**Scientific Journals**
**of the Maritime University of Szczecin**

**Zeszyty Naukowe**
**Akademii Morskiej w Szczecinie**

**2023, 74 (146), 65–74**
**ISSN 2392-0378 (Online)**
**DOI: 10.17402/565**

*Received:* *12.09.2022*
*Accepted:* *28.02.2023*
*Published:* *30.06.2023*

# Modeling safety of port and maritime transportation systems

**Beata Magryta-Mut**

https://orcid.org/0000-0002-4189-1853

Gdynia Maritime University, Department of Mathematics
81-87 Morska St., 81-225 Gdynia, Poland
e-mail: b.magryta@wn.umg.edu.pl

**Keywords:** port transport, maritime transport, safety, operation impact, resilience, operation costs, optimization

**JEL Classification:** C60, C61, C63

**Abstract**

This study aims to improve an earlier safety analysis of port and maritime transportation systems in two cases. The first case does not consider outside impacts and the second case operates under the assumption that they are impacted by their operation processes. New and original suggestions on separate and joint system safety and operation cost optimization are also described and future research is also outlined. Probabilistic modeling methods are used as the research methods. The proposed research procedures enable the determination of the safety function and risk function for the port oil terminal critical infrastructure and the maritime ferry technical system in both examined cases, based on the strictly exact statistical data about their operation processes and on the improved approximate evaluations of their components safety parameters through expert opinion methods that originate directly from the users of these systems. Other proposed practically significant safety and resilience indicators are the mean lifetime up to the exceeding of a critical safety state, the moment when the risk function value exceeds the acceptable safety level, the intensity of ageing/degradation in both cases, the coefficient of operation process impact on system safety, and the coefficient of system resilience to operation process impact in the second case. As a result of this research, it is originally found that the proposed cost optimization procedures and the finding of the corresponding system safety indicators deliver an important possibility for the system total operation cost minimizing and keep fixed the corresponding conditional safety indicators during the operation. It was also established that the proposed system safety optimization procedures, and corresponding system operation total costs, deliver an important possibility for the system safety indicators maximization and keep fixed the corresponding system operation total costs during the operation.

## Introduction

The complex technical system, especially the critical infrastructure operating in a designated area, may be prone to damage and degradation induced by external threats. Although it might cause threats to other systems and critical infrastructures (Gouldby, et al., 2010; Lauge, Hernantes & Sarriegi, 2015). The required and practically very important safety indicators for such systems can be obtained by using an original and innovative probabilistic approach to their safety multistate modeling, while considering their operation process impact. At first, the approach can be focused on the simplest, pure system safety multistate ageing model based on the primary introduced in multistate system reliability analysis from previous research (Xue, 1985; Xue & Yang, 1995), without considering outside impacts and defining the critical infrastructure and its subsystems practically useful safety indicators. This set of safety indicators can be completed by linking the safety pure model with the model of the critical infrastructure operation process (Kołowrocki, 2014). This way of creating the joint safety model of the critical infrastructure related to its operation process can offer additional resilience indicators, which are measures of the critical infrastructure operation impact on its safety and its resilience to operation.

The critical infrastructure safety indicators improvement is of high importance in industrial practice. Hence, there is a need to find the means for searching for the critical infrastructure safety and resilience indicators and their optimal forms, and the procedures allowing for the changing of the critical infrastructure operation process by replacing the values of these indicators with their values after the critical infrastructure operation process optimization in order to improve its safety (Tang, Yin & Xi, 2007).

In the paper, after the presentation of the improved general approach to the safety of the multistate ageing system, based on the strictly exact statistical data about their operation processes and on the more exact expert opinions originating directly from the users of these systems approximate evaluations of their components safety parameters, we determine safety function and risk function for the port oil terminal and the maritime ferry technical system (Kołowrocki & Magryta-Mut, 2020; Magryta-Mut, 2023a) and other, practically significant safety and resilience indicators such as the mean lifetime up to the exceeding of a critical safety state, the moment when risk function exceeds the acceptable safety level, the intensity of ageing/degradation, the coefficient of operation process impact intensity of ageing, and the coefficient of resilience to operation process impact. At the end, the new and original general suggestions on critical infrastructure safety and operation cost optimization of complex technical systems, including critical infrastructure, are given (Magryta-Mut, 2020, 2022).

## General approach to the safety of a multistate ageing system

Similar to the case of a multistate approach to system reliability (Xue, 1985; Xue & Yang, 1995; Kołowrocki, 2014) in the multistate system safety analysis, to define safety indicators of the system with degrading/ageing components we assume that:

- $n$ is the number of system components;
- $E_i$, with $i = 1,2,...,n$, are the system components;
- all components and the system have the safety state set $\{0,1,...,z\}$, for $z \geq 1$;
- the safety states are ordered: the safety state 0 is the worst and the safety state $z$ is the best;
- $r$, where $r \in \{1,2,\ldots,z\}$, is the critical safety state (the system and its components remain in the safety states less than the critical state, i.e., in safety states $0, 1, 2, \ldots, r-1$, is highly dangerous for them and for their operating environment);
- $T_i(u)$, for $i = 1,2,...,n$, are random variables representing the lifetimes of components $E_i$ in the safety state subset $\{u,u+1,...,z\}$, where $u = 0,1,2,...,z$, while they were in the safety state $z$ at the moment $t = 0$;
- $T(u)$ is a random variable representing the lifetime of the system in the safety state subset $\{u,u+1,...,z\}$, where $u = 0,1,2,...,z$, while it was in the safety state $z$ at the moment $t = 0$;
- the components and the system safety states degrade with time $t$;
- $s_i(t)$ is the component $E_i$, where $i = 1,2,...,n$, safety state at the moment $t$, with $t \in \langle 0,\infty)$, while it was in the safety state $z$ at the moment $t = 0$;
- $s(t)$ is the system safety state at the moment $t$, for $t \in \langle 0,\infty)$, given that it was in the safety state $z$ at the moment $t = 0$.

The above assumptions mean that the safety states of the system with degrading components may be changed in time only from better to worse.

