Optica Applicata, Vol. XLVIII, No. 4, 2018
10.5277/oa180411

Reciprocity relations for light wave scattered by a particulate collection

ZHENFEI JIANG, ZHANGHANG ZHU, TAO WANG*

Department of Physics, Sichuan Normal University, Chengdu 610068, China

*Corresponding author: towerwang@126.com

Two Fourier relations of light waves scattered by a random-distributed particulate medium have been investigated. We find that the scattered field and the particulate collection satisfy two Fourier relations, *i.e.* the spectral density is directly proportional to a Fourier transform of a convolution of correlation coefficient of each particle and correlation coefficient of distribution function of the whole collection, and the spectral degree of coherence is directly proportional to a Fourier transform of a convolution of strength of the scattering potential of each particle and strength of the distribution function of the whole collection. To illustrate these relations, behaviors of the far-field generated by Gaussian-correlated particles with Gaussian-correlated distributions have been discussed.

Keywords: light scattering, particulate collection, spectral density, spectral degree of coherence.

1. Introduction

As one of important methods to determine the structural information of an unknown object, the weak scattering theory is always a topic that has attracted much attention. To properly describe the characteristic of the scattering medium, a lot of models of scattering media were constructed, for example, the quasi-homogeneous medium [1–6], the anisotropic medium [7–9], the semisoft boundary medium [10–13], and the particulate medium [14–19]. It has been shown that there is some important structural information which can be obtained by measuring scattered field [20]. This phenomenon may provide a way to determine structural characteristic of an unknown object from the measurement of scattered field (see, for examples, [21–27]).

When we discuss light scattering, a model of quasi-homogeneous medium is usually considered. It is well-known that there are special Fourier relations between distribution of scattered field and characteristic of the medium, which is known as reciprocity relations [28]. These relations attracted much attention because they could provide available ways to measure structural information of scatterer. For example, XINYUE DU and DAOMU ZHAO discussed the reciprocity relations of an anisotropic medium [29], and

JIMING YU and JIA LI discussed the reciprocity relations of two incident beams which are generated by Young's pinhole [30]. Recently, the scattering behaviors of light wave from a particulate medium were discussed extensively. For example, the spectral degree of coherence of light wave on scattering from a particulate medium was discussed, and the particle-related coherence changes and the distribution-related coherence changes were investigated [31]; the scattering behavior of light wave from a mixed collection composed by different types was discussed, and it is shown that both the distribution characteristic of random-distributed particles and the location of determinate-distributed particles play a role in the behaviors of the far-zone scattered field [32]. In this manuscript, based on the random-distributed identical particles collection, the reciprocity relations between the scattered field and the particulate collection will be studied. To illustration this relations, an example to illustrate behaviors of light waves scattered from Gaussian-correlated particles with Gaussian-correlated distributions will be discussed.

2. Theory

As shown in Fig. 1, assume that a spatially coherent plane light wave, propagating in a direction of $\mathbf{s}_0$, is incident on a collection of particles. To analyze the statistical properties of incident field, we employ the cross-spectral density function located in two position vectors $\mathbf{r}'_1$ and $\mathbf{r}'_2$, which is defined as [33]

$$W^{(i)}(\mathbf{r}'_1, \mathbf{r}'_2, \mathbf{s}_0, \omega) = \langle U^{(i)*}(\mathbf{r}'_1, \mathbf{s}_0, \omega) U^{(i)}(\mathbf{r}'_2, \mathbf{s}_0, \omega)\rangle \tag{1}$$

where $*$ denotes complex conjugate, and $\langle\cdot\rangle$ denotes the ensemble average, with $U^{(i)}$ being the incident field, *i.e.*

$$U^{(i)}(\mathbf{r}', \mathbf{s}_0, \omega) = a(\omega)\exp(ik\mathbf{s}_0 \cdot \mathbf{r}') \tag{2}$$

where $a(\omega)$ is a random amplitude, and $k = \omega/c$ is wave number.

