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# Reverse logistic model for deteriorating item with preservation technology investment and learning effect in an inflationary environment*

by

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**Abstract:** Recycling is essential for the existence of the environment. Nowadays, recycling is also being promoted by the governments of numerous countries in the world. In the recycling process, the used items are collected from the customers and sent back to the remanufacturing unit. In the literature, many authors have assumed that the remanufactured items are as good as the freshly produced ones, but this is not really practical. The remanufactured items are of lower quality than the freshly produced ones. For purposes of studying the recycling process we have developed a reverse logistic model for the deteriorating items. The deterioration rate is treated as a controllable variable, which is controlled by investing in preservation technology. The demand rate is taken as exponential function of time with demand dependent manufacturing and remanufacturing rates. The fresh products are sold in the primary market and the remanufactured products are sold in the secondary market at a low selling price, in view of the low quality of remanufactured products. The whole study is carried out under the effect of learning in an inflationary environment and the learning coefficients are associated with production cost and set-up cost. The main objective of this study is to find the optimal cost value and the optimal operating times. At the end of this article numerical illustration is presented and sensitivity analysis is performed. The whole of the mathematical calculations is done with the use of the mathematical software Mathematica7.

**Keywords:** reverse logistics, deterioration, preservation technology, learning, inflation

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## 1. Introduction

Deterioration means decay, spoilage, evaporation, degradation in quality etc. Ghare and Schrader (1963) were among the forst to have used the concept of deterioration in inventory control modeling. Due to rapid changes in environmental conditions, deteriorating items need extra protection. By actively using the preservation technologies one can provide extra protection for the deteriorating items. Thus, for instance, Hsu, Wee and Teng (2010) have studied the effect of preservation technology, while Dye and Hsieh (2012) have developed an inventory model for deteriorating items under the effect of preservation technology. Similarly, many authors have focused on this concept, like Hsieh and Dye (2013), Dye (2013), Shah, Shah and Patel (2014) etc. Further, for instance, Singh and Rathore (2014a,b) have studied the effect of preservation technology for the deteriorating items under the effect of inflation.

Recycling is another concept, which plays an important role in the protection of the environment. It is the process, in which used items are collected from the customers and are returned back to the organization, essentially for the remanufacturing process. In the inventory control modeling literature this concept is termed reverse logistics. Schrady (1967) was the first to introduce the concept of reverse logistics. For further study we refer the Reader to such works as Alamri (2010), Singh and Saxena (2012), Singh, Prasher and Saxena (2013), Yang et al. (2013), etc.

Many authors have assumed that the quality of the remanufactured items is as good as the quality of the freshly produced items. This assumption, though, does not always hold real circumstances, meaning that there many cases, in which the quality of the remanufactured items is lower than the quality of the newly produced items. These lower quality items are sold in the secondary market. For further review one can refer to the research work of Jaber and El Saadany (2009), Konstantaras and Skouri (2010), Hasanov et al. (2012), Singh and Saxena (2013, 2014), etc.

It is often seen that through the repetitive procedures an improvement takes place in either the product or in the process itself, and this happens owing to the action of the so-called learning effect. The reverse logistics model has been developed under the effect of learning by Maity et al. (2009). Later, Singh, Jain and Pareek (2013) have developed an inventory model for deteriorating items also including the effect of learning. On the other hand, Singh, Rathore and Saxena (2014) and Singh and Rathore (2014c) have studied the effect of inflation on inventory control modeling in the case of deteriorating items.

In the present article we have formulated a reverse logistics inventory model for deteriorating items. The deterioration rate is assumed to be under control by applying some preservation technology. The time dependent demand on the primary market is satisfied by the fresh products, whereas the remanufactured items are sold on the secondary market at a lower price. The rates of remanufacturing and manufacturing processes are directly related to the demand rates. The used items are collected and returned back to the remanufacturing unit for

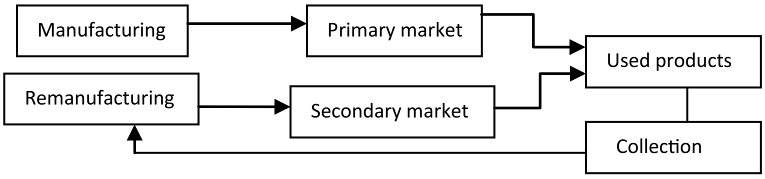


Figure 1. Forward and backward supply chain

recycling. In the next sections the mathematical model is formulated by using the notations and assumptions that are introduced at the outset.