We define the first basic safety indicator, i.e., the system safety function via the vector (Magryta-Mut, 2023a):

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2),\ldots, \boldsymbol{S}(t,z)],\ \ t \in \langle 0,\infty) \quad (1)$$

where

$$\boldsymbol{S}(t,u) = P(s(t) \geq u \mid s(0) = z) = P(T(u) > t) \\ t \in \langle 0,\infty),\ \ u = 1,2,\ldots,z \quad (2)$$

is the probability that the multistate system is in the safety state subset $\{u,u+1,\ldots,z\}$, where $u = 1,2,...,z$, at the moment $t$, for $t \in \langle 0,\infty)$, while it was in the safety state $z$ at the moment $t = 0$.

We do not consider in vector (1) the function $\boldsymbol{S}(t,0)$ as:

$$\boldsymbol{S}(t,0) = P(s(t) \geq 0 \mid s(0) = z) = P(T(0) > t) = 1 \\ \text{for } t \in \langle 0,\infty),$$

which means that it is constant.

The safety functions $\boldsymbol{S}(t,u)$ for $t \in \langle 0,\infty)$, where $u = 1,2,\ldots z$, defined by equation (2), are called the coordinates of the system safety function $\boldsymbol{S}(t,\cdot)$, for $t \in \langle 0,\infty)$, given by equation (1). Thus, the relationship between the distribution function $\boldsymbol{F}(t,u)$ of the system lifetime $T(u)$, where $u = 1,2,\ldots,z$, in the safety state subset $\{u,u+1,\ldots,z\}$, in which $u = 1,2,\ldots,z$, and the coordinate $\boldsymbol{S}(t,u)$, for $t \in \langle 0,\infty)$ and $u = 1,2,\ldots z$, of its safety function is given by:

$$\boldsymbol{F}(t,u) = P(T(u) \leq t) = 1 - P(T(u) > t) = 1 - \boldsymbol{S}(t,u) \\ t \in \langle 0,\infty),\ u = 1,2,\ldots z.$$

The exemplary graph of a four-state ($z = 3$) system safety function is given by:

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3)],\ \ t \in \langle 0,\infty)$$

and is shown in Figure 1.

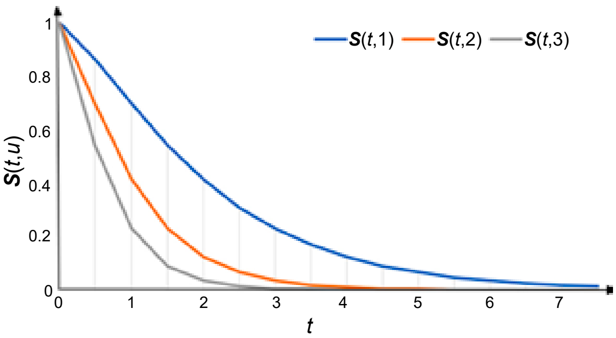


**Figure 1. Graphs of a four-state system safety function coordinates**

The second basic safety indicator, i.e., the multistate system risk function, is written as:

$$\boldsymbol{r}(t) = P(s(t)) < r \mid s(0) = z = P(T(r) \le t)$$
$$t \in \langle 0,\infty) \qquad (3)$$

which is defined as the probability that the system is in the subset of safety states worse than the critical safety state $r$, for $r \in \{1,2,\dots,z\}$. While in the best safety state $z$ at the moment $t = 0$, it is given by (Magryta-Mut, 2023a):

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,r),\ \ t \in \langle 0,\infty) \qquad (4)$$

where $\boldsymbol{S}(t,r)$ is the coordinate of the multistate system safety function (1) given by equation (2) for $u = r$. The graph of the exemplary system risk function is presented in Figure 2.

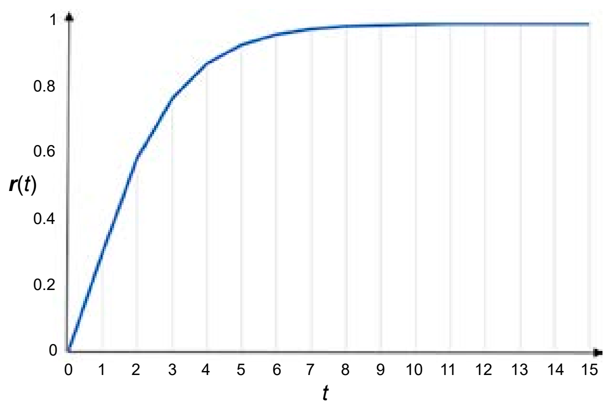


**Figure 2. Graph of the exemplary system risk function**

The moment, $\tau$, when the system risk function exceeds a permitted level $\delta$, for $\delta \in (0,1)$, is defined by:

$$\tau = \boldsymbol{r}^{-1}(\delta) \qquad (5)$$

where $\boldsymbol{r}^{-1}(t)$, for $t \in \langle 0,\infty)$, is the inverse function of the risk function $\boldsymbol{r}(t)$, for $t \in \langle 0,\infty)$, given by equation (4). The intensities of ageing of a multistate ageing system, i.e., the intensities of this system departure from the safety state subsets $\{u,u+1,\dots,z\}$, where $u = 1,2,\dots,z$, are defined by:

$$\boldsymbol{\lambda}(t,u) = \frac{-\dfrac{d\boldsymbol{S}(t,u)}{dt}}{\boldsymbol{S}(t,u)}$$
$$t \in \langle 0,\infty),\ u = 1,2,\dots,z \qquad (6)$$

where $\boldsymbol{S}(t,u)$, in which $u = 1,2,\dots,z$, are the coordinates of this system safety function (1) given by equation (2). The multistate ageing system approximate mean intensities of ageing are defined by:

$$\boldsymbol{\lambda}(u) = \frac{1}{\boldsymbol{\mu}(u)},\ \ u = 1,2,\dots,z \qquad (7)$$

where $\boldsymbol{\mu}(u)$, in which $u = 1,2,\dots,z$, are the mean values of this system lifetimes in the safety state subsets $\{u,u+1,\dots,z\}$, for $u = 1,2,\dots,z$.

The coefficients of the outside impact on the multistate ageing system safety are defined by:

$$\boldsymbol{\rho}(t,u) = \frac{\boldsymbol{\lambda}(u)}{\boldsymbol{\lambda}^0(u)},\ \ u = 1,2,\dots,z \qquad (8)$$

where $\boldsymbol{\lambda}(u)$ and $\boldsymbol{\lambda}^0(u)$, with $u = 1,2,\dots,z$, are the intensities of ageing of this system with and without impacts, respectively, determined according to equation (6) or (7). Finally, we define the multistate ageing system resilience indicators, i.e., the coefficients of the system resilience to the outside impact, with the following:

$$\boldsymbol{RI}(t,u) = \frac{1}{\boldsymbol{\rho}(t,u)},\ \ u = 1,2,\dots,z \qquad (9)$$

where $\boldsymbol{\rho}(t,u)$, with $u = 1,2,\dots,z$, are the coefficients of the outside impact on this system safety, which is given by equation (8).

