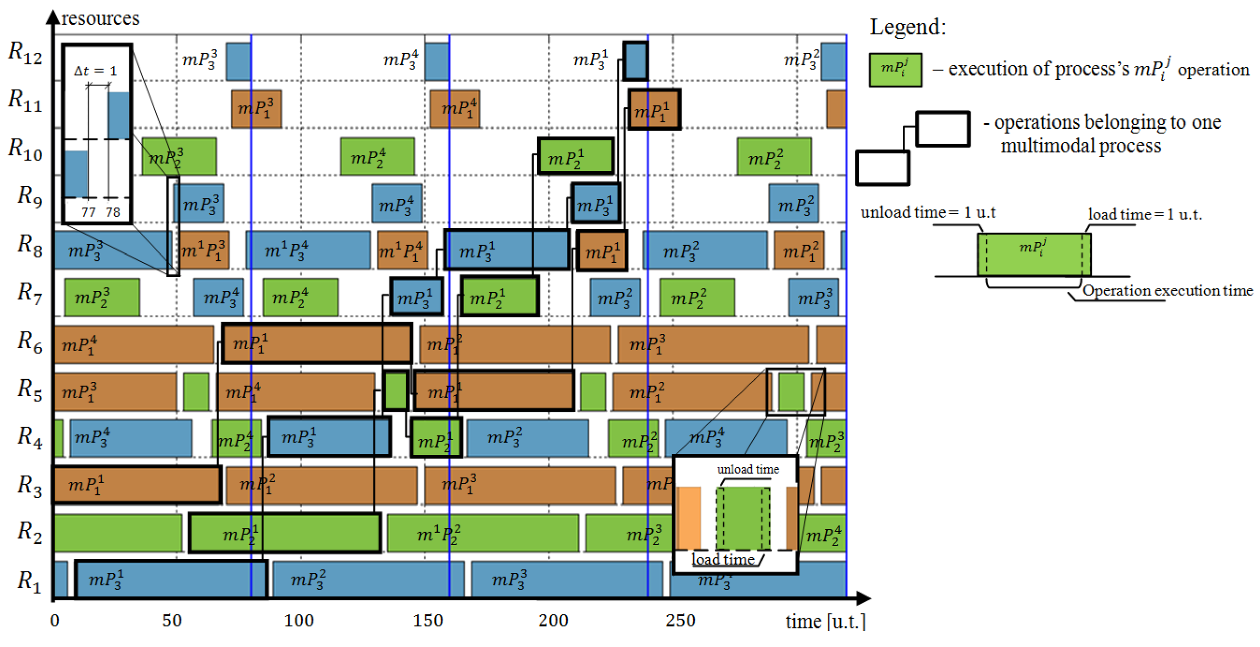

Fig. 6. Cyclic schedule of multimodal processes representing accepted production plan

In given SCMCP with known:

- resources R,
- local processes P (level SL),
- multimodal processes mP (level SM),

there is a search for conditions guaranteeing an existence of cyclic schedule X' of local and multimodal realizations.

The cyclic schedule X' means [5]:

$$X' = ((X, \alpha), (mX, m\alpha)), \qquad (6)$$

where X – set of dates of beginning $x^h_{i,j}$ of operations of all local processes realized in SCMCP, defined like in (1), α – period of local processes realization $\alpha \in \mathbb{N}^+$, defined like in (1), mX – set of dates of beginning $mx^h_{i,j}$ of operations of all multimodal processes realized in SCMCP, $mx^h_{i,j} = mx^h_{i,j}(0)$ – date of beginning of operation of the first h-th stream of process mP_i, (for $k = 0$); $mx^h_{i,j}(k) = mx^h_{i,j}(0) + m\alpha \cdot k$, $k \in C$, $mx^h_{i,j}(k) \in \mathbb{C}$, $m\alpha$ – period of multimodal processes realization $m\alpha \in \mathbb{N}^+$.

5. Declarative model

5.1. Constraints satisfaction problem. Introduced concepts of the structure SC (5) and the cyclic schedule X' (6) describing SCMCP behavior allow to look at the analysis problem as the problem of reachability (in a system of a given structure SC) of specific cyclic schedule X', while at the synthesis problem as the problem of SC structure existence which guarantees the assumed cyclic schedule X'. The questions related to these issues concern: the occurrence of cyclic steady states, desired length of transient states, structures characterized by an established number of process executions, etc. This kind of problems (e.g. a problem of SCMCP synthesis, see Fig. 5), assuming many levels of SCMCP behavior (SL, SM) as well as encompassing concurrent while collision- and deadlock-free processes execution cannot be solved using currently available approaches, for instance as proposed in [16, 18, 23–25]; simply because they do not allow to consider the multimodal processes.

Presented types of problems can be considered in terms of constraints satisfaction problems. In the general form, the Constraints Satisfaction Problem (CSP) [30] related to the systems of concurrent cyclic processes, takes the following form:

$$PS = ((\{SC, X'\}, \{D_{SC}, D_X\}), C_{PS}), \qquad (7)$$

where $\{SC, X'\}$ – set of decision variables, SC – SCMCP structure (5), X' – cyclic schedule of SCMCP, $\{D_{SC}, D_P\}$ – set of decision variables domains, D_{SC} – domain determining admissible SCMCP structures, D_X – domain determining admissible cyclic schedules, $C_{PS} = C_{SC} \cup C_X \cup C_D$ – set of constraints determining relationships between decision variables, C_{SC} – set of constraints determining kind of the structure, C_X – set of constraints characterizing processes realization: conditions determining processes access to shared resources, conditions of local processes using, C_D – set of additional constraints determining user's needs (e.g. existence of cyclic steady states).

CSP solution means determination of such values of decision variables from the set of domains, for which all assumed constraints are satisfied [30]. In case of problem (7), the solution is in a form of structure SC and cyclic schedule X', satisfying the assumptions put on the structure and reachability of cyclic states, etc.

In case of the example presented in the previous point, the considered problem may be expressed as a simplified form of CSP (7), in which the parameters characterizing structure SC and cyclic schedule X are only known in part. In particular, known is the structure SM of multimodal processes, as well as their behavior (represented by cyclic schedule – $(mX, m\alpha)$ – Fig. 6).

This means, that the sought solution comes down to determination of the form of local processes level SL, or more precisely, variable times of execution of operation $t^h_{i,j}$ of local processes. These times must be selected so that the realization of local processes (X, α) was consistent with assumed schedule of multimodal processes $(mX, m\alpha)$. This means, that in the considered case, the realization of multimodal processes level SM determined allowable realization of local processes SL.