Assume that the scattering medium is random-distributed random particles, *i.e.* the scattering potential of each particle is a random function and the location of each par-

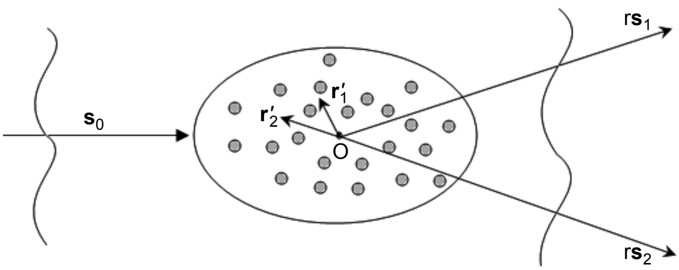

Fig. 1. Illustration of the notations.

ticle in the collection is also a random function. In the situation, characteristics of collection should be described by its correlation function, *i.e.* [24]

$$C_F(\mathbf{r}_1', \mathbf{r}_2', \omega) = \langle F^*(\mathbf{r}_1', \omega) F(\mathbf{r}_2', \omega) \rangle \tag{3}$$

where

$$F(\mathbf{r}', \omega) = \sum_n f(\mathbf{r}' - \mathbf{r}_n', \omega) \tag{4}$$

denotes scattering potential of collection with $\mathbf{r}_n'$ being the location vectors of the particles, and n denotes the sum of particles [19].

If the refractive index only slightly differs from unity, the magnitude of the scattered field may be smaller enough than the incident one. In this case, the scattering process could be analyzed within the first-order Born approximation [34]. Assume that the ensemble averages of that over the Fourier transform of particle's scattering potential and of that over the Fourier transform of the distribution function are independent. Then the far-zone cross-spectral density function at two location vectors $r\mathbf{s}_1$ and $r\mathbf{s}_2$ can be expressed as [31]

$$W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2} \tilde{C}_f\Big(-k(\mathbf{s}_1 - \mathbf{s}_0), k(\mathbf{s}_2 - \mathbf{s}_0), \omega\Big) \times \tilde{C}_n\Big(-k(\mathbf{s}_1 - \mathbf{s}_0), k(\mathbf{s}_2 - \mathbf{s}_0), \omega\Big) \tag{5}$$

where $S^{(i)}(\omega)$ is the spectrum of the incident field, and

$$\tilde{C}_f(\mathbf{K}_1, \mathbf{K}_2, \omega) = \iint_D C_f(\mathbf{r}_1', \mathbf{r}_2', \omega) \exp\Big[-i(\mathbf{K}_1 \cdot \mathbf{r}_1' + \mathbf{K}_2 \cdot \mathbf{r}_2')\Big] \mathrm{d}^3 r_1' \, \mathrm{d}^3 r_2' \tag{6}$$

and

$$\tilde{C}_n(\mathbf{K}_1, \mathbf{K}_2, \omega) = \iint_D C_n(\mathbf{r}_1', \mathbf{r}_2', \omega) \exp\Big[-i(\mathbf{K}_1 \cdot \mathbf{r}_1' + \mathbf{K}_2 \cdot \mathbf{r}_2')\Big] \mathrm{d}^3 r_1' \, \mathrm{d}^3 r_2' \tag{7}$$

are two spatially Fourier transforms of C_f and C_n, respectively, with

$$C_f(\mathbf{r}_1', \mathbf{r}_2', \omega) = \langle f^*(\mathbf{r}_1', \omega) f(\mathbf{r}_2', \omega) \rangle \tag{8}$$

and

$$C_n(\mathbf{r}_1', \mathbf{r}_2', \omega) = \langle \sum_m \sum_n \delta^*(\mathbf{r}_1' - \mathbf{r}_m', \omega) \delta(\mathbf{r}_2' - \mathbf{r}_n', \omega) \rangle \tag{9}$$

When the two unit vectors of scattering directions are the same (*i.e.*, $\mathbf{s}_1 = \mathbf{s}_2 = \mathbf{s}$), the spectral density in the far field can be given by the expression