## Safety of port and maritime transportation systems without considering outside impacts

The general approach to system safety analysis (introduced in the previous section) is applied to the safety evaluation of a port oil terminal and a maritime ferry technical system without considering the outside impacts.

### Safety of port oil terminal critical infrastructure

We now consider the port oil terminal critical infrastructure positioned at the Baltic seaside, which is designated for receiving oil products from ships, storing them, and sending them by carriages and trucks to the recipients inland (Kołowrocki & Magryta-Mut, 2020; Magryta-Mut, 2023a).

In previous research (Magryta-Mut, 2023a), it was assumed that the port oil terminal critical infrastructure and its components have the following three safety states:

- a safety state 2 – the operation of component and the port oil terminal is fully safe,
- a safety state 1 – the operation of component and the port oil terminal is less safe and more dangerous because of the possibility of environment pollution,
- a safety state 0 – the component and the port oil terminal are destroyed.

Under this assumption, using a date originating from experts and the safety model considered in the previous section, it can be fixed that the port oil terminal safety function is given by the vector (Magryta-Mut, 2013a):

$$\boldsymbol{S}^0(t,\cdot) = [\boldsymbol{S}^0(t,1), \boldsymbol{S}^0(t,2)],\ \ t \in \langle 0,\infty) \qquad (10)$$

with coordinates:

$$\boldsymbol{S}^0(t,1) = \exp[-0.015873t]\exp[-0.0125t] \exp[-0.0125t]\exp[-0.0125t]\exp[-0.0125t] \exp[-0.0125t]\exp[-0.0125t]\exp[-0.0125t] \exp[-0.0125t] = \exp[-0.115873t]$$

$$\boldsymbol{S}^0(t,2) = \exp[-0.021739t]\exp[-0.02t] \exp[-0.02t]\exp[-0.02t]\exp[-0.02t] \exp[-0.02t]\exp[-0.02t]\exp[-0.02t] \exp[-0.02t] = \exp[-0.18739t] \qquad (11)$$

The graph of this three-state port oil terminal system safety function is presented in Figure 3.

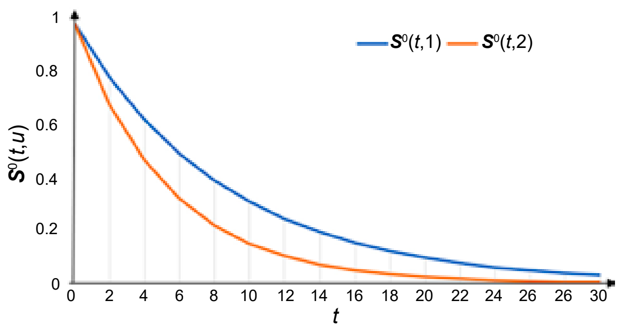


**Figure 3. Graph of the port oil terminal system safety function coordinates**

The expected values and the standard deviations of the terminal system lifetimes in the safety state subsets {1, 2} and {2}, expressed in years, are respectively:

$$\boldsymbol{\mu}^0(1) \cong 8.63,\ \ \boldsymbol{\mu}^0(2) \cong 5.50$$
$$\boldsymbol{\sigma}^0(1) \cong 8.63,\ \ \boldsymbol{\sigma}^0(2) \cong 5.50 \qquad (12)$$

and the mean values of the lifetimes in the particular safety states 1 and 2 are respectively:

$$\bar{\boldsymbol{\mu}}^0(1) \cong 3.13,\ \ \bar{\boldsymbol{\mu}}^0(2) \cong 5.50 \text{ years} \qquad (13)$$

Assuming that the critical safety state is $r = 1$, the system risk function, according to equations (4) and (11), is given by:

$$\boldsymbol{r}^0(t) = 1 - \boldsymbol{S}^0(t,1) = 1 - \exp[-0.115873t] \quad \text{for } t \geq 0 \qquad (14)$$

Hence, from equation (5), the moment when the system risk function exceeds a permitted level $\delta = 0.05$ is given by:

$$\boldsymbol{\tau}^0 = \boldsymbol{r}^{0^{-1}}(t) \cong 0.44 \text{ year} \qquad (15)$$

The graph of the port oil terminal system risk function $\boldsymbol{r}^0(t)$ is presented in Figure 4.

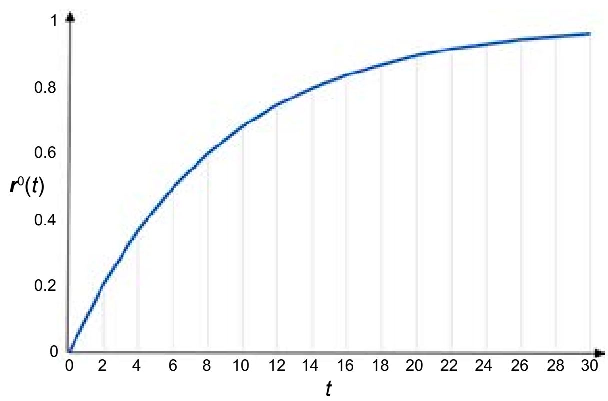


**Figure 4. Graph of the port oil terminal system risk function**

The port oil terminal approximate mean intensities of ageing, after considering equations (7) and (12), are:

$$\lambda^0(1) \cong 0.115873,\ \ \lambda^0(2) \cong 0.181739 \qquad (16)$$

### Safety of maritime ferry technical system

The considered maritime ferry is a passenger ship operating in the Baltic Sea between Gdynia and Karlskrona ports on a regular everyday line (Kołowrocki & Magryta-Mut, 2020; Magryta-Mut, 2023a). In earlier work (Magryta-Mut, 2023a), it was assumed that the maritime technical system and its components have the following five safety states:

- a safety state 4 – the component and the ferry operation are completely safe,
- a safety state 3 – the component and the ferry operation are less safe and more dangerous because of the possibility of environment pollution,
- a safety state 2 – the component and the ferry operation are less safe and more dangerous because of the possibility of environment pollution and causing small accidents,
- a safety state 1 – the component and the ferry operation are much less safe and much more dangerous because of the possibility of serious environment pollution and causing extensive accidents,
- a safety state 0 – the component and the ferry technical system are destroyed.