Figure 7 presents the idea of the proposed approach searching for solutions of lower levels forms based on upper levels known parameters, the idea which corresponds to top-down strategy. This means, that the solution of PS (7) problem should take into account the constraints resulting from multimodal processes schedule. In the considered case, the problem (7) may be presented in a following form:

$$PS_R = ((T \cup X \cup \{\alpha\}, D_R), C'_X(SM, mX)), \qquad (8)$$

where T – set of times of execution $t^h_{i,j}$ of streams operations P^h_i of local processes; X, α – set of beginning moments $x^h_{i,j}$ and period of local processes, defined like in (1), $D_R = \{D_T, D_X, D_\alpha\}$ – set of decision variables domains: D_T – domain determining admissible values of the times of operation execution: $t^h_{i,j} \in \mathbb{N}^+$, D_X – domain of moments of operation beginning: $x^h_{i,j} \in \mathbb{C}$, D_α – domain of the period $\alpha : \alpha \in \mathbb{N}^+$, $C'_X(SM, mX)$ – constraints determining relationships between moments $x^h_{i,j}$ of local processes operation beginning guaranteeing realization of multimodal processes according to the schedule mX.

The key role in such a way defined problem is played by constraints $C'_X(SM, mX)$ assuring deadlock-free (i.e., cyclic) realization of local processes, and guaranteeing realization of multimodal processes according to established schedule mX. Detailed description of these constraints is presented in further section.

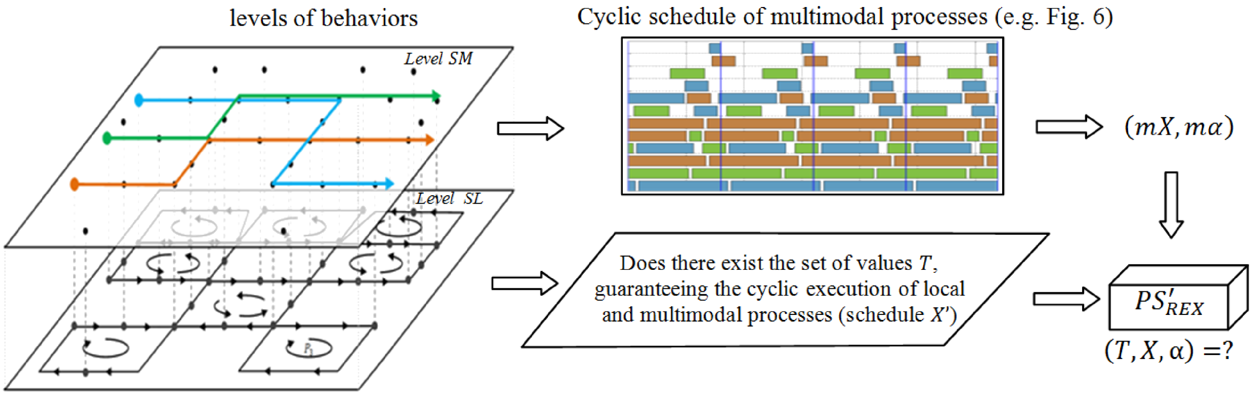

Fig. 7. Determination of parameters of SC structure for the system from Fig. 5 in top-down strategy

5.2. Constraints in cyclic execution of the processes. In the systems of SCMCP class, each cyclic schedule X (determining local processes execution) may be represented by so called **precedence digraph** $G = (V_G, E_G)$. Sample digraph $G_1 = (V_{G,1}, E_{G,1})$ from Fig. 9b corresponds to the realization of operation in accordance to the schedule X from Fig. 9a of the system from Fig. 8.

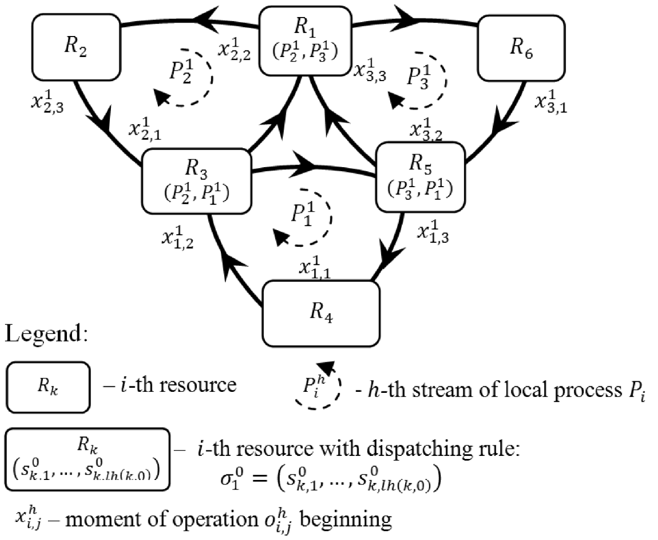

Fig. 8. Example of SCCP described by dispatching rules: $\sigma_1^0 = (P_2^1, P_3^1)$ $\sigma_2^0 = (P_2^1)$, $\sigma_3^0 = (P_2^1, P_1^1)$, $\sigma_4^0 = (P_1^1)$ $\sigma_5^0 = (P_3^1, P_1^1)$, $\sigma_6^0 = (P_3^1)$

The states of the system, identified with processes allocation in time, are marked as: Sl^0, Sl^1, Sl^2, Sl^3, Sl^4. These states correspond to time intervals represented by subsequent levels of cuboids, marked with the symbols of executed processes: P_1, P_2, P_3 (Fig. 9b). Each cuboid P_i with polygon base with $lr(i)$ vertexes ($lr(i)$ – number of operations of the stream P_i^h of process P_i), represents the time axis, with respect to which the operations of stream P_i^h of process P_i are realized. The vertexes of cuboid base represent resources used during execution of stream P_i^h operations.

The precedence digraph G_1 is spread on the set of cuboids (where one cuboid corresponds to one process). The vertexes $v_{i,j}^h(k)$ of that digraph marked with symbols: ●○● and related to operations $o_{i,j}^h$ of streams P_i^h, are placed on the cuboids, at a height corresponding to the moments of their beginning $x_{i,j}^h(k)$ (moment of operation beginning $o_{i,j}^h$ in k-th cycle). The vertexes of subsequent operations are connected with arcs (always directed according to time axis direction), which lengths determine the time of occupation of resources used by the streams (represented by the vertexes of the base). These arcs also reflect the order of operations execution specified by processes routes P_i.