## 2. Assumptions and notations

The assumptions and notations, which are used in mathematical model formulation are given as follows. The subscripts $r$, $m$ and $c$ are used for remanufacturing, manufacturing and collection cycle, respectively.

### 2.1. Notations

- $P_m$, $P_r$ are the manufacturing and remanufacturing rates, respectively
- $R_c$: rate of returning the items
- $D_m(t)$: the time dependent demand rate on the primary market
- $W$: $(= D_r(t))$ is the demand rate of customers on the secondary market
- $\theta_r$: deterioration rate of the remanufactured product
- $\theta_m$: deterioration rate of the freshly manufactured product
- $\theta_c$: deterioration rate of the collected items
- $\xi$: the preservation technology cost
- $m_i(\xi)$: reduced deterioration rate due to the use of preservation technology: $m_i(\xi) = \theta_i(1 - e^{-a_1^{\xi}})$; where $i = r, m, c$; $a_1 > 0$ and $0 < \theta_i < 1$;
- $\tau_i$: resultant deterioration rate at the wholesaler's end, $\tau_i = (\theta_i - m(\xi))$; where $i = r, m, c$
- $T$: total cycle length
- $t_i$: time intervals for $i = 1, 2, 3$
- $K_m$: set up cost for manufacturing
- $K_r$: set up cost for remanufacturing
- $K_c$: set up cost for collection
- $C_c$: unit acquisition cost.
- $S_m$: unit procurement cost
- $C_m$: unit production cost
- $C_r$: unit remanufacturing cost
- $h_r$: holding cost (per item unit per unit time) of remanufactured item
- $h_m$: holding cost (per item unit per unit time) of manufactured item
- $h_c$: cost for holding returned items per item unit per unit time

- $I_{mi}(t)$: inventory level during manufacturing cycle, $i = 1, 2$
- $I_{ri}(t)$: inventory level during remanufacturing cycle, $i = 1, 2$
- $I_{ci}(t)$: inventory level during collection cycle, $i = 1, 2$.

### 2.2. Assumptions

- Items are returnable and returned items are remanufactured. Concerning quality, both manufactured and remanufactured products are of the same quality.
- The system is proposed for a single item only with a constant deterioration rate.
- A preservation technology is used to preserve the items.
- Items are manufactured / remanufactured at a finite rate of manufacturing and remanufacturing, which is directly related to the demand rate, $P_r = k_r D_r(t)$; $P_m = k_m D_m(t)$, where $k_r$ and $k_m > 1$.
- The demand rate is $D_m(t) = ae^{bt}$, where $a$ and $b > 0$, and this demand is satisfied by the fresh products sold at the primary market. Then, $W(= D_r(t) = \alpha e^{\beta t}$, with coefficients $\alpha$ and $\beta > 0)$ is the rate of demand at the secondary market, which is satisfied by the remanufactured products, these products being of lower quality.
- The cycle length is infinite, but we discuss only a typical cycle of length $T$. All other cycles are identical in duration to the cycle $T$.

$C_r + C_r'/\,n^{\lambda}$ is the remanufacturing cost under the effect of learning, where $\lambda > 0$ is the learning coefficient and $n$ is the learning factor.

- $C_m + C_m'/\; n^{\gamma}$ is the manufacturing cost under the effect of learning, where $\gamma > 0$ is the learning coefficient and $n$ is the learning factor.

## 3. Mathematical model formulation

The inventory depletion in different cycles is presented in Fig. 2 and can be described as in the following relations.