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2}\tilde{C}_f\Big(-k(\mathbf{s}-\mathbf{s}_0), k(\mathbf{s}-\mathbf{s}_0), \omega\Big)\tilde{C}_n\Big(-k(\mathbf{s}-\mathbf{s}_0), k(\mathbf{s}-\mathbf{s}_0), \omega\Big) \tag{10}$$

On the other hand, the spectral degree of coherence in the far field is defined as

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{W^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega)}{\sqrt{S^{(s)}(r\mathbf{s}_1, \mathbf{s}_0, \omega)}\sqrt{S^{(s)}(r\mathbf{s}_2, \mathbf{s}_0, \omega)}} \tag{11}$$

Then on employing Eqs. (5) and (10), the spectral degree of coherence defined by Eq. (11) can be rewritten as

$$\begin{aligned}\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) &\\ = \frac{\tilde{C}_f\Big(-k(\mathbf{s}_1-\mathbf{s}_0), k(\mathbf{s}_2-\mathbf{s}_0), \omega\Big)}{\sqrt{\tilde{C}_f\Big(-k(\mathbf{s}_1-\mathbf{s}_0), k(\mathbf{s}_1-\mathbf{s}_0), \omega\Big)}\sqrt{\tilde{C}_f\Big(-k(\mathbf{s}_2-\mathbf{s}_0), k(\mathbf{s}_2-\mathbf{s}_0), \omega\Big)}} &\\ \times \frac{\tilde{C}_n\Big(-k(\mathbf{s}_1-\mathbf{s}_0), k(\mathbf{s}_2-\mathbf{s}_0), \omega\Big)}{\sqrt{\tilde{C}_n\Big(-k(\mathbf{s}_1-\mathbf{s}_0), k(\mathbf{s}_1-\mathbf{s}_0), \omega\Big)}\sqrt{\tilde{C}_n\Big(-k(\mathbf{s}_2-\mathbf{s}_0), k(\mathbf{s}_2-\mathbf{s}_0), \omega\Big)}} &\end{aligned} \tag{12}$$

In the following discussion, we will discuss Fourier relations between distributions of scattered field and properties of particulate collection. Hypothesize that all of particles in the collection are quasi-homogeneous, *i.e.*, its strength of the scattering potential $S_f(\mathbf{r})$ is a "slow" function of $\mathbf{r}$, whereas the normalized correlation coefficient of the scattering potential $\mu_f(\mathbf{r}')$ is a "fast" function of $\mathbf{r}'$ with $\mathbf{r}' = \mathbf{r}_2 - \mathbf{r}_1$ [28]. In this case, the correlation of scattering potential of the collection has a form [31]

$$C_f(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = S_f\left(\frac{\mathbf{r}'_1+\mathbf{r}'_2}{2}, \omega\right)\mu_f(\mathbf{r}'_2-\mathbf{r}'_1, \omega) \tag{13}$$

where S_f and μ_f denote strength and normalized correlation coefficient of scattering potential of a particle, respectively. Moreover, assume that distribution functions of particles in the collection are also quasi-homogeneous, *i.e.*

$$C_n(\mathbf{r}'_1, \mathbf{r}'_2, \omega) = S_n\left(\frac{\mathbf{r}'_1+\mathbf{r}'_2}{2}, \omega\right)\mu_n(\mathbf{r}'_2-\mathbf{r}'_1, \omega) \tag{14}$$

where S_n and μ_n denote strength and normalized correlation coefficient of distribution function, respectively.

Substituting Eq. (13) into Eq. (6), after manipulating integration, one obtains that

$$\tilde{C}_f(\mathbf{K}_1, \mathbf{K}_2, \omega) = \tilde{S}_f(\mathbf{K}_1 + \mathbf{K}_2, \omega)\, \tilde{\mu}_f\left(\frac{\mathbf{K}_2 - \mathbf{K}_1}{2}, \omega\right) \tag{15}$$

where

$$\tilde{S}_f(\mathbf{K}, \omega) = \int_D S_f(\mathbf{r}, \omega) \exp(-i\mathbf{K} \cdot \mathbf{r}) \mathrm{d}^3 r \tag{16}$$