Except the arcs related to the order of operations realized within one stream (arcs composing the walls of cuboids) there are also the arcs connecting the vertexes of various cuboids. This kind of arcs describes the precedence of operations being a consequence of accepted form of dispatching rules Θ^0.

This kind of arc is always directed towards the vertex related to further operations (of higher value $x_{i,j}^h(k)$). The delay Δt, caused by such arc occurrence, determines the delay related to the change of streams on a given resource. In the example considered, it was assumed that $\Delta t = 1$.

For example, the value of variable $x_{1,2}^1(k) = 2$ (vertex $v_{1,2}^1(k)$ is placed on level 2 – Fig. 9b), is higher than value $x_{1,1}^1(k) = 0$ since the operation $o_{1,2}^1$ occurs in the route p_1 after operation $x_{1,1}^1$. Moreover, $x_{2,2}^1(k) = 1 < x_{1,2}^1(k) = 2$ since according to the dispatching rule σ_3^0 (sequence of operation on R_3: …, $o_{2,1}^1$, $o_{1,2}^1$,…) operation $o_{1,2}^1$ performed by the stream P_1^1 may initiate its realization just after the resource R_3 (operation $o_{2,1}^1$) occupied by stream P_2^1, will be released, i.e. after execution of operation $o_{2,2}^1$

Such an understanding of a cyclic digraph G_1 is composed with an infinite number of vertexes $v_{i,j}^h(k)$ localized on the cuboids according to the order determined by the set of routes of the processes, and the dispatching rules. In the considered system, the cyclic schedule X (Fig. 9a) corresponds to the following sequence of transitions between the states Sl^i:

$$\ldots \to Sl^0 \to Sl^1 \to Sl^2 \to Sl^3 \to Sl^4 \to Sl^0 \to \ldots$$

in which realization of certain operations is related to each state Sl^i:

$$\to (o_{1,3}^1, o_{3,1}^1) \to (o_{1,1}^1, o_{2,1}^1) \to (o_{2,2}^1, o_{3,2}^1) \to (o_{2,3}^1, o_{1,2}^1) \to (o_{3,3}^1) \to (o_{1,3}^1, o_{3,1}^1) \to$$

Each cyclic schedule X may be thus described by the set of sequences of operations and related set of the moments of these operations beginning. Operations and moments of their beginning corresponding to schedule X are presented in Table 4. Each column of the table contains the state, attributed to it sequence of operations, and the moments of these operations beginning for subsequent cycles.

Table 4
Moments of beginning of operations realized in subsequent states of cyclic schedule X

States:	Sl^0	Sl^1	Sl^2	Sl^3	Sl^4
Operations:	$(o^1_{1,3}, o^1_{3,1})$	$(o^1_{1,1}, o^1_{2,1})$	$(o^1_{2,2}, o^1_{3,2})$	$(o^1_{2,3}, o^1_{1,2})$	$(o^1_{3,3})$
$x^h_{i,j}(k)$ for k:	−1	0	1	2	3
$k+1$:	4	5	6	7	8
$k+2$:	9	10	11	12	13

In other words, Table 4 contains values $x^h_{i,j}(k)$ attributed to vertexes $v^h_{i,j}(k)$ of digraph G_1, vertexes corresponding to execution of the operations observed in subsequent (k-th) cycles of the system. For example, values $x^1_{2,2}(k)$ and $x^1_{3,2}(k)$ determine the moments of beginning $o^1_{2,2}$, $o^1_{3,2}$ respectively for the cycles: k, $k+1$, $k+2$, $\ldots$, etc., values of these moments are as follows: 1, 6, 11, $\ldots$ Operations $o^1_{2,2}$, $o^1_{3,2}$ (as state Sl^2) are thus repeated with period equal 5 time units ($\alpha = 5$).

Generally, this means that the digraph of operations precedence may be used for the reconstruction of operation execution of each cyclic schedule X.

The order of operation execution illustrated by digraph G_1 will characterize the constraint (9):

$$x^h_{i,j}(k) = \max\{x^h_{i,(j-1)}(k') + t_{i,(j-1)}; x^c_{a,b}(k'') + \Delta t,\} \quad (9)$$

where $x^h_{i,j}(k)$, $x^h_{i,(j-1)}(k')$, $x^c_{a,b}(k'')$ mean the values of operation beginning moments $o^h_{i,j}$, $o^h_{i,(j-1)}$, $o^c_{a,b}$ related to vertexes $v^h_{i,j}(k)$, $v^h_{i,(j-1)}(k')$, $v^c_{a,b}(k'')$ of operation precedence digraph. Vertexes $v^h_{i,(j-1)}(k')$, $v^c_{a,b}(k'')$ are the predecessors of the vertex $v^h_{i,j}(k)$, where $v^h_{i,(j-1)}(k')$ is the vertex of the same cuboid like $v^h_{i,j}(k)$ (i.e. representing operation of the same stream), while $v^c_{a,b}(k'')$ is a vertex of other cuboid (i.e. vertex representing preceding operation according to the accepted dispatching rule). Values k, k' and k'' determine in turn the number of cycles the operations $o^h_{i,j}$, $o^h_{i,(j-1)}$, $o^c_{a,b}$ are performed within. Vertexes $v^h_{i,(j-1)}(k')$, $v^c_{a,b}(k'')$ may be related to operations executed within the current cycle: $k', k'' = k$, previous cycle: $k', k'' = k-1$ and in case of vertex $v^c_{a,b}(k'')$ to the next cycle (with respect to operation $o^h_{i,j}$): $k' = k+1$. Values $t_{i,(j-1)}$ and Δt mean the time of operation execution $o_{i,(j-1)}$ and delay related to the change of processes on the resource, respectively.