### 3.1. The remanufacturing cycle

$$\frac{dI_{r1}(t)}{dt} + \tau_r I_{r1}(t) = P_r - D_r; \quad I_{r1}(t=0) = 0, \quad 0 \leqslant t \leqslant t_1 \tag{1}$$

$$\frac{dI_{r2}(t)}{dt} + \tau_r I_{r2}(t) = -D_r; I_{r1}(t=t_1) = I_{r2}(t=t_1), t_1 \leqslant t \leqslant t_2. \tag{2}$$

By solving (1) and (2) using the boundary conditions, we get

$$I_{r1}(t) = \left(\frac{\alpha(k_r-1)}{\beta+\tau_r}\right)(e^{\beta t} - e^{-\tau_r t}); \tag{3}$$

$$I_{r2}(t) = \alpha\left(\frac{e^{(\tau_r+\beta)t_2}e^{-\tau_r t} - e^{\beta t}}{(\tau_r+\beta)}\right); \tag{4}$$

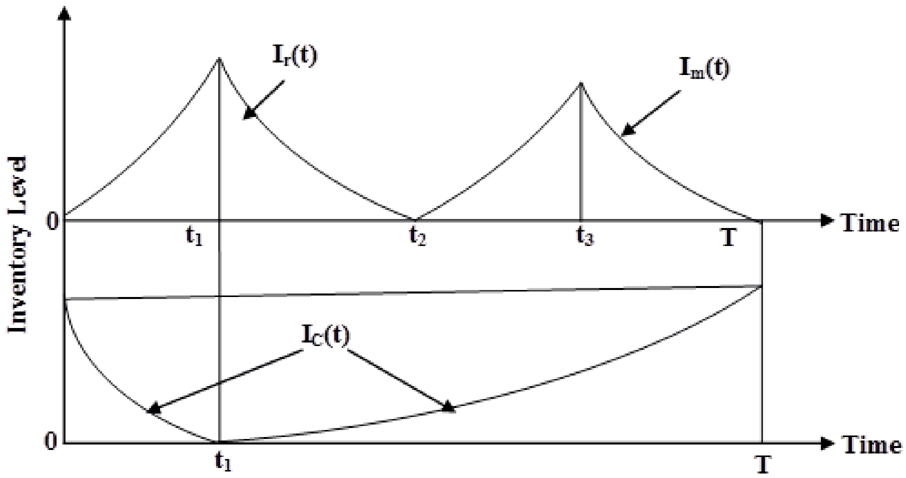


Figure 2. Functioning of the inventory during different time intervals

$$t_2 = \left(\frac{1}{\tau_r + \beta}\right) \log\left(1 - k_r + k_r e^{(\tau_r+\beta)t_1}\right). \tag{5}$$

### 3.2. The manufacturing cycle

$$\frac{dI_{m1}(t)}{dt} + \tau_m I_{m1}(t) = P_m - D_m(t); \quad I_{m1}(t = t_2) = 0, \quad t_2 \leqslant t \leqslant t_3 \tag{6}$$

$$\frac{dI_{m2}(t)}{dt} + \tau_m I_{m2}(t) = P_m; I_{m1}(t = t_3) = I_{m2}(t = t_3), t_3 \leqslant t \leqslant T. \tag{7}$$

By solving (6) and (7) using the boundary conditions, we get

$$I_{m1}(t) = a(k_m - 1)\left(\frac{e^{bt} - e^{(\tau_m+b)t_2} e^{-\tau_r t}}{(\tau_m+b)}\right); \tag{8}$$

$$I_{m2}(t) = \alpha\left(\frac{e^{(\tau_m+b)T} e^{-\tau_m t} - e^{bt}}{(\tau_m+b)}\right); \tag{9}$$

$$T = \left(\frac{1}{\tau_m + b}\right) \log\left(k_m e^{(\tau_m+b)t_3} - (k_m - 1)\, e^{(\tau_m+b)t_2}\right). \tag{10}$$