Under this assumption, using a date originating from experts and the safety model considered in the previous section, it was fixed that the maritime technical system safety function is given by the vector:

$$\boldsymbol{S}^0(t,\cdot) = [\boldsymbol{S}^0(t,1), \boldsymbol{S}^0(t,2), \boldsymbol{S}^0(t,3), \boldsymbol{S}^0(t,4)] \quad t \in \langle 0,\infty) \tag{17}$$

with coordinates:

$$\boldsymbol{S}^0(t,1) = \exp[-0.033t]\,[12\exp[-0.33t] + 8\exp[-0.429t] - 16\exp[-0.363t] - 3\exp[-0.462t]]\cdot \exp[-0.139t]\exp[-0.083t]\exp[-0.099t] = 12\exp[-0.684t] + 8\exp[-0.783t] + - 16\exp[-0.717t] - 3\exp[-0.816t]$$

$$\boldsymbol{S}^0(t,2) = \exp[-0.040t]\,[12\exp[-0.38t] + 8\exp[-0.49t] + 6\exp[-0.46t] - 16\exp[-0.42t] + - 6\exp[-0.45t] - 3\exp[-0.53t]]\cdot \exp[-0.175t]\exp[-0.100t]\exp[-0.12t] = 12\exp[-0.815t] + 8\exp[-0.925t] + 6\exp[-0.895t] + - 16\exp[-0.855t] - 6\exp[-0.885t] - 3\exp[-0.965t]$$

$$\boldsymbol{S}^0(t,3) = \exp[-0.045t]\,[12\exp[-0.43t] + 8\exp[-0.555t] + 6\exp[-0.53t] - 16\exp[-0.48t] + - 6\exp[-0.505t] - 3\exp[-0.605t]]\cdot \exp[-0.200t]\exp[-0.110t]\exp[-0.145t] = 12\exp[-0.930t] + 8\exp[-1.055t] + 6\exp[-1.030t] + - 16\exp[-0.980t] - 6\exp[-1.005t] - 3\exp[-1.105t]$$

$$\boldsymbol{S}^0(t,4) = \exp[-0.05t]\,[12\exp[-0.47t] + 8\exp[-0.605t] + 6\exp[-0.58t] - 16\exp[-0.525t] + - 6\exp[-0.55t] - 3\exp[-0.66t]]\cdot \exp[-0.230t]\exp[-0.120t]\exp[-0.165t] = 12\exp[-1.035t] + 8\exp[-1.170t] + 6\exp[-1.145t] + - 16\exp[-1.090t] - 6\exp[-1.115t] - 3\exp[-1.225t] \tag{18}$$

The graph of this five-state ferry technical system safety function is shown in Figure 5.

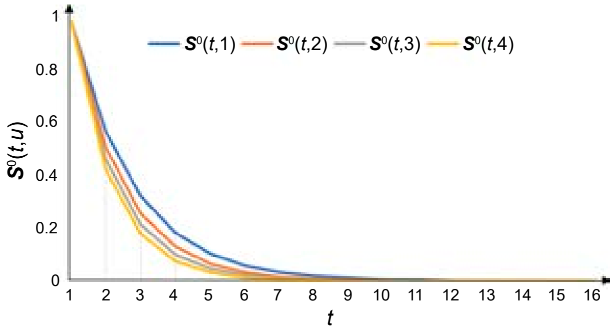


**Figure 5. Graph of the ferry technical system safety function coordinates**

The expected values and the standard deviations of the ferry lifetimes in the safety state subsets {1, 2, 3, 4}, {2, 3, 4}, {3, 4}, and {4}, expressed in years, are respectively:

$$\boldsymbol{\mu}^0(1) \cong 1.770,\ \boldsymbol{\mu}^0(2) \cong 1.476$$
$$\boldsymbol{\mu}^0(3) \cong 1.300,\ \boldsymbol{\mu}^0(4) \cong 1.164$$
$$\boldsymbol{\sigma}^0(1) \cong 1.733,\ \boldsymbol{\sigma}^0(2) \cong 1.447$$
$$\boldsymbol{\sigma}^0(3) \cong 1.277,\ \boldsymbol{\sigma}^0(4) \cong 1.144 \tag{19}$$

and further, the unconditional lifetimes in the particular safety states 1, 2, 3, and 4, respectively, are:

$$\bar{\boldsymbol{\mu}}^0(1) \cong 0.294,\ \bar{\boldsymbol{\mu}}^0(2) \cong 0.176$$
$$\bar{\boldsymbol{\mu}}^0(3) \cong 0.136,\ \bar{\boldsymbol{\mu}}^0(4) \cong 1.164 \text{ years} \tag{20}$$

Under the assumption that the critical safety state is $r = 2$, the system risk function, according to equations (4) and (18), is given by:

$$\boldsymbol{r}^0(t) = 1 - \boldsymbol{S}^0(t,2) = 1 - 12\exp[-0.815t] + 8\exp[-0.925t] + 6\exp[-0.895t] - 16\exp[-0.855t] + - 6\exp[-0.885t] - 3\exp[-0.965t], \text{ for } t \geq 0 \tag{21}$$

Hence, from equation (5), the moment when the system risk function exceeds a permitted level (for instance, $\delta = 0.05$) is found as:

$$\boldsymbol{\tau}^0 = \boldsymbol{r}^{0^{-1}}(t) \cong 0.077 \text{ year} \tag{22}$$

The graph of the ferry technical system risk function, $\boldsymbol{r}^0(t)$, is presented in Figure 6.