and

$$\tilde{\mu}_f(\mathbf{K}', \omega) = \int_D \mu_f(\mathbf{r}', \omega) \exp(-i\mathbf{K}' \cdot \mathbf{r}') \mathrm{d}^3 r' \tag{17}$$

denote the Fourier transform of S_f and μ_f, respectively. Similarly, substituting Eq. (14) into Eq. (7), the Fourier transform of distribution function is given as

$$\tilde{C}_n(\mathbf{K}_1, \mathbf{K}_2, \omega) = \tilde{S}_n(\mathbf{K}_1 + \mathbf{K}_2, \omega)\, \tilde{\mu}_n\left(\frac{\mathbf{K}_2 - \mathbf{K}_1}{2}, \omega\right) \tag{18}$$

where $\tilde{S}_n$ and $\tilde{\mu}_n$ denote the Fourier transform of S_n and μ_n, respectively. Substituting Eqs. (15) and (18) into Eq. (10), one gets the spectral density in the far field as

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2} \tilde{S}_f(0, \omega)\, \tilde{S}_n(0, \omega)\, \tilde{\mu}_f\Big(k(\mathbf{s} - \mathbf{s}_0), \omega\Big)\, \tilde{\mu}_n\Big(k(\mathbf{s} - \mathbf{s}_0), \omega\Big) \tag{19}$$

Next, on employing Eqs. (15) and (18), and after the rearrangement, the spectral degree of coherence given by Eq. (12) can be rewritten as

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{\tilde{S}_f\Big(k(\mathbf{s}_2 - \mathbf{s}_1), \omega\Big)\, \tilde{\mu}_f\left(k\left(\frac{\mathbf{s}_1 + \mathbf{s}_2}{2} - \mathbf{s}_0\right), \omega\right)}{\tilde{S}_f(0, \omega) \sqrt{\tilde{\mu}_f\Big(k(\mathbf{s}_1 - \mathbf{s}_0), \omega\Big)} \sqrt{\tilde{\mu}_f\Big(k(\mathbf{s}_2 - \mathbf{s}_0), \omega\Big)}}$$
$$\times \frac{\tilde{S}_n\Big(k(\mathbf{s}_2 - \mathbf{s}_1), \omega\Big)\, \tilde{\mu}_n\left(k\left(\frac{\mathbf{s}_1 + \mathbf{s}_2}{2} - \mathbf{s}_0\right), \omega\right)}{\tilde{S}_n(0, \omega) \sqrt{\tilde{\mu}_n\Big(k(\mathbf{s}_1 - \mathbf{s}_0), \omega\Big)} \sqrt{\tilde{\mu}_n\Big(k(\mathbf{s}_2 - \mathbf{s}_0), \omega\Big)}} \tag{20}$$

For a quasi-homogeneous medium, $\tilde{\mu}_n$ should be a slow function of $k\mathbf{s}$. In this case, one can obtain the approximation relation [33], *i.e.*

$$\tilde{\mu}_i\Big(k(\mathbf{s}_1 - \mathbf{s}_0), \omega\Big) \approx \tilde{\mu}_i\Big(k(\mathbf{s}_2 - \mathbf{s}_0), \omega\Big) \approx \tilde{\mu}_i\left(k\left(\frac{\mathbf{s}_1 + \mathbf{s}_2}{2} - \mathbf{s}_0\right), \omega\right) \tag{21}$$

By employing the approximation relationship given by Eq. (21), the spectral degree of coherence can be obtained, *i.e.*

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{\tilde{S}_f\Big(k(\mathbf{s}_2 - \mathbf{s}_1), \omega\Big)}{\tilde{S}_f(0, \omega)} \frac{\tilde{S}_n\Big(k(\mathbf{s}_2 - \mathbf{s}_1), \omega\Big)}{\tilde{S}_n(0, \omega)} \tag{22}$$

Next, let us investigate the relations between the scattered field and the collection of particles. After some rearrangements, one finds spectral density denoted by Eq. (19) and spectral degree of coherence denoted by Eq. (22) can be presented as