### 3.3. The collection cycle

$$\frac{dI_{c1}(t)}{dt} + \tau_c I_{c1}(t) = R_c - P_r; \quad I_{c1}(t = t_1) = 0, \quad 0 \leqslant t \leqslant t_1 \tag{11}$$

$$\frac{dI_{c2}(t)}{dt} + \tau_c I_{c2}(t) = R_c; I_{c2}(t = t_1) = 0 t_1 \leqslant t \leqslant T. \tag{12}$$

By solving (11) and (12) using the boundary conditions, we get

$$I_{c1}(t) = \left( \frac{R_c}{\tau_c}(1 - e^{\tau_c t_1} e^{-\tau_c t}) - \frac{ak_r}{(\tau_c + \beta)}(e^{\beta t} - e^{(\tau_c + \beta)t_1} e^{-\tau_c t}) \right); \tag{13}$$

$$I_{c2}(t) = \frac{R_c}{\tau_c}\left(1 - e^{\tau_c t_1} e^{-\tau_c t}\right). \tag{14}$$

### 3.4. The cost parameters

Now, the acceptable return quantity for used items is $Q = R_c T$and the different cost parameters are as follows:

1. Present worth of procurement (POC) and acquisitions (AC) cost is

$$= S_m \int_{t_2}^{t_3} P_m e^{-Rt} dt + C_c \int_0^T R_c e^{-Rt} dt =$$

$$aS_m k_m \left( \frac{e^{(b-R)t_3} - e^{(b-R)t_2}}{(b - R)} \right) + C_c R_c \left( \frac{1 - e^{-RT}}{R} \right). \tag{15}$$

2. Present worth of manufacturing cost (MC) and remanufacturing (RC) cost is

$$\left( C_m + \frac{C'_m}{n^\gamma} \right) \int_{t_2}^{t_3} P_m e^{-Rt} dt + \left( C_r + \frac{C'_r}{n^\lambda} \right) \int_0^{t_1} P_r e^{-Rt} dt MC + RC =$$

$$ak_m \left( C_m + \frac{C'_m}{n^\gamma} \right) \left( \frac{e^{(b-R)t_3} - e^{(b-R)t_2}}{(b - R)} \right) + \alpha k_r \left( C_r + \frac{C'_r}{n^\lambda} \right) \left( \frac{e^{(\beta-R)t_1} - 1}{(\beta - R)} \right). \tag{16}$$

3. Present worth of holding cost is (HC) = [Holding cost for remanufactured items + Holding cost for manufactured items + Holding cost for collected items], that is

$$HC =$$

$$\int_0^{t_1} h_r I_{r1}(t) e^{-Rt} dt + \int_{t_1}^{t_2} h_r I_{r2}(t) e^{-Rt} dt + \int_{t_2}^{t_3} h_m I_{m1}(t) e^{-Rt} dt + \int_{t_3}^{T} h_m I_{m2}(t) e^{-Rt} dt$$

$$+ \int_0^{t_1} h_c I_{c1}(t) e^{-Rt} dt + \int_{t_1}^{T} h_c I_{c2}(t) e^{-Rt} dt$$