The ferry technical system approximate mean intensities of ageing, after considering equations (7) and (19), are:

$$\lambda^0(1) \cong 0.564972,\ \lambda^0(2) \cong 0.677507$$
$$\lambda^0(3) \cong 0.769231,\ \lambda^0(4) \cong 0.859107 \tag{23}$$

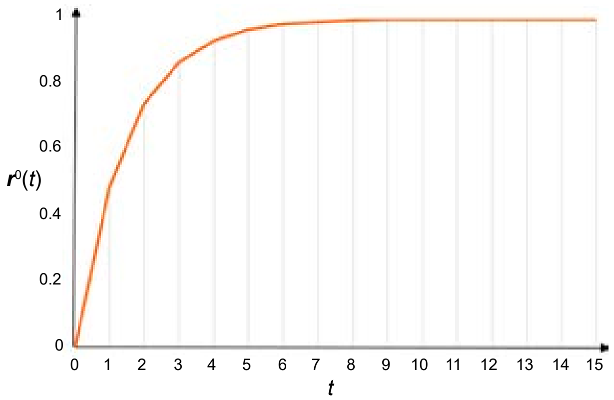


**Figure 6. Graph of the ferry technical system risk function**

## Safety of port oil terminal and maritime technical system impacted by their operation processes

The general approach to system safety analysis introduced in the section *General approach…* is modified by assuming that the system safety is related to its operation process having ν, in which $v > 1$. While changing in time operation states $z_b$, where $b = 1,2,\dots,v$, affects the changes of the system functional and safety structures and the system components' safety parameters. After considering the limit transient probabilities $p_b$, where $b = 1,2,\dots,v$, and the system conditional safety functions at particular operation states, the system safety is given by vector (1) with the modified coordinates in the form:

$$\boldsymbol{S}(t,u) \cong \sum_{b=1}^{v} p_b [\boldsymbol{S}(t,u)]^{(b)}$$

$$t \in \langle 0,\infty), \ u = 1,2,\dots z \qquad (24)$$

### Safety of port oil terminal critical infrastructure

The limit transient probabilities of the port oil terminal operation process at seven particular operation states $z_b$, where $b = 1,2,\dots,7$, which is fixed in previous research (Magryta-Mut, 2023a), are:

$$p_1 = 0.395, p_2 = 0.060, p_3 = 0.003, p_4 = 0.002$$
$$p_5 = 0.20, p_6 = 0.058, p_7 = 0.282.$$

Hence, from equations (1), (10), and (24), the port oil terminal system safety function is given by the vector:

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2)], \ t \in \langle 0,\infty) \qquad (25)$$

with coordinates:

$$\boldsymbol{S}(t,1) = 0.395\,[\boldsymbol{S}(t,1)]^{(1)} + 0.060\,[\boldsymbol{S}(t,1)]^{(2)} + 0.003\,[\boldsymbol{S}(t,1)]^{(3)} + 0.002\,[\boldsymbol{S}(t,1)]^{(4)} + 0.2\,[\boldsymbol{S}(t,1)]^{(5)} + 0.058\,[\boldsymbol{S}(t,1)]^{(6)} + 0.282\,[\boldsymbol{S}(t,1)]^{(7)}, \ t \in \langle 0,\infty)$$

$$\boldsymbol{S}(t,2) = 0.395\,[\boldsymbol{S}(t,2)]^{(1)} + 0.060\,[\boldsymbol{S}(t,2)]^{(2)} + 0.003\,[\boldsymbol{S}(t,2)]^{(3)} + 0.002\,[\boldsymbol{S}(t,2)]^{(4)} + 0.2\,[\boldsymbol{S}(t,2)]^{(5)} + 0.058\,[\boldsymbol{S}(t,2)]^{(6)} + 0.282\,[\boldsymbol{S}(t,2)]^{(7)}, \ t \in \langle 0,\infty) \qquad (26)$$

where the conditional coordinates $[\boldsymbol{S}(t,u)]^{(b)}$ with $t \in \langle 0,\infty)$, in which $u = 1,2,\dots,z$ and $b = 1,2,\dots,7$, are determined in earlier work (Magryta-Mut, 2023a). The graph of this three-state port oil terminal system safety function is presented in Figure 7.

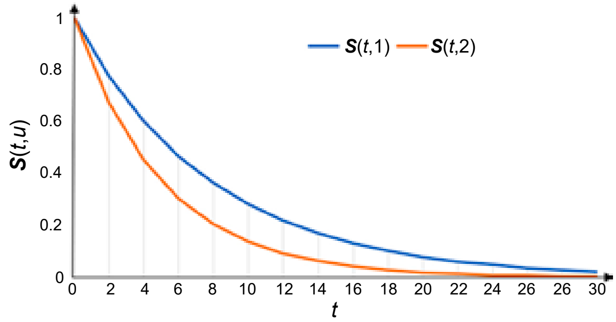


**Figure 7. Graph of the port oil terminal system safety function coordinates**

The expected values and standard deviations of the terminal unconditional lifetimes in the safety state subsets {1, 2} and {2}, in years, are respectively:

$$\boldsymbol{\mu}(1) = 0.395 \cdot 8.08342 + 0.060 \cdot 8.16593 + 0.003 \cdot 7.60179 + 0.002 \cdot 6.80805 + 0.2 \cdot 7.60179 + 0.058 \cdot 6.80805 + 0.282 \cdot 8.00256 \cong 7.89$$
$$\boldsymbol{\sigma}(1) \cong 7.91$$
$$\boldsymbol{\mu}(2) = 0.395 \cdot 5.15695 + 0.060 \cdot 5.21069 + 0.003 \cdot 4.85232 + 0.002 \cdot 4.34292 + 0.2 \cdot 4.85232 + 0.058 \cdot 4.3429 + 0.282 \cdot 5.10431 \cong 5.03$$
$$\boldsymbol{\sigma}(2) \cong 5.03 \qquad (27)$$

and the mean values of the unconditional lifetimes in the particular safety states 1 and 2 are respectively:

$$\bar{\boldsymbol{\mu}}(1) = \boldsymbol{\mu}(1) - \boldsymbol{\mu}(2) = 2.86$$
$$\bar{\boldsymbol{\mu}}(2) = \boldsymbol{\mu}(2) = 5.03 \text{ years} \qquad (28)$$

Since in previous research (Magryta-Mut, 2023a) it was assumed that the critical safety state is $r = 1$, the system risk function according to equations (4) and (25) is given by:

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,1) \ \text{ for } \ t \geq 0 \qquad (29)$$

Hence, from equation (5), the moment when the system risk function exceeds a permitted level (for instance, $\delta = 0.05$) is found as:

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.40 \text{ year} \qquad (30)$$

The graph of the port oil terminal system risk function $\boldsymbol{r}(t)$ is presented in Figure 8.