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2} \tilde{S}(0, \omega) \tilde{\mu}\Big(k(\mathbf{s} - \mathbf{s}_0), \omega\Big) \tag{23}$$

and

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \frac{1}{\tilde{S}(0, \omega)} \tilde{S}\Big(k(\mathbf{s}_2 - \mathbf{s}_1), \omega\Big) \tag{24}$$

where $\tilde{\mu}$ is the Fourier transform of μ, with

$$\mu(\mathbf{r}_2' - \mathbf{r}_1', \omega) = \mu_f(\mathbf{r}_2' - \mathbf{r}_1', \omega) \otimes \mu_n(\mathbf{r}_2' - \mathbf{r}_1', \omega) \tag{25}$$

and $\tilde{S}$ is the Fourier transform of S, with

$$S\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \omega\right) = S_f\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \omega\right) \otimes S_n\left(\frac{\mathbf{r}_1 + \mathbf{r}_2}{2}, \omega\right) \tag{26}$$

where $\otimes$ represents the convolution. Based on Eqs. (23) and (24), when light waves are scattered from a particulate collection, two new reciprocity relations can be found, which may be expressed as:

1) Spectral density produced by light waves incident on random-distributed random particles is directly proportional to the Fourier transform of a convolution of correlation coefficient of each particle and correlation coefficient of distribution function of whole collection;

2) Spectral degree of coherence produced by light waves incident on random-distributed random particles is directly proportional to the Fourier transform of a convolution of strength of scattering potential of each particle and strength of distribution function of whole collection.

3. Numerical results

As an example, let us further suppose that strength function and correlation coefficient, *i.e.* Eqs. (13) and (14), are Gaussian-centered [31], *i.e.*

$$S_f(\mathbf{r}, \omega) = A\exp\left(-\frac{\mathbf{r}^2}{2\sigma_{S_f}^2}\right) \tag{27a}$$

$$\mu_f(\mathbf{r}', \omega) = \exp\left(-\frac{\mathbf{r}'^2}{2\sigma_{\mu_f}^2}\right) \tag{27b}$$

and

$$S_n(\mathbf{r}, \omega) = B\exp\left(-\frac{\mathbf{r}^2}{2\sigma_{S_n}^2}\right) \tag{28a}$$

$$\mu_n(\mathbf{r}', \omega) = \exp\left(-\frac{\mathbf{r}'^2}{2\sigma_{\mu_n}^2}\right) \tag{28b}$$

where σ_{S_f} and σ_{μ_f} are effective width and effective correlation width of scattering potential of each particle, respectively, and σ_{S_n} and σ_{μ_n} are effective width and effective correlation width of distribution function, respectively. Substituting Eqs. (27) and (28) first into Eqs. (25) and (26), and then into Eqs. (23) and (24), after manipulating the Fourier transform, one finds the far-zone spectral density and far-zone spectral degree of coherence as

$$S^{(s)}(r\mathbf{s}, \mathbf{s}_0, \omega) = \frac{S^{(i)}(\omega)}{r^2}AB(2\pi)^6\sigma_{S_f}^3\sigma_{S_n}^3\sigma_{\mu_f}^3\sigma_{\mu_n}^3\exp\left[-\frac{(\sigma_{\mu_f}^2+\sigma_{\mu_n}^2)k^2}{2}(\mathbf{s}-\mathbf{s}_0)^2\right] \tag{29}$$

$$\mu^{(s)}(r\mathbf{s}_1, r\mathbf{s}_2, \mathbf{s}_0, \omega) = \exp\left[-\frac{(\sigma_{S_f}^2+\sigma_{S_n}^2)k^2}{2}(\mathbf{s}_2-\mathbf{s}_1)^2\right] \tag{30}$$