$$HC =$$
$$\frac{\alpha h_r}{(\tau_r+\beta)}\left((\tau_r+\beta)(k_r-1)\frac{t_1^2}{2}+(t_2-t_1)\left(e^{(\tau_r+\beta)t_2}-1\right)-\right.$$
$$\left.\left(\beta-\tau_r e^{(\tau_r+\beta)t_2}-R(e^{(\tau_r+\beta)t_2}+1)\right)\left(\frac{t_2^2-t_1^2}{2}\right)\right)+$$
$$\frac{ah_m(k_m-1)}{(\tau_m+b)}\left[(t_3-t_2)\left(1-e^{(\tau_m+b)t_2}\right)+\right.$$
$$\left.\left(b+\tau_m e^{(\tau_m+b)t_2}+R(e^{(\tau_m+b)t_2}-1)\right)\left(\frac{t_3^2-t_2^2}{2}\right)\right]+$$
$$\frac{ah_m}{(\tau_m+b)}\left[(t_3-T)\left(e^{(\tau_m+b)T}-1\right)-\right.$$
$$\left.\left(b+\tau_m e^{(\tau_m+b)T}+R(e^{(\tau_m+b)T}-1)\right)\left(\frac{t_3^2-T^2}{2}\right)\right]+$$
$$h_c\left[\frac{R_c}{\tau_c}(t_1-R)\left(1-e^{\tau_c t_1}\right)+\tau_c e^{\tau_c t_1}-\left(\frac{\alpha k_r}{\tau_c+\beta}\right)\left(t_1\left(1-e^{(\tau_c+\beta)t_1}\right)+\right.\right.$$
$$\left.\left.\frac{t_1^2}{2}\left(\beta+\tau_c e^{(\tau_c+\beta)t_1}+R(e^{(\tau_c+\beta)t_1}-1)\right)\right)\right]+$$
$$\frac{R_c h_c}{\tau_c}\left[(T-t_1)\left(1-e^{\tau_c t_1}\right)-\left(e^{\tau_c t_1}\tau_c+R(e^{(\tau_m+b)T}-1)\right)\left(\frac{T^2-t_1^2}{2}\right)\right]. \tag{17}$$

Hence, by using the continuity of functions for values of $t_1$, $t_2$, $t_3$ and $T$ in total cost function, we can calculate the total cost per unit time as:

$$TC(t_1, t_3\xi) = (1/T)[K_r + K_m + K_C + POC + AC + PC + RC + HC]. \tag{18}$$

To minimize total relevant cost, we differentiate $TC$ $(t_1,t_2,\xi)$ with respect to $t_1$, $t_2$ and $\xi$, and for the optimal value the necessary conditions are

$$\frac{\partial TC(t_1,t_3,\xi)}{\partial t_1}=0;\frac{\partial TC(t_1,t_3,\xi)}{\partial t_3}=0;\frac{\partial TC(t_1,t_3,\xi)}{\partial \xi}=0.$$

Provided the determinants of the principal minor of Hessian matrix are positive definite, i.e. det($H1$)>0, det($H2$)>0 where $H1$, $H2$ are the principal minors of the Hessian-matrix, the Hessian matrix of the total cost function can be expressed as follows:

$$\begin{bmatrix} \frac{\partial^2 TC}{\partial t_1^2} & \frac{\partial^2 TC}{\partial t_1 \partial t_3} & \frac{\partial^2 TC}{\partial t_1 \partial \xi} \\ \frac{\partial^2 TC}{\partial t_1 \partial t_3} & \frac{\partial^2 TC}{\partial t_3^2} & \frac{\partial^2 TC}{\partial t_3 \partial \xi} \\ \frac{\partial^2 TC}{\partial \xi \partial t_1} & \frac{\partial^2 TC}{\partial \xi \partial t_3} & \frac{\partial^2 TC}{\partial \xi^2} \end{bmatrix}.$$

## 4. Numerical illustration and sensitivity analysis

For the numerical illustration, the following numerical values are used to calculate the optimal value of total cost: $K_m = 1500$, $K_r = 800$, $K_c = 1000$,

$h_r = 1.2$, $h_m = 1.2$, $h_c = 1$, $R_c = 50$, $a = 250$, $b = 50$, $\alpha = 100$, $\beta = 2$, $k_r = 2$, $k_m = 2$, $n = 4$, $\theta_r = 0.055$, $\theta_m = 0.05$, $\theta_c = 0.057$, $R = 0.02$, $S_m = 8$, $C_m = 3$, $C_m^{'} = 3.2$, $C_r = 2$, $C_r^{'} = 2.2$, $C_c = 2$, $a_1 = 2$. The optimal values, obtained with the help of the mathematical software Mathematica7 are as follows: $TC^* = 79311.5$, $\xi^* = 2480.83$, $t_1^* = 0.58442$, $t_2^* = 1.16884$, $t_3^* = 1.83136$, $T^* = 2.49388$.