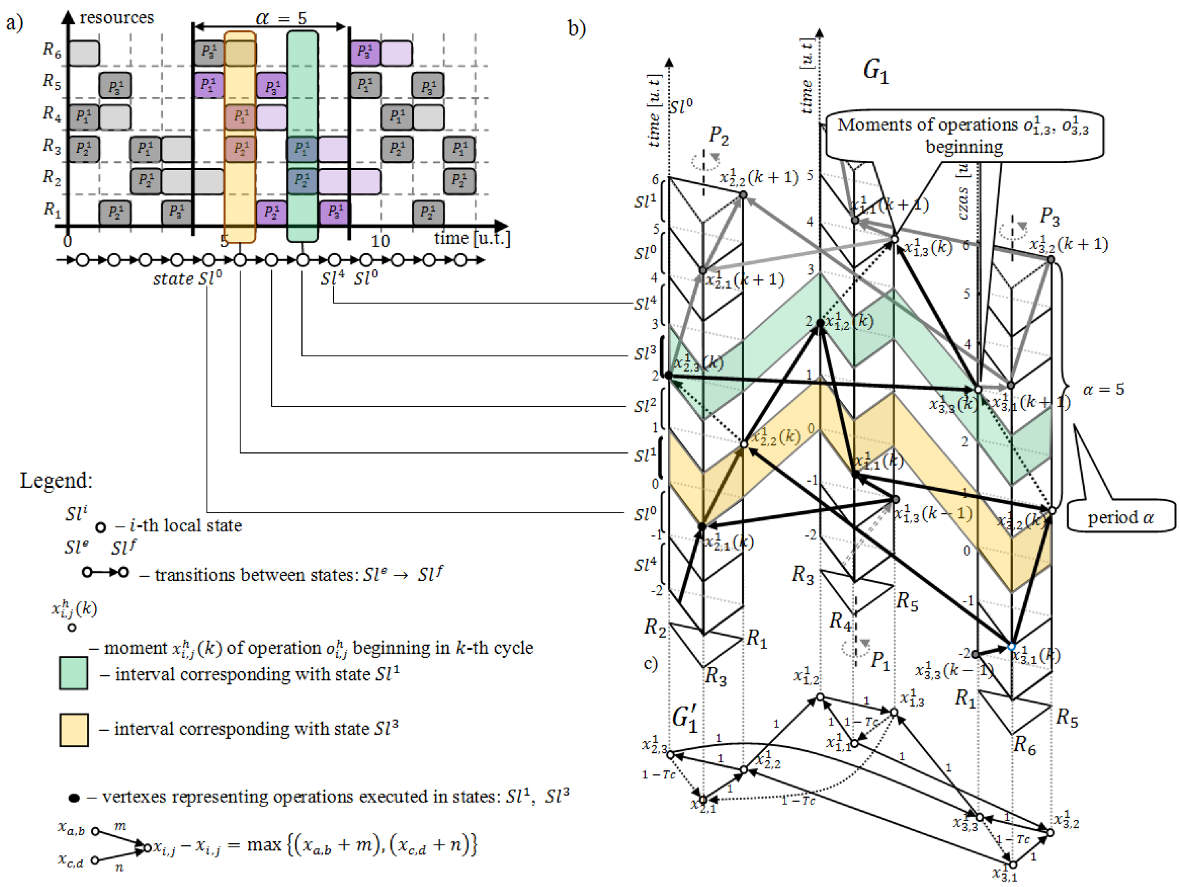

Fig. 9. Cyclic schedule X of the system from Fig. 8 (a), digraph of G_1 operation precedence representing schedule X (b), G'_1 – projection of G_1 digraph on the plane (c)