In what follows, some necessarily numerical results relating to scattered spectral density and scattered spectral degree of coherence will be presented to further illustrate the reciprocity relations. Figure 2 presents the normalized spectral density *vs*. the scattering angle θ. It should be noted that the scattering angle is the angle between the incident direction $\mathbf{s}_0$ and the scattering direction $\mathbf{s}$. Figure 2**a** plots spectral density with three different effective correlation widths of distribution functions, and Fig. 2**b** plots spectral density with three different effective correlation widths of scattering potential of particles. It is shown from Fig. 2 that both effective correlation width of scattering potential of particles and effective correlation width of distribution function of collection may affect the distribution of the far field. Figure 3 presents the degree of the

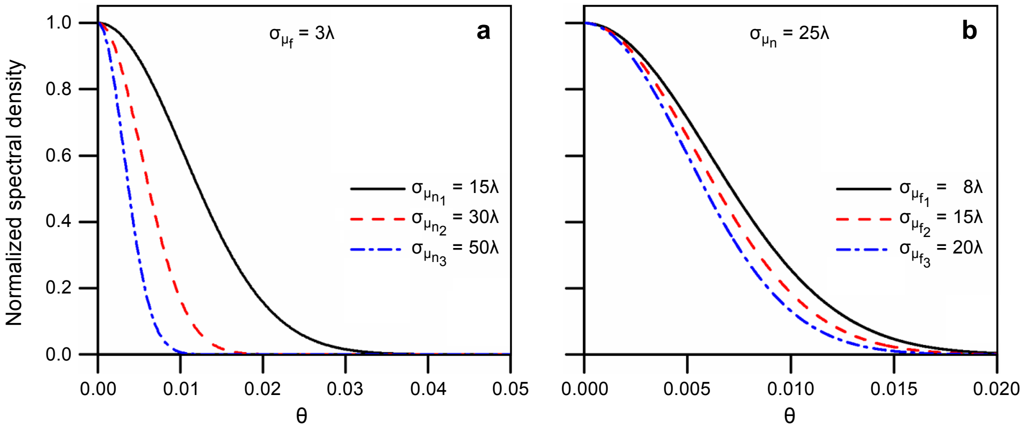

Fig. 2. Normalized spectral density generated by light waves scattered from three different collections with three different effective correlation widths of distribution functions (**a**) and three different effective correlation widths of scattering potentials of particles (**b**).

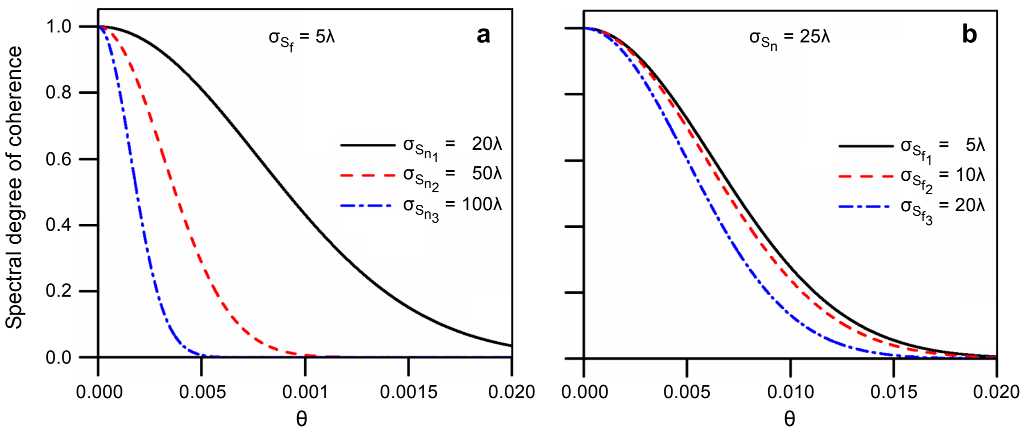

Fig. 3. Spectral degree of coherence generated by light waves scattered from three different collections with three different effective widths of distribution functions (**a**) and three different effective widths of scattering potentials of particles (**b**).

coherence in the far field. Figures 3**a** and 3**b** plot spectral degree of coherence with different effective widths of distribution functions and that with three different effective widths of scattering potentials of particles, respectively. It is shown from Figs. 3**a** and 3**b** that the spectral degree of coherence is affected by effective width of scattering potential of particles and by effective width of distribution function of collection.