We have performed a kind of sensitivity analysis by changing the values of important parameters, such as the demand factors $a$, $b$, $\alpha$, $\beta$, the deterioration rates $\theta_r$, $\theta_m$, $\theta_c$, the learning coefficients $\lambda$, $\gamma$, as well as the inflation rate $R$. The resultant optimal values of $t_1{}^*$, $t_2{}^*$, $t_3{}^*$, $T^*$ and $TC^*$ are given in Table 1. The observations that can be made on the basis of Table 1 are as follows:

- Increment in the value of $a$ results in the increment in $\xi^*$, $t_1^*, t_2^*, t_3^*, T^*$ and $TC^*$.
- Increment in the values of $b$ and $\beta$ results in the decrement in $\xi^*$, $t_1^*, t_2^*, t_3^*, T^*$ and $TC^*$.
- Increment in the value of $\alpha$ results in the increment in $\xi^*$, $t_3^*, T^*$ and $TC^*$, and in the decrement in $t_1^*, t_2^*$.
- Increment in the values of $\gamma$ and $\lambda$ results in the increment in $\xi^*$, $t_1^*, t_2^*, t_3^*$ and $T^*$, and, at the same time, in the decrement in $TC^*$.
- Increment in the values of $R$ and $\theta_c$ results in the decrement in $t_1^*, t_2^*, t_3^*$ and $T^*$, and, simultaneously, in the increment in $TC^*$.
- Increment in the values of $\theta_r$ and $\theta_m$ results in the increment in $\xi^*$, $t_1^*, t_2^*, t_3^*, T^*$ and $TC^*$.

The convexity of the function of the optimal total cost value $(TC^*)$ with respect to the time interval $(t_1^*)$ and the preservation technology cost $(\xi^*)$ is well illustrated by the convex surface shown in Fig. 3. Similarly, the convex graph of the optimal total cost value $(TC^*)$ function with respect to time interval $(t_3^*)$ and preservation technology cost $(\xi^*)$ is illustrated in Fig. 4. Here, $t_1$ is the time, at which the remanufacturing process has stopped and $t_3$ is the time, at which the manufacturing process has stopped.