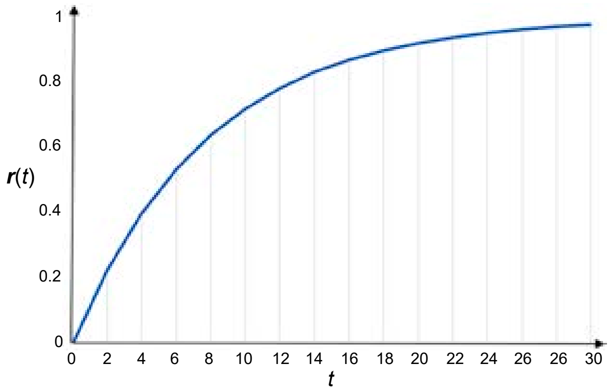


**Figure 8. Graph of the port oil terminal system risk function**

The port oil terminal critical infrastructure mean intensities of ageing, after considering equation (27) and applying equation (7), are:

$$\boldsymbol{\lambda}(1) \cong 0.126743, \ \boldsymbol{\lambda}(2) \cong 0.198807 \qquad (31)$$

The coefficients of the operation process impact on the port oil terminal critical infrastructure intensities of ageing are:

$$\boldsymbol{\rho}(1) \cong 1.09381, \ \boldsymbol{\rho}(2) \cong 1.09391 \qquad (32)$$

Finally, for $u = r = 1$, the port oil terminal critical infrastructure resilience indicator, i.e., the coefficient of the port oil terminal critical infrastructure resilience to the operation process impact is written as:

$$\boldsymbol{RI}(1) = 1/\boldsymbol{\rho}(1) \cong 0.9142 = 91.42\% \qquad (33)$$

### Safety of maritime ferry technical system

The limit transient probabilities of the maritime ferry technical system at the 18 particular operation states $z_b$, in which $b = 1,2,\ldots,18$, fixed in earlier work (Magryta-Mut, 2023a), are:

$$p_1 = 0.038, p_2 = 0.002, p_3 = 0.026, p_4 = 0.036,$$
$$p_5 = 0.363, p_6 = 0.026, p_7 = 0.005, p_8 = 0.016,$$
$$p_9 = 0.037, p_{10} = 0.002, p_{11} = 0.003, p_{12} = 0.016,$$
$$p_{13} = 0.351, p_{14} = 0.034, p_{15} = 0.024, p_{16} = 0.003,$$
$$p_{17} = 0.005, p_{18} = 0.013.$$

Hence, from equation (1), the maritime technical system safety function is given by the vector:

$$\boldsymbol{S}(t,\cdot) = [\boldsymbol{S}(t,1), \boldsymbol{S}(t,2), \boldsymbol{S}(t,3), \boldsymbol{S}(t,4)] \quad t \in \langle 0,\infty) \qquad (34)$$

with coordinates given by:

$$\begin{aligned}\boldsymbol{S}(t,1) = {} & 0.038\cdot[\boldsymbol{S}(t,1)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,1)]^{(2)} + \\ & + 0.026\cdot[\boldsymbol{S}(t,1)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,1)]^{(4)} + \\ & + 0.363\cdot[\boldsymbol{S}(t,1)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,1)]^{(6)} + \\ & + 0.005\cdot[\boldsymbol{S}(t,1)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,1)]^{(8)} + \\ & + 0.037\cdot[\boldsymbol{S}(t,1)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,1)]^{(10)} + \\ & + 0.003\cdot[\boldsymbol{S}(t,1)]^{(11)} + 0.016\cdot[\boldsymbol{S}(t,1)]^{(12)} + \\ & + 0.351\cdot[\boldsymbol{S}(t,1)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,1)]^{(14)} + \\ & + 0.024\cdot[\boldsymbol{S}(t,1)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,1)]^{(16)} + \\ & + 0.005\cdot[\boldsymbol{S}(t,1)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,1)]^{(18)}\end{aligned}$$

$$\begin{aligned}\boldsymbol{S}(t,2) = {} & 0.038\cdot[\boldsymbol{S}(t,2)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(2)} + \\ & + 0.026\cdot[\boldsymbol{S}(t,2)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,2)]^{(4)} + \\ & + 0.363\cdot[\boldsymbol{S}(t,2)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,2)]^{(6)} + \\ & + 0.005\cdot[\boldsymbol{S}(t,2)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,2)]^{(8)} + \\ & + 0.037\cdot[\boldsymbol{S}(t,2)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,2)]^{(10)} \\ & + 0.003\cdot[\boldsymbol{S}(t,2)]^{(11)} + 0.016\cdot[\boldsymbol{S}(t,2)]^{(12)} + \\ & + 0.351\cdot[\boldsymbol{S}(t,2)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,2)]^{(14)} + \\ & + 0.024\cdot[\boldsymbol{S}(t,2)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,2)]^{(16)} + \\ & + 0.005\cdot[\boldsymbol{S}(t,2)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,2)]^{(18)}\end{aligned}$$

$$\begin{aligned}\boldsymbol{S}(t,3) = {} & 0.038\cdot[\boldsymbol{S}(t,3)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,3)]^{(2)} + \\ & + 0.026\cdot[\boldsymbol{S}(t,3)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,3)]^{(4)} + \\ & + 0.363\cdot[\boldsymbol{S}(t,3)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,3)]^{(6)} + \\ & + 0.005\cdot[\boldsymbol{S}(t,3)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,3)]^{(8)} + \\ & + 0.037\cdot[\boldsymbol{S}(t,3)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,3)]^{(10)} + \\ & + 0.003\cdot[\boldsymbol{S}(t,3)]^{(11)} + 0.016\cdot[\boldsymbol{S}(t,3)]^{(12)} + \\ & + 0.351\cdot[\boldsymbol{S}(t,3)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,3)]^{(14)} + \\ & + 0.024\cdot[\boldsymbol{S}(t,3)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,3)]^{(16)} + \\ & + 0.005\cdot[\boldsymbol{S}(t,3)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,3)]^{(18)}\end{aligned}$$

$$\begin{aligned}\boldsymbol{S}(t,4) = {} & 0.038\cdot[\boldsymbol{S}(t,4)]^{(1)} + 0.002\cdot[\boldsymbol{S}(t,4)]^{(2)} + \\ & + 0.026\cdot[\boldsymbol{S}(t,4)]^{(3)} + 0.036\cdot[\boldsymbol{S}(t,4)]^{(4)} + \\ & + 0.363\cdot[\boldsymbol{S}(t,4)]^{(5)} + 0.026\cdot[\boldsymbol{S}(t,4)]^{(6)} + \\ & + 0.005\cdot[\boldsymbol{S}(t,4)]^{(7)} + 0.016\cdot[\boldsymbol{S}(t,4)]^{(8)} + \\ & + 0.037\cdot[\boldsymbol{S}(t,4)]^{(9)} + 0.002\cdot[\boldsymbol{S}(t,4)]^{(10)} + \\ & + 0.003\cdot[\boldsymbol{S}(t,4)]^{(11)} + 0.016\cdot[\boldsymbol{S}(t,4)]^{(12)} + \\ & + 0.351\cdot[\boldsymbol{S}(t,4)]^{(13)} + 0.034\cdot[\boldsymbol{S}(t,4)]^{(14)} + \\ & + 0.024\cdot[\boldsymbol{S}(t,4)]^{(15)} + 0.003\cdot[\boldsymbol{S}(t,4)]^{(16)} + \\ & + 0.005\cdot[\boldsymbol{S}(t,4)]^{(17)} + 0.013\cdot[\boldsymbol{S}(t,4)]^{(18)} \qquad (35)\end{aligned}$$