4. Conclusions

In conclusion, new reciprocity relations relating to light waves incident on random-distributed random particles were discussed. We show that the spectral density in the far field is affected by correlation coefficient of scattering potential of each particle and correlation coefficient of particle's distribution function, while the spectral degree of coherence in the far field is affected by strength of scattering potential of particles and strength of particle's distribution function. These results may provide a simple way to determine the structural characteristics of a particulate medium from the measurements of the scattered field. Specifically, one can determine the density information of the scattering potentials of a particulate medium from the measurement of the spectral degree of coherence of the scattered field, and one can determine the correlation information of the scattering potentials of a particulate medium from the measurement of the spectral density of the scattered field.

Acknowledgements – One of the authors T. Wang is obliged to Dr. Y. Cai at Soochow University and Dr. Z. Shi at University of South Florida for their valuable discussion. This work was supported by the National Natural Science Foundation of China (NSFC) under grants 11404231, 61475105.

References

[1] Carter W.H., Wolf E., *Scattering from quasi-homogeneous media*, Optics Communications 67(2), 1988, pp. 85–90.
[2] Yu Xin, YanRu Chen, Qi Zhao, MuChun Zhou, *Beam radiated from quasi-homogeneous uniformly polarized electromagnetic source scattering on quasi-homogeneous media*, Optics Communications 278(2), 2007, pp. 247–252.
[3] Yu Xin, Yingjun He, Yanru Chen, Jia Li, *Correlation between intensity fluctuations of light scattered from a quasi-homogeneous random media*, Optics Letters 35(23), 2010, pp. 4000–4002.
[4] Tao Wang, Xiaoling Ji, Daomu Zhao, *Equivalence theorem for the spectral density of light waves on weak scattering*, Optics Letters 39(13), 2014, pp. 3837–3840.
[5] Jia Li, Liping Chang, *Spectral shifts and spectral switches of light generated by scattering of arbitrary coherent waves from a quasi-homogeneous media*, Optics Express 23(13), 2015, pp. 16602–16616.
[6] Jia Li, Yali Qin, Shouli Zhou, *Fourth-order correlation statistics of an electromagnetic plane wave scattered by a quasi-homogeneous medium*, Journal of Optics 13(11), 2011, article ID 115702.
[7] Jia Li, Pinghui Wu, Liping Chang, *Condition for invariant spectrum of an electromagnetic wave scattered from an anisotropic random media*, Optics Express 23(17), 2015, pp. 22123–22133.
[8] Xinyue Du, Daomu Zhao, *Scattering of light by Gaussian-correlated quasi-homogeneous anisotropic media*, Optics Letters 35(3), 2010, pp. 384–386.
[9] Hao Wu, Xiaoning Pan, Zhanghang Zhu, Xiaoling Ji, Tao Wang, *Reciprocity relations of an electromagnetic light wave on scattering from a quasi-homogeneous anisotropic medium*, Optics Express 25(10), 2017, pp. 11297–11305.
[10] Sahin S., Gbur G., Korotkova O., *Scattering of light from particles with semisoft boundaries*, Optics Letters 36(20), 2011, pp. 3957–3959.
[11] HaiXia Wang, ChaoLiang Ding, BaoHong Ma, Cunhua Zhao, LiuZhan Pan, *The intensity properties of a multi-Gaussian Schell-model pulse scattering from a sphere with semisoft boundaries*, Optical and Quantum Electronics 48(6), 2016, article ID 335.
[12] Tao Wang, Xiaoqing Li, Xiaoling Ji, Daomu Zhao, *Spectral changes and spectral switches of light waves on scattering from a semisoft boundary medium*, Optics Communications 324, 2014, pp. 152–156.
[13] Tao Wang, Xiaoqing Li, Xiaoling Ji, *Factors affecting the spectrum of an electromagnetic light wave on scattering from a semisoft boundary medium*, Chinese Optics Letters **12**(12), 2014, article ID 122901.