Table 5
Constraints determining local processes execution in SCMCP from Fig. 5

Constraints C_I	
$x^1_{1,1} = x^1_{1,4} + t^1_{1,4} - \alpha$	$x^1_{5,6} = \max\left\{\left(x^1_{8,3} + \Delta t\right), \left(x^1_{5,5} + t^1_{5,5}\right)\right\}$
$x^1_{1,2} = x^1_{1,1} + t^1_{1,1}$	$x^1_{5,7} = \max\left\{\left(x^1_{8,2} + \Delta t\right), \left(x^1_{5,6} + t^1_{5,6}\right)\right\}$
$x^1_{1,3} = x^1_{1,2} + t^1_{1,2}$	$x^1_{5,8} = \max\left\{\left(x^2_{5,4} + \Delta t\right), \left(x^1_{5,7} + t^1_{5,7}\right)\right\}$
$x^1_{1,4} = \max\left\{\left(x^1_{2,7} + \Delta t\right), \left(x^1_{1,3} + t^1_{1,3}\right)\right\}$	$x^1_{3,1} = x^1_{3,4} + t^1_{3,4} - \alpha$
$x^1_{2,1} = \max\left\{\left(x^3_{2,6} + \Delta t - \alpha\right), \left(x^1_{2,8} + t^1_{2,8} - \alpha\right)\right\}$	$x^1_{3,2} = x^1_{3,1} + t^1_{3,1}$
$x^1_{2,2} = \max\left\{\left(x^2_{4,7} + \Delta t - \alpha\right), \left(x^1_{2,1} + t^1_{2,1}\right)\right\}$	$x^1_{3,3} = \max\left\{\left(x^2_{2,7} + \Delta t\right), \left(x^1_{3,2} + t^1_{3,2}\right)\right\}$
$x^1_{2,3} = \max\left\{\left(x^3_{2,8} + \Delta t - \alpha\right), \left(x^1_{2,2} + t^1_{2,2}\right)\right\}$	$x^1_{3,4} = x^1_{3,3} + t^1_{3,3}$
$x^1_{2,4} = \max\left\{\left(x^3_{2,1} + \Delta t\right), \left(x^1_{2,3} + t^1_{2,3}\right)\right\}$	$x^1_{4,1} = \max\left\{\left(x^1_{7,4} + \Delta t - \alpha\right), \left(x^1_{4,8} + t^1_{4,8} - \alpha\right)\right\}$
$x^1_{2,5} = \max\left\{\left(x^3_{2,2} + \Delta t\right), \left(x^1_{2,4} + t^1_{2,4}\right)\right\}$	$x^1_{4,2} = \max\left\{\left(x^2_{6,1} + \Delta t\right), \left(x^1_{4,1} + t^1_{4,1}\right)\right\}$
$x^1_{2,6} = \max\left\{\left(x^3_{2,3} + \Delta t\right), \left(x^1_{2,5} + t^1_{2,5}\right)\right\}$	$x^1_{4,3} = \max\left\{\left(x^2_{4,6} + \Delta t - \alpha\right), \left(x^1_{4,2} + t^1_{4,2}\right)\right\}$
$x^1_{2,7} = \max\left\{\left(x^3_{2,4} + \Delta t\right), \left(x^1_{2,6} + t^1_{2,6}\right)\right\}$	$x^1_{4,4} = \max\left\{\left(x^1_{2,3} + \Delta t\right), \left(x^1_{4,3} + t^1_{4,3}\right)\right\}$
$x^1_{2,8} = \max\left\{\left(x^2_{5,7} + \Delta t\right), \left(x^1_{2,7} + t^1_{2,7}\right)\right\}$	$x^1_{4,5} = \max\left\{\left(x^2_{4,8} + \Delta t - \alpha\right), \left(x^1_{4,4} + t^1_{4,4}\right)\right\}$
$x^2_{2,1} = \max\left\{\left(x^1_{2,8} + \Delta t - \alpha\right), \left(x^2_{2,8} + t^2_{2,8} - \alpha\right)\right\}$	$x^1_{4,6} = \max\left\{\left(x^2_{4,1} + \Delta t\right), \left(x^1_{4,5} + t^1_{4,5}\right)\right\}$
$x^2_{2,2} = \max\left\{\left(x^1_{2,1} + \Delta t\right), \left(x^2_{2,1} + t^2_{2,1}\right)\right\}$	$x^1_{4,7} = \max\left\{\left(x^2_{4,2} + \Delta t\right), \left(x^1_{4,6} + t^1_{4,6}\right)\right\}$
$x^2_{2,3} = \max\left\{\left(x^1_{2,2} + \Delta t\right), \left(x^2_{2,2} + t^2_{2,2}\right)\right\}$	$x^1_{4,8} = \max\left\{\left(x^1_{7,1} + \Delta t - \alpha\right), \left(x^1_{4,7} + t^1_{4,7}\right)\right\}$
$x^2_{2,4} = \max\left\{\left(x^1_{4,5} + \Delta t\right), \left(x^2_{2,3} + t^2_{2,3}\right)\right\}$	$x^2_{5,1} = \max\left\{\left(x^1_{5,7} + \Delta t - \alpha\right), \left(x^2_{5,8} + t^2_{5,8} - \alpha\right)\right\}$
$x^2_{2,5} = \max\left\{\left(x^1_{2,4} + \Delta t\right), \left(x^2_{2,4} + t^2_{2,4}\right)\right\}$	$x^2_{5,2} = \max\left\{\left(x^1_{5,8} + \Delta t - \alpha\right), \left(x^2_{5,1} + t^2_{5,1}\right)\right\}$
$x^2_{2,6} = \max\left\{\left(x^1_{2,5} + \Delta t\right), \left(x^2_{2,5} + t^2_{2,5}\right)\right\}$	$x^2_{5,3} = \max\left\{\left(x^1_{5,1} + \Delta t\right), \left(x^2_{5,2} + t^2_{5,2}\right)\right\}$
$x^2_{2,7} = \max\left\{\left(x^1_{2,1} + \Delta t\right), \left(x^2_{2,6} + t^2_{2,6}\right)\right\}$	$x^2_{5,4} = \max\left\{\left(x^1_{5,2} + \Delta t\right), \left(x^2_{5,3} + t^2_{5,3}\right)\right\}$
$x^2_{2,8} = \max\left\{\left(x^1_{1,1} + \Delta t - \alpha\right), \left(x^2_{2,7} + t^2_{2,7}\right)\right\}$	$x^2_{5,5} = \max\left\{\left(x^1_{5,3} + \Delta t\right), \left(x^2_{5,4} + t^2_{5,4}\right)\right\}$
$x^3_{2,1} = \max\left\{\left(x^2_{2,8} + \Delta t - \alpha\right), \left(x^3_{2,8} + t^3_{2,8} - \alpha\right)\right\}$	$x^2_{5,6} = \max\left\{\left(x^1_{5,4} + \Delta t\right), \left(x^2_{5,5} + t^2_{5,5}\right)\right\}$
$x^3_{2,2} = \max\left\{\left(x^2_{2,1} + \Delta t\right), \left(x^3_{2,1} + t^3_{2,1}\right)\right\}$	$x^2_{5,7} = \max\left\{\left(x^1_{5,5} + \Delta t\right), \left(x^2_{5,6} + t^2_{5,6}\right)\right\}$
$x^3_{2,3} = \max\left\{\left(x^2_{2,2} + \Delta t\right), \left(x^3_{2,2} + t^3_{2,2}\right)\right\}$	$x^2_{5,8} = \max\left\{\left(x^1_{5,6} + \Delta t\right), \left(x^2_{5,7} + t^2_{5,7}\right)\right\}$
$x^3_{2,4} = \max\left\{\left(x^2_{2,3} + \Delta t\right), \left(x^3_{2,3} + t^3_{2,3}\right)\right\}$	$x^1_{6,1} = \max\left\{\left(x^2_{6,3} + \Delta t - \alpha\right), \left(x^1_{6,5} + t^1_{6,5} - \alpha\right)\right\}$
$x^3_{2,5} = \max\left\{\left(x^2_{2,4} + \Delta t\right), \left(x^3_{2,4} + t^3_{2,4}\right)\right\}$	$x^1_{6,2} = \max\left\{\left(x^2_{5,1} + \Delta t\right), \left(x^1_{6,1} + t^1_{6,1}\right)\right\}$
$x^3_{2,6} = \max\left\{\left(x^2_{2,5} + \Delta t\right), \left(x^3_{2,5} + t^3_{2,5}\right)\right\}$	$x^1_{6,3} = \max\left\{\left(x^2_{6,5} + \Delta t - \alpha\right), \left(x^1_{6,2} + t^1_{6,2}\right)\right\}$
$x^3_{2,7} = \max\left\{\left(x^2_{2,6} + \Delta t\right), \left(x^3_{2,6} + t^3_{2,6}\right)\right\}$	$x^1_{6,4} = \max\left\{\left(x^2_{4,5} + \Delta t\right), \left(x^1_{6,3} + t^1_{6,3}\right)\right\}$
$x^3_{2,8} = \max\left\{\left(x^2_{2,7} + \Delta t\right), \left(x^3_{2,7} + t^3_{2,7}\right)\right\}$	$x^1_{6,5} = \max\left\{\left(x^2_{6,2} + \Delta t\right), \left(x^1_{6,4} + t^1_{6,4}\right)\right\}$
$x^2_{4,1} = \max\left\{\left(x^1_{4,8} + \Delta t - \alpha\right), \left(x^2_{4,8} + t^2_{4,8} - \alpha\right)\right\}$	$x^2_{6,1} = \max\left\{\left(x^2_{6,3} + \Delta t\right), \left(x^2_{6,5} + t^2_{6,5} - \alpha\right)\right\}$
$x^2_{4,2} = \max\left\{\left(x^1_{4,1} + \Delta t\right), \left(x^2_{4,1} + t^2_{4,1}\right)\right\}$	$x^2_{6,2} = \max\left\{\left(x^1_{6,2} + \Delta t\right), \left(x^2_{6,1} + t^2_{6,1}\right)\right\}$
$x^2_{4,3} = \max\left\{\left(x^1_{4,2} + \Delta t\right), \left(x^2_{4,2} + t^2_{4,2}\right)\right\}$	$x^2_{6,3} = \max\left\{\left(x^1_{8,4} + \Delta t\right), \left(x^2_{6,2} + t^2_{6,2}\right)\right\}$
$x^2_{4,4} = \max\left\{\left(x^1_{4,3} + \Delta t\right), \left(x^2_{4,3} + t^2_{4,3}\right)\right\}$	$x^2_{6,4} = \max\left\{\left(x^1_{6,4} + \Delta t\right), \left(x^2_{6,3} + t^2_{6,3}\right)\right\}$
$x^2_{4,5} = \max\left\{\left(x^1_{4,4} + \Delta t\right), \left(x^2_{4,4} + t^2_{4,4}\right)\right\}$	$x^2_{6,5} = \max\left\{\left(x^1_{7,3} + \Delta t\right), \left(x^2_{6,4} + t^2_{6,4}\right)\right\}$
$x^2_{4,6} = \max\left\{\left(x^3_{2,7} + \Delta t\right), \left(x^2_{4,5} + t^2_{4,5}\right)\right\}$	$x^1_{7,1} = x^1_{7,4} + t^1_{7,4} - \alpha$
$x^2_{4,7} = \max\left\{\left(x^1_{4,6} + \Delta t\right), \left(x^2_{4,6} + t^2_{4,6}\right)\right\}$	$x^1_{7,2} = \max\left\{\left(x^1_{6,5} + \Delta t\right), \left(x^1_{7,1} + t^1_{7,1}\right)\right\}$
$x^2_{4,8} = \max\left\{\left(x^1_{4,7} + \Delta t\right), \left(x^2_{4,7} + t^2_{4,7}\right)\right\}$	$x^1_{7,3} = \max\left\{\left(x^2_{4,4} + \Delta t\right), \left(x^1_{7,2} + t^1_{7,2}\right)\right\}$
$x^1_{5,1} = \max\left\{\left(x^2_{5,5} + \Delta t - \alpha\right), \left(x^1_{5,8} + t^1_{5,8} - \alpha\right)\right\}$	$x^1_{7,4} = \max\left\{\left(x^2_{4,3} + \Delta t\right), \left(x^1_{7,3} + t^1_{7,3}\right)\right\}$
$x^1_{5,2} = \max\left\{\left(x^2_{5,6} + \Delta t - \alpha\right), \left(x^1_{5,1} + t^1_{5,1}\right)\right\}$	$x^1_{8,1} = \max\left\{\left(x^2_{5,3} + \Delta t\right), \left(x^1_{8,4} + t^1_{8,4} - \alpha\right)\right\}$
$x^1_{5,3} = \max\left\{\left(x^3_{2,5} + \Delta t\right), \left(x^1_{5,2} + t^1_{5,2}\right)\right\}$	$x^1_{5,4} = \max\left\{\left(x^2_{5,8} + \Delta t - \alpha\right), \left(x^1_{5,3} + t^1_{5,3}\right)\right\}$