The graph of this five-state ferry technical system safety function is shown in Figure 9.

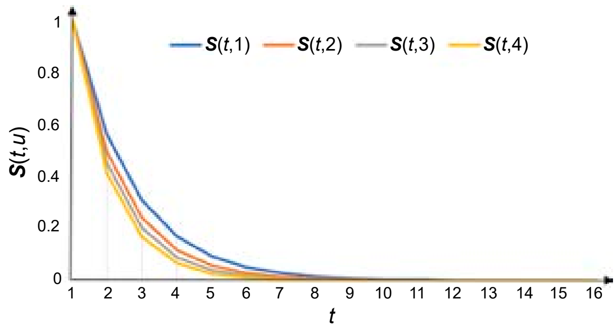


**Figure 9. Graph of the ferry technical system safety function coordinates**

The expected values and the standard deviations of the ferry technical system lifetimes in the safety state subsets {1, 2, 3, 4}, {2, 3, 4}, {3, 4}, and {4} (expressed in years) are respectively:

$$\boldsymbol{\mu}(1) \cong 0.038 \cdot 1.70476 + 0.002 \cdot 1.60772 + 0.026 \cdot 1.68087 + 0.036 \cdot 1.6956 + 0.363 \cdot 1.69547 + 0.026 \cdot 1.67434 + 0.005 \cdot 1.54736 + 0.016 \cdot 1.72871 + 0.037 \cdot 1.72871 + 0.002 \cdot 1.60772 + 0.003 \cdot 1.6102 + 0.016 \cdot 1.70148 + 0.351 \cdot 1.69547 + 0.034 \cdot 1.6863 + 0.024 \cdot 1.68087 + 0.003 \cdot 1.61025 + 0.005 \cdot 1.54736 + 0.013 \cdot 1.70476 \cong 1.694$$

$$\boldsymbol{\sigma}(1) \cong 1.66811$$

$$\boldsymbol{\mu}(2) \cong 0.038 \cdot 1.41708 + 0.002 \cdot 1.32879 + 0.026 \cdot 1.3912 + 0.036 \cdot 1.39303 + 0.363 \cdot 1.39292 + 0.026 \cdot 1.37699 + 0.005 \cdot 1.27865 + 0.016 \cdot 1.43719 + 0.037 \cdot 1.43719 + 0.002 \cdot 1.32879 + 0.003 \cdot 1.3336 + 0.016 \cdot 1.39692 + 0.351 \cdot 1.39292 + 0.034 \cdot 1.3854 + 0.024 \cdot 1.3912 + 0.003 \cdot 1.3336 + 0.005 \cdot 1.27865 + 0.013 \cdot 1.41708 \cong 1.395$$

$$\boldsymbol{\sigma}(2) \cong 1.37645$$

$$\boldsymbol{\mu}(3) \cong 0.038 \cdot 1.22861 + 0.002 \cdot 1.18936 + 0.026 \cdot 1.24553 + 0.036 \cdot 1.24632 + 0.363 \cdot 1.24619 + 0.026 \cdot 1.23228 + 0.005 \cdot 1.15851 + 0.016 \cdot 1.26722 + 0.037 \cdot 1.26722 + 0.002 \cdot 1.18936 + 0.003 \cdot 1.19593 + 0.016 \cdot 1.24985 + 0.351 \cdot 1.24619 + 0.034 \cdot 1.23945 + 0.024 \cdot 1.24553 + 0.003 \cdot 1.19593 + 0.005 \cdot 1.15851 + 0.013 \cdot 1.22861 \cong 1.244$$

$$\boldsymbol{\sigma}(3) \cong 1.23042$$

$$\boldsymbol{\mu}(4) \cong 0.038 \cdot 1.11601 + 0.002 \cdot 1.06574 + 0.026 \cdot 1.11512 + 0.036 \cdot 1.11522 + 0.363 \cdot 1.1151 + 0.026 \cdot 1.10301 + 0.005 \cdot 1.02847 + 0.016 \cdot 1.13163 + 0.037 \cdot 1.13163 + 0.002 \cdot 1.06574 + 0.003 \cdot 1.07262 + 0.016 \cdot 1.11836 + 0.351 \cdot 1.1151 + 0.034 \cdot 1.1091 + 0.024 \cdot 1.11512 + 0.003 \cdot 1.07262 + 0.005 \cdot 1.02847 + 0.013 \cdot 1.11601 \cong 1.114$$

$$\boldsymbol{\sigma}(4) \cong 1.10170 \quad (36)$$

Furthermore, the lifetimes in the particular safety states 1, 2, 3, and 4, respectively, are:

$$\bar{\boldsymbol{\mu}}(1) = \boldsymbol{\mu}(1) - \boldsymbol{\mu}(2) = 0.299816$$
$$\bar{\boldsymbol{\mu}}(2) = \boldsymbol{\mu}(2) - \boldsymbol{\mu}(3) = 0.149614$$
$$\bar{\boldsymbol{\mu}}(3) = \boldsymbol{\mu}(3) - \boldsymbol{\mu}(4) = 0.130199$$
$$\bar{\boldsymbol{\mu}}(4) = \boldsymbol{\mu}(4) = 1.114243 \text{ years} \quad (37)$$

Since previous research (Magryta-Mut, 2023a) assumed that the critical safety state is $r = 2$, then the system risk function according to equations (4) and (34) is given by:

$$\boldsymbol{r}(t) = 1 - \boldsymbol{S}(t,2) \text{ for } t \geq 0 \quad (38)$$

The graph of the ferry technical system risk function $\boldsymbol{r}(t)$ is presented in Figure 10.