[14] SAHIN S., KOROTKOVA O., *Effect of the pair-structure factor of a particulate medium on scalar wave scattering in the first Born approximation*, Optics Letters 34(12), 2009, pp. 1762–1764.
[15] ZHANGRONG MEI, KOROTKOVA O., *Random light scattering by collections of ellipsoids*, Optics Express 20(28), 2012, pp. 29296–29307.
[16] SAHIN S., KOROTKOVA O., *Scattering of scalar light fields from collections of particles*, Physical Review A 78(6), 2008, article ID 063815.
[17] XINYUE DU, DAOMU ZHAO, *Scattering of light by a system of anisotropic particles*, Optics Letters 35(10), 2010, pp. 1518–1520.
[18] DOGARIU A., WOLF E., *Spectral changes produced by static scattering on a system of particles*, Optics Letters 23(17), 1998, pp. 1340–1342.
[19] TAO WANG, YI DING, XIAOLING JI, DAOMU ZHAO, *The influence of the characteristics of a collection of particles on the scattered spectral density and its applications*, Chinese Optics Letters **13**(10), 2015, article ID 102901.
[20] DAOMU ZHAO, TAO WANG, *Direct and inverse problems in the theory of light scattering*, [In] *Progress in Optics*, [Ed.] E. Wolf, Chapter 5, Vol. 57, 2012, pp. 261–308.
[21] JIA LI, *Determination of correlation function of scattering potential of random medium by Gauss vortex beam*, Optics Communications 308, 2013, pp. 164–168.
[22] TAO WANG, DAOMU ZHAO, *Determination of correlation function of scattering potentials of a random medium from the scattered spectral density*, Physics Letters A 375(3), 2011, pp. 780–783.
[23] GBUR G., WOLF E., *Determination of density correlation functions from scattering of polychromatic light*, Optics Communications 168(1–4), 1999, pp. 39–45.
[24] LAHIRI M., WOLF E., FISCHER D.G., SHIRAI T., *Determination of correlation functions of scattering potentials of stochastic media from scattering experiments*, Physical Review Letters 102(12), 2009, article ID 123901.
[25] TAO WANG, DAOMU ZHAO, *Determination of pair-structure factor of scattering potential of a collection of particles*, Optics Letters 35(3), 2010, pp. 318–320.
[26] BALEINE E., DOGARIU A., *Variable-coherence tomography for inverse scattering problems*, Journal of the Optical Society of America A 21(10), 2004, pp. 1917–1923.
[27] FISCHER D.G., WOLF E., *Inverse problems with quasi-homogeneous random media*, Journal of the Optical Society of America A 11(3), 1994, pp. 1128–1135.
[28] VISSER T.D., FISCHER D.G., WOLF E., *Scattering of light from quasi-homogeneous sources by quasi-homogeneous media*, Journal of the Optical Society of America A 23(7), 2006, pp. 1631–1638.
[29] XINYUE DU, DAOMU ZHAO, *Reciprocity relations for scattering from quasi-homogeneous anisotropic media*, Optics Communications 284(16–17), 2011, pp. 3808–3810.
[30] JIMING YU, JIA LI, *Reciprocal relations for light from Young's pinholes scattering upon a quasi-homogeneous medium*, Laser Physics Letters 14(5), 2017, article ID 056003.
[31] TAO WANG, HAO WU, YI DING, XIAOLING JI, DAOMU ZHAO, *Changes in the spectral degree of coherence of a light wave on scattering from a particulate medium*, Optics Communications 381, 2016, pp. 210–213.
[32] YI DING, DAOMU ZHAO, *Far-zone behaviors of light waves on scattering from a particulate medium with different types of particles*, Journal of the Optical Society of America B 34(11), 2017, pp. 2376–2380.
[33] WOLF E., *Introduction to the Theory of Coherence and Polarization of Light*, Cambridge University Press, 2007.
[34] BORN M., WOLF E., *Principles of Optics*, Cambridge University Press, 1995.

Received November 30, 2017
in revised form February 1, 2018