In the context of such defined operations precedence digraphs, the following theorem occurs:

Theorem. If an acyclic digraph of operations precedence G is observed in the system of SC structure, then exits the cyclic schedule X.

Proof. According to (9), for any arch vertexes $\left(v_{i,j}^{h}(k), v_{a,b}^{c}(k)\right) \in \mathbb{E}_G$ of digraph G, the following relation is satisfied: $x_{i,j}^{h}(k) < x_{a,b}^{c}(k)$ (vertex $v_{a,b}^{c}(k)$ is located higher than $v_{i,j}^{h}(k)$). Let digraph G to be cyclic, that means an existence of closed pathway: $\left(v_{i,j}^{h}(k), v_{a,b}^{c}(k), \ldots, v_{i,j}^{h}(k)\right)$. Relations related to this pathway accept the form: $x_{i,j}^{h}(k) < x_{a,b}^{c}(k) < \ldots < x_{i,j}^{h}(k)$,which leads to the contradiction: $x_{i,j}^{h}(k) < x_{i,j}^{h}(k)$ That contradictions means that cyclic schedule X cannot exist (constraints (9) are not satisfied), and thus digraph vertexes cannot be placed according to the established order (vertex $v_{i,j}^{h}(k)$ should be located above itself). In case when digraph G is acyclic, the closed pathways are not observed, and thus relations of $x_{i,j}^{h}(k) < x_{i,j}^{h}(k)$ type do not occur. Thus, all vertexes are placed according to (9) which means an existence of cyclic schedule X.

It may be concluded from above theorem, that an existence of schedule X requires that corresponding to it precedence digraph G must be acyclic.

The question related to the cyclicality of operations precedence digraph G seems to be natural in the context of the introduced Theorem. It was demonstrated in the Theorem evidence that the consequence of the cyclicality operations precedence digraph G is a contradictions in constraints of type (9). In other words, the digraph is acyclic, when the constraints (9) are not contrary.

Thus, the constraints (9) may be treated as the conditions which satisfaction guarantees the cyclic realization of local processes. In that context, the constraints $C'_X(SM, mX)$ of the problem (8) involve the following set of constraints:

$$C_I = \{x_{i,j}^{h} + k \cdot \alpha = \max\left\{\left(x_{i,(j-1)}^{h} + k' \cdot \alpha\right) + t_{i,(j-1)}\,; \left(x_{a,b}^{c} + k'' \cdot \alpha\right) + \Delta t\right\} \mid \left(v_{i,(j-1)}^{h}(k'), v_{i,j}^{h}(k)\right) \in \mathbb{E}_G; \left(v_{a,b}^{c}(k''), v_{i,j}^{h}(k)\right) \in \mathbb{E}_G; i = 1, \ldots, ln;\ j = 1, \ldots, lr(i); h = 1, \ldots, ls(i);\ k, k', k'' \in \mathbb{C}\}. \tag{10}$$

The above constraints may be simplified taking into account that: $x_{i,j}^{h}(k) = x_{i,j}^{h} + k \cdot \alpha$, $k \in \mathbb{C}$. Set of constraints C_I for the system from Fig. 4 is illustrated in Table 5.

Table 6 presents in turn the constraints C_{II}, guaranteeing realization of local processes in accordance with established realization of multimodal processes. It was accepted within the constraints, that the moment of beginning of operation $x_{i,j}^{h}$ of stream P_i^h of a local process (vehicle) must have the same value like the moment of the beginning of operation $mx_{a,b}^{c}$ of a multimodal process requiring stream P_i^h for its realization. In other words, the moment of vehicle work beginning (unload/transport operations) at a given resource overlaps with the moment of beginning of suitable operation of a given element production process. For example: $x_{1,2}^{1} = mx_{1,1}^{h} + k \cdot m\alpha$ means that the second operation of the stream P_1^1 (realized on resource R_3) starts together with operation $mo_{1,1}^{h}$ of h-th stream of multimodal process mP_1. Moreover, cyclic realization of the operations of local and multimodal processes requires satisfaction of the constraint of mutual multiplicity of periods $m\alpha$, α: $mod(m\alpha, \alpha) = 0$.