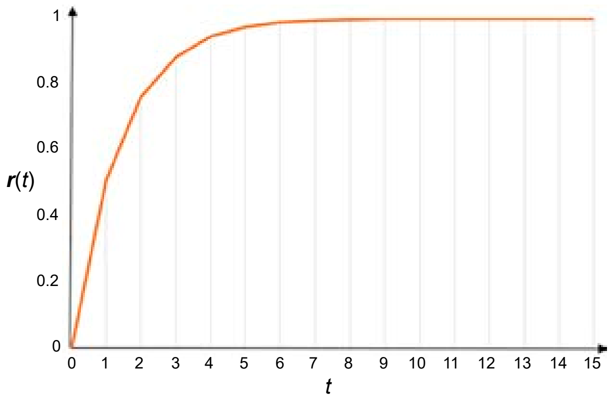


**Figure 10. Graph of the ferry technical system risk function**

Hence, from equation (5), the moment when the system risk function exceeds a permitted level (for instance, $\delta = 0.05$) is determined as:

$$\tau = \boldsymbol{r}^{-1}(\delta) \cong 0.073 \text{ year} \quad (39)$$

The ferry technical system mean intensities of ageing, after considering equation (36) and applying equation (7), are:

$$\lambda(1) \cong 0.590363, \ \lambda(2) \cong 0.716869, \ \lambda(3) \cong 0.803573, \ \lambda(4) \cong 0.897470 \quad (40)$$

The coefficients of the operation process impact on the ferry technical system intensities of ageing are:

$$\boldsymbol{\rho}(1) \cong 1.044942, \ \boldsymbol{\rho}(2) \cong 1.058098, \ \boldsymbol{\rho}(3) \cong 1.044645, \ \boldsymbol{\rho}(4) \cong 1.044655 \quad (41)$$

Finally, for $u = r = 2$, the ferry technical system resilience indicator, i.e., the coefficient of the ferry technical system resilience to the operation process impact, is given as:

$$\boldsymbol{RI}(2) = 1/\boldsymbol{\rho}(2) \cong 0.9451 = 94.51\% \quad (42)$$

## Remarks on system safety and operation cost optimization

The results from two previous sections are the basis for the system safety and operation cost optimization considered in previous works (Magryta-Mut, 2020, 2022, 2023a, 2023b). The procedures of using the general safety analytical model and the operation cost models (Kołowrocki & Magryta, 2020; Kołowrocki & Magryta-Mut, 2022) of a complex multistate technical system, related to its operation process and the linear programming (Klabjan & Adelman, 2006), are presented and proposed for a separate and joint analysis of this system safety maximization and its operation cost minimization (Magryta-Mut, 2023a, 2023b).

The proposed separate system safety optimization is based on the mean value of the complex multistate system lifetime in the system safety state subset not worse than the system critical safety state maximization through the system operation process modification. This operation process modification ensures the corresponding best forms and values of the system safety indicators.

The proposed separate system operation cost optimization depends on the complex multistate system mean value of the operation total costs, during the fixed operation time minimization or the operation total costs in the safety state subset not worse than a critical safety state through the system operation process modification. This operation process modification ensures that the corresponding minimal system operation total costs during the fixed operation time, or in the safety state subset not worse than a critical safety state.

The procedure of joint system safety, and its operation cost optimization, enables us to first perform the system safety maximization and next determine its conditional operation total costs during the fixed operation time or in the safety state subset not worse than a critical safety state, which corresponds to this system maximal safety. In this case, the operation process modification enables us to fix the complex system conditional operation total costs during the fixed operation time, or in the safety state subset not worse than a critical safety state that corresponds to the system best safety indicators. The proposed system safety optimization procedure, and the corresponding system operation total costs finding, deliver practically important possibilities for the system safety indicators maximization and keep fixed the corresponding system operation total costs during the operation via the system's new operation strategy.

Alternatively, the procedure of joint system safety and its operation cost optimization also enable us to perform, firstly, the system operation total costs during the fixed operation time, or in the safety state subset not worse than a critical safety state minimization, and next determine its conditional safety function and remaining safety indicators corresponding to this system minimal operation total costs. In this case, the operation process modification enables us to fix the complex system conditional safety indicators, which corresponds to the system minimal operation total costs during the fixed operation time or in the safety state subset not worse than a critical safety state. The proposed cost optimization procedure, and finding corresponding system safety indicators, deliver practically important possibilities for the system total operation costs that minimize and keep fixed the corresponding conditional safety indicators during the operation through the system's new operation strategy.

## Conclusions

The proposed system safety, system operation costs separate and joint analysis, and optimization models and procedures can be successfully applied to the port oil terminal critical infrastructure and the maritime ferry technical system safety maximization and operation cost minimization (Magryta-Mut, 2023a, 2023b). Moreover, it can be applied to the very wide class of complex technical systems that change their functional structures and their components' safety and operation costs during the exploitation. The only limitation of the proposed universal model application, in practice, is to obtain sufficiently exact statistical data originating from the users of these systems to evaluate their components' safety and operation costs with unknown parameters. Further developments of the presented research should be focused on improving the proposed safety and operation cost models by considering the very significant impact of the changing climate-weather conditions and creating more general complex technical systems safety and operation cost models related to the joint impact of the operation process and the climate-weather change process at these systems operation areas.

Thus, in particular, further research can be related to considering other impacts on the system's safety, for instance, the weather impacts (Kołowrocki, 2021), and solving the problems of their safety optimization and finding optimal values of their safety and resilience indicators. These results can also help to mitigate complex systems accident consequences and to enhance their resilience to operations and other impacts (Bogalecka, 2020).

The suggested optimization safety procedures and perspective of future research applied to operation costs, and to safety and resilience optimization of complex systems and critical infrastructures, can provide practically important possibilities for these systems. This allows an effectiveness improvement through a proposing of their new operation strategy application.

## Acknowledgment

This paper presents the results developed within the scope of the research project WN/PI/2022/05: "Safety and cost optimization of port and maritime transportation systems", granted by Gdynia Maritime University in 2022.

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**Cite as:** Magryta-Mut, B. (2023) Modeling safety of port and maritime transportation systems. *Scientific Journals of the Maritime University of Szczecin, Zeszyty Naukowe Akademii Morskiej w Szczecinie* 74 (146), 65–74.