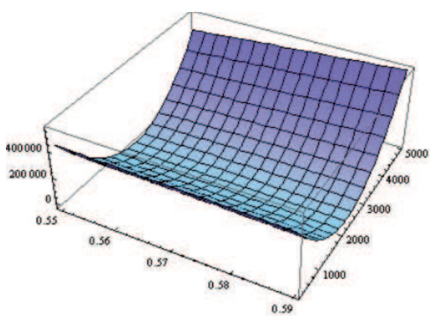


Figure 3. Convexity of TC* with respect to $t_1^*$ and $\xi^*$

Table 1. Sensitivity analysis

| Change in $a$ | $\xi^*$ | $t_1^*$ | $t_2^*$ | $t_3^*$ | $T^*$ | $TC^*$ |
|---|---|---|---|---|---|---|
| 200 | 1788.39 | 0.44403 | 0.8881 | 1.31412 | 1.74018 | 136159 |
| 270 | 2778.63 | 0.64126 | 1.2825 | 2.05383 | 2.82515 | 455867 |
| 280 | 2932.19 | 0.66981 | 1.3396 | 2.16855 | 2.99747 | 28408 |
| Change in $b$ | | | | | | |
| 45 | 2636.42 | 0.66145 | 1.3229 | 2.16495 | 3.00699 | 117874 |
| 55 | 2374.73 | 0.52389 | 1.0478 | 1.59228 | 2.13683 | 56837 |
| 65 | 2242.97 | 0.43481 | 0.8696 | 1.271 | 1.67238 | 33701 |
| Change in $\alpha$ | | | | | | |
| 90 | 2469.7 | 0.58693 | 1.1739 | 1.82303 | 2.47221 | 72907 |
| 120 | 2500.69 | 0.57933 | 1.1587 | 1.84624 | 2.53382 | 91328 |
| 150 | 2524.05 | 0.57148 | 1.143 | 1.86375 | 2.58453 | 108207 |
| Change in $\beta$ | | | | | | |
| 5 | 2476.63 | 0.58436 | 1.1688 | 1.82823 | 2.48766 | 79007 |
| 10 | 2469.63 | 0.58426 | 1.1685 | 1.82299 | 2.47745 | 78758 |
| 15 | 2462.6 | 0.58416 | 1.1684 | 1.81773 | 2.46707 | 78453 |
| Change in $\lambda$ | | | | | | |
| 0.01 | 2480.8 | 0.58442 | 1.1688 | 1.83134 | 2.49384 | 79201 |
| 0.03 | 2480.85 | 0.58442 | 1.1688 | 1.83138 | 2.49392 | 79200 |
| 0.04 | 2480.88 | 0.58442 | 1.1688 | 1.8314 | 2.49396 | 79199 |
| Change in $\gamma$ | | | | | | |
| 0.01 | 2476.93 | 0.58232 | 1.1646 | 1.82276 | 2.48096 | 80961 |
| 0.03 | 2484.71 | 0.58651 | 1.173 | 1.83993 | 2.50684 | 80077 |
| 0.04 | 2488.59 | 0.58858 | 1.1772 | 1.84847 | 2.51978 | 78343 |
| Change in $R$ | | | | | | |
| 0.01 | 2486.98 | 0.58471 | 1.1694 | 1.83549 | 2.50155 | 78206 |
| 0.025 | 2477.76 | 0.58427 | 1.1686 | 1.8293 | 2.49006 | 78704 |
| 0.03 | 2474.69 | 0.58413 | 1.1683 | 1.82724 | 2.48623 | 80200 |
| Change in $\theta_r$ | | | | | | |
| 0.045 | 2369.57 | 0.57603 | 1.1521 | 1.7482 | 2.34434 | 61458 |
| 0.06 | 2539.5 | 0.58858 | 1.1772 | 1.87522 | 2.57328 | 89109 |
| 0.65 | 2600.25 | 0.59271 | 1.1854 | 1.92063 | 2.65584 | 99763 |
| Change in $\theta_m$ | | | | | | |
| 0.045 | 2038.21 | 0.45536 | 0.9107 | 1.3486 | 1.78648 | 34499 |
| 0.055 | 3034.63 | 0.73408 | 1.4682 | 2.47172 | 3.47527 | 181365 |
| 0.065 | 4671.64 | 1.09843 | 2.1969 | 4.51593 | 6.835 | 892156 |
| Change in $\theta_c$ | | | | | | |
| 0.05 | 1369.78 | 0.81103 | 1.6221 | 2.94854 | 4.27502 | 312625 |
| 0.06 | 2113.04 | 0.51561 | 1.0312 | 1.55667 | 2.08211 | 49095 |
| 0.07 | 3976.08 | 0.35675 | 0.7135 | 1.00167 | 1.28984 | 14848 |

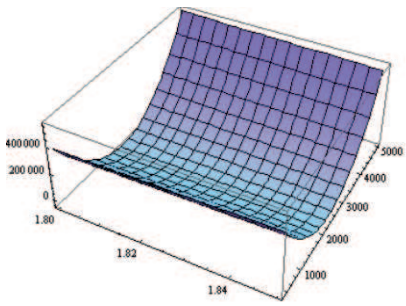


Figure 4. Convexity of $TC^*$ with respect to $t_3^*$ and $\xi^*$

## 5. Conclusion

In this paper, we proposed a recycling model, in which manufacturing and remanufacturing rates are directly related to the time dependent demand rates of primary and secondary markets, respectively. The freshly manufactured products are sold on the primary market, whereas the recycled products are being sold through the secondary market. The used items are returned back to the collection unit and then go to the remanufacturing unit. The items are deteriorating in nature and preserved under the effect of preservation technology. The whole study has been performed with the assumption of effect of two parameters, related to learning and inflation. The main objective of the formulated problem is to find the optimal value of total cost, and preservation cost, and the optimal operating time. The model is numerically illustrated and a kind of sensitivity analysis has been performed. The convexity of the model is also illustrated for the verification of the study premises. This study can be further extended by incorporating other parameters of inventory control modeling.

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