Introduced constraints of C_I and C_{II} type compose the set of constraints $C'_X(SM, mX)$ of the problem (8). The following section presents the results of the experiment of problem (8) solving for the sample SCMCP.

Table 6
Constraints determining admissible values of the moments of local SCMCP processes beginning from Fig. 5

Constraints C_{II}		
mP_1^h	mP_2^h	mP_3^h
$x_{1,2}^{1} = mx_{1,1}^{h} + k \cdot m\alpha$	$x_{5,1}^{1} = mx_{2,1}^{h} + k \cdot m\alpha$	$x_{8,1}^{1} = mx_{3,1}^{h} + k \cdot m\alpha$
$x_{1,3}^{1} = mx_{1,2}^{h} + k \cdot m\alpha$	$x_{5,2}^{1} = mx_{2,2}^{h} + k \cdot m\alpha$	$x_{8,2}^{1} = mx_{3,2}^{h} + k \cdot m\alpha$
$x_{1,4}^{1} = mx_{1,3}^{h} + k \cdot m\alpha$	$x_{5,3}^{1} = mx_{2,3}^{h} + k \cdot m\alpha$	$x_{8,3}^{1} = mx_{3,3}^{h} + k \cdot m\alpha$
$x_{2,7}^{1} = mx_{1,4}^{h} + k \cdot m\alpha$	$x_{5,7}^{2} = mx_{2,4}^{h} + k \cdot m\alpha$	$x_{6,4}^{2} = mx_{3,4}^{h} + k \cdot m\alpha$
$x_{2,8}^{1} = mx_{1,5}^{h} + k \cdot m\alpha$	$x_{5,8}^{2} = mx_{2,5}^{h} + k \cdot m\alpha$	$x_{6,5}^{2} = mx_{3,5}^{h} + k \cdot m\alpha$
$x_{2,5}^{3} = mx_{1,6}^{h} + k \cdot m\alpha$	$x_{6,3}^{1} = mx_{2,6}^{h} + k \cdot m\alpha$	$x_{4,3}^{1} = mx_{4,3}^{h} + k \cdot m\alpha$
$x_{2,6}^{3} = mx_{1,7}^{h} + k \cdot m\alpha$	$x_{6,4}^{1} = mx_{2,7}^{h} + k \cdot m\alpha$	$x_{4,4}^{1} = mx_{3,7}^{h} + k \cdot m\alpha$
$x_{4,7}^{2} = mx_{1,8}^{h} + k \cdot m\alpha$	$x_{7,3}^{1} = mx_{2,8}^{h} + k \cdot m\alpha$	$x_{2,5}^{2} = mx_{3,8}^{h} + k \cdot m\alpha$
$x_{4,8}^{2} = mx_{1,9}^{h} + k \cdot m\alpha$	$x_{7,4}^{1} = mx_{2,9}^{h} + k \cdot m\alpha$	$x_{2,6}^{2} = mx_{3,9}^{h} + k \cdot m\alpha$
$k \in C, \quad h = 1 \ldots 4$	$k \in C, \quad h = 1 \ldots 4$	$x_{3,3}^{1} = mx_{3,10}^{h} + k \cdot m\alpha$
$mod(m\alpha, \alpha) = 0, k \in C, \quad h = 1 \ldots 4$		$x_{3,4}^{1} = mx_{3,11}^{h} + k \cdot m\alpha$

6. Computational experiment

In this section an effectiveness of constraints programming environment for solving the CSP (e.g. PS_R) aimed at SCM-CP synthesis problem is emphasized. The considered problem PS_R (8) (where set of constraints contained constraints from Table 5 and 6), implemented in the constraints programming environment with OzMozart (Intel Core Duo2 3.00 GHz, 4 GB RAM), was solved over a time not exceeding 7 s. The determined values of the times of operation T execution (collected in a form of sequence T_i^h) of local processes (vehicles), guaranteeing performance of production plan from Fig. 6, are as follows:

$$T = \{T_1^1, T_2^1, T_2^2, T_2^3, T_3^1, T_4^1, T_4^2, T_5^1, T_5^2, T_6^1, T_6^2, T_7^1, T_8^1\},$$

$$T_1^1 = (1, 76, 1, 2), \quad T_2^1 = (1, 1, 1, 1, 1, 18, 1, 4),$$

$$T_2^1 = (53, 1, 1, 1, 1, 1, 1, 1), \quad T_2^3 = (1, 1, 1, 3, 1, 1, 1, 1),$$

$$T_3^1 = (1, 77, 1, 1), \quad T_4^1 = (1, 4, 1, 1, 1, 1, 1, 1),$$

$$T_4^2 = (1, 1, 1, 1, 71, 1, 1, 1), \quad T_5^1 = (61, 1, 1, 1, 1, 1, 1, 1),$$

$$T_5^2 = (1, 1, 1, 1, 8, 1, 1, 1), \quad T_6^1 = (13, 1, 1, 1, 64),$$

$$T_6^2 = (1, 51, 1, 1, 1), \quad T_7^1 = (44, 1, 1, 34),$$

$$T_8^1 = (1, 1, 1, 77),$$

$T_i^h = (t_{i,1}^h, \ldots, t_{i,j}^h, \ldots, t_{i,lr(i)}^h)$ – sequence of the times of execution of operation of stream P_i^h, $t_{i,j}^h$ – time of execution of j-th operation of h-th stream of process P_i.

The obtained values of moments $x_{i,j}^h$ (sequence X'_{DC}) guaranteeing cyclic execution of local processes are presented in Table 7.

Table 7

Moments of beginning of operation X of local processes guaranteeing execution of the schedule from Fig. 6

No.	$x_i^h = (x_{i,1}^h, \ldots, x_{i,j}^h, \ldots, x_{i,lr(i)}^h)$
1	$x_1^1 = (-10, -9, 67, 68)$
2	$x_2^1 = (-10, -8, -7, -6, -5, 47, 65, 66)$
3	$x_2^1 = (-8, 45, 46, 47, 48, 49, 50, 71)$
4	$x_2^3 = (-8, -7, 46, 47, 50, 51, 52, 70)$
5	$x_3^1 = (-10, -9, 68, 69)$
6	$x_4^1 = (-8, -7, -3, -2, -1, 0, 1, 71)$
7	$x_4^2 = (-7, -6, -5, -2, -1, 70, 71, 72)$
8	$x_5^1 = (-9, 52, 53, 54, 57, 58, 69, 70)$
9	$x_5^2 = (-10, -9, -8, 53, 54, 62, 63, 64)$
10	$x_6^1 = (-10, 3, 4, 5, 6)$
11	$x_6^2 = (-9, 4, 55, 56, 57)$
12	$x_7^1 = (-10, 34, 35, 36)$
13	$x_8^1 = (5, 6, 7, 8)$
14	$\alpha = 80$

The schedule from Fig. 10 illustrates concurrent realization of local and multimodal processes. As can be noticed, the operations are executed in accordance to the accepted assumptions. For example, the product realized in process mP_2^3 starting its operation on resource R_{10}, requires the transport (using vehicle P_7^1) from resource R_7 at the 36^{th} time unit. It is thus required, that the vehicle P_7^1 is present at the moment of product load on the resource R_7 (last unit of operation mP_2^3 on R_7), and at the moment of unload on resource R_{10} (first unit of operation mP_2^3 on R_{10}).

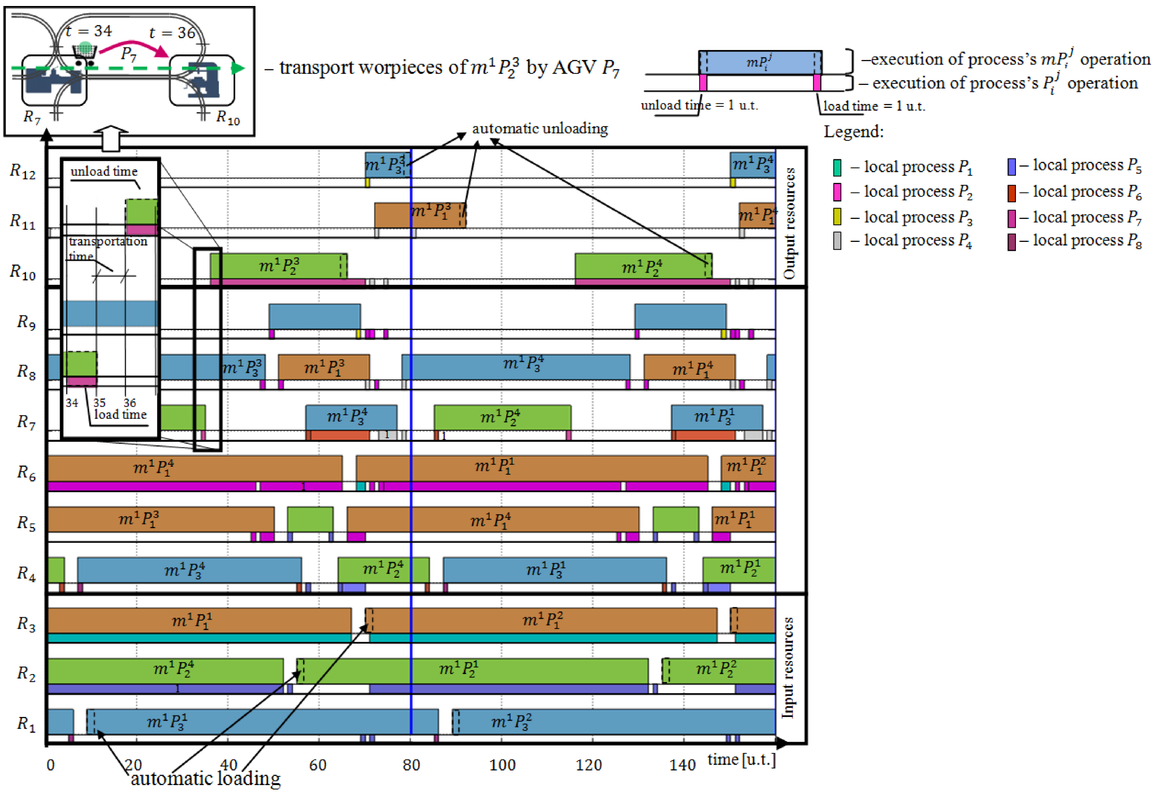

Fig. 10. Cyclic schedule of the system from Fig. 5

This means that both streams mP_2^3 and P_7^1 move concurrently between resources R_7 and R_{10} (through resource R_{29} representing sector connecting R_7 with R_{10} – see Fig. 4 and 5), which means that the product manufactured in process mP_2^3 is transported using vehicle P_7^1. The transport between other positions is realized in an analogical manner.

To sum up, the determined times of execution of operations T of local processes allow to perform the assumed production plan without delays and conflicts. Moreover, the used constraints programming environment enables to solve the real size instances of reverse problems.

7. Summary

The elaborated declarative SCMCP model distinguishes two basic elements: structure SC and cyclic schedule X' representing processes execution. In that context, the problems of analysis and synthesis searching for responses to the following questions are considered: Is there a cyclic schedule in the SCMCP of a specified structure? Is there a structure guaranteeing a cyclic schedule of the SCMCP?

These problems may be formulated in the categories of constraints satisfaction problems (CSP). Thus, it is possible to apply commercially available programming environments with constraints for their solving. The key role in the proposed CSP model is played by the constraints, satisfaction of which guarantees cyclic, disruptions-free (collision-free and deadlock-free) execution of SCMCP processes. In that context, the elaborated constraints may be treated as the sufficient conditions of reachability of cyclic schedule in the SCMCP.

The discussed example of a synthesis problem demonstrates that the proposed approach allows to obtain the solutions (evaluate an existence of SCMCP parameters assuring an existence of a cyclic schedule of local/multimodal processes) in a time not exceeding 10 s. for the systems of practically observed size.

To sum up, the proposed approach to SCMCP modeling enables:

- description of more complex concurrent processes systems (in contrast to the papers do not treating about multimodal processes [16, 18, 23–25]),
- solving decision problems (analysis/synthesis problems) instead of optimization ones,
- simplification of considered problems solving due to constraints programming techniques, i.e. CSP implementation in constraints programming environment,
- conditions (constraints) guaranteeing cyclic, disruptions-free processes execution.

In real cases of multimodal systems, a significant role is played by imprecise character of information available: times of operations execution, moments of their beginning, etc. The presented expectations determine the directions of the presented model and related method development taking into account the possibility of an inclusion of imprecise character of the decision variables. The models of Fuzzy Constraints Satisfaction Problems will be applied for that purpose [31, 32].

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