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OPTIMIZATION OF TRANSPORT POTENTIAL OF THE TRANSPORTATION COMPANY TAKING INTO ACCOUNT RANDOM DEMAND FOR TRANSPORT SERVICES

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Is considered a transport company, which operates a uniform in the sense of destination, means of transport for example tankers, with random exploitation characteristics. The company's transport potential is measured by the number of transportable means of transport for a sufficiently long period of time. The company operates on the market of transport services, where the demand for transport services is also random. The problem of optimizing the transport potential of a transport company is being considered, taking into account the random demand for transport services.

Keywords: transport potential, demand, stochastic process.

1. Introduction

One of the basic problems of transport company management is to ensure its continuous presence on the market of transport services. This is achieved mainly by ensuring the appropriate transport potential of the means of transport available, in line with the anticipated demand for transport services [8, 18].

The transport potential of a transport company is usually equated with the number of means of transport (ST) capable of providing transport services at a given time. Reducing the number of ST below a certain minimum (threshold value) will reduce the company's transport potential, hence the loss of its competitiveness and, as a consequence, the company's falling out of the transport services market.

The maladjustment of the transportation company's transport potential to the demand for transport services, which will be shaped on the market of these services in the time horizon anticipated by the company, naturally leads to the following two situations:

- reduce the company's competitiveness in the transport services market, and even exit the market when transport capacity is lower than demand,
- incurring additional costs by the company due to not utilizing the transport potential exceeding the demand for transport services.

One of the measure of matching the transportation potential of a transport company to the demand for transport services may be the probability that in the forecast time horizon the transport potential will not be exceeded by demand - in the first situation or transport potential is exceeded by demand - in the second situation.

Usually, the first situation among the above-mentioned generates greater consequences for the transport company and will therefore be considered further, while the random nature of both the demand for transport services created by the market and the supply of transport services by the company will be taken into account [5].

2. Transport potential – supply of transport services

Consider the transport company [5], which has $\boldsymbol{I}$ of of means of transport (ST) for the same destiny (e.g. trucks) and used to meet the demand for homogeneous type of transport services (e.g. transport of bulk cargo). Let $\boldsymbol{\mathcal{I}} = \{\boldsymbol{1, 2,. . I}\}$ be the set of numbers of ST that do not need to be the same, i.e. they do not have to have the same design solutions.

It is assumed that from the point of view of the transport company the process of each ST can be considered as a succession over time of independent states:

- the state of fitness of the ST for the implementation of transport services,
- the state of unfitness of the ST for the implementation of transport services (e.g. ST repair).

Thus, the process of exploitation of each ST can be considered as a two-state stochastic process $\boldsymbol{X(t)}$ (Fig. 1), which is a sequence of consecutive (not overlapping in time) states fitness (rectangular pulses), separated states of unfitness.

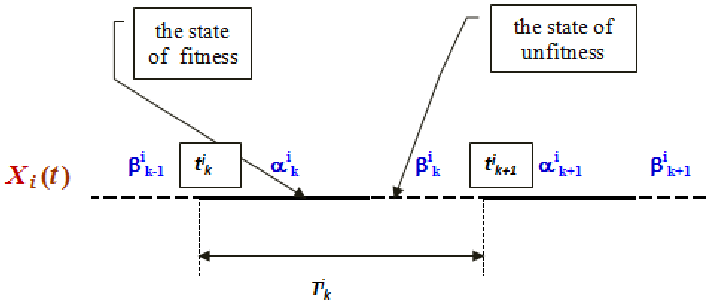

Figure 1. Example of a exploitation process of the ***i***-th, ($i \in \boldsymbol{I}$) of ST.

Fig. 1 are symbolized $\alpha^i_k, \alpha^i_{k+1}, \ldots, (k = 1,2,\ldots)$ durations of states fitness of ***i***-th ST, and symbols $\beta^i_k, \beta^i_{k+1}, \ldots, (k = 1,2,\ldots)$ - durations his of states unfitness.

Let that α^i_k, $(k = 1,2,\ldots)$ are realizations of continuous random variables, A^i_k respectively, with the same probability distributions. For simplify the notation, each of these random variables will be denoted by symbol A_i. Let that β^i_k, $(k = 1,2,\ldots)$ are realizations of continuous random variables, B^i_k respectively, with the same probability distributions. For simplify the notation, each of these random variables will be denoted by symbol B_i. With the use of t^i_k and t^i_{k+1}, the moments of occurrence of two successive states of fitness, ***i***-th ST, were determined, and with the help of T^i_k - the length of the time interval between occurrences of these states.

Using the designations shown in Fig. 1, exploitation process of ***i***-th ST you can be represented as a stochastic process, in which the condition is satisfied:

$$T^i_k = t^i_{k+1} - t^i_k > \alpha^i_k \; . \tag{1}$$

It is assumed that the processes of exploitation of all ST are stochastic processes, which are independent and stationary in a broader sense. Thus, for the ***i***-th ST can be determined the expected length of time between occurrences of two consecutive states of fitness, which is expressed in the following formula:

$$ET_i = \int_0^\infty T \cdot f_i(T)\, dT \tag{2}$$

where $f_i(T)$ is the density function of the probability distribution of the random variable describing the length of time between occurrences of two consecutive states of fitness) the process of exploitation the i-th ST.

Is assumed that are known density functions $f_i^{\alpha}(t)$ and $f_i^{\beta}(t)$ of probability distributions of random variables A_i and B_i respectively. It is also assumed that the random variables A_i and B_i are independent from each other and that have finite variances and finite expected values Ea_i and Eb_i expressed by the following formulas:

$$Ea_i = \int_0^{\infty} \alpha \cdot f_i^{\alpha}(\alpha)\, d\alpha, \qquad Eb_i = \int_0^{\infty} \beta \cdot f_i^{\beta}(\beta)\, d\beta. \tag{3}$$

If the process exploitation of car is stationary, the probability that in randomly chosen time moment ξ there occurs the state of fitness is given by the formula:

$$p_i = \frac{Ea_i}{ET_i} = E\mu_i \cdot Ea_i \tag{4}$$

where $E\mu_i$ - expected frequency of occurrence state of fitness, wherein

$$E\mu_i = \frac{1}{ET_i} = \frac{1}{Ea_i + Eb_i}. \tag{5}$$

It is assumed that the transport company will have the required potential of lading when in the required period of time in a state of fitness would be no less cars than the threshold number s, resulting from the estimated level of demand for transport services.

Due to the fact that the transport companies can include a different number of different transport means and to exploit them under different conditions of the threshold number of means of transport in those companies will also be different. The threshold number of ST should be set so that:

- was the smallest possible under the given conditions,
- took into account the process of shaping the demand for transport services in the area of the company's operations.

The independence of the process of ST exploitation, this means that it is possible that a randomly chosen moment in a state of alertness may also be more than one ST. Let $X(t)$ is the resultant of a process exploitation of cars. It is a process binary (the state of fitness and the state of unfitness), in which the state of fitness, means the state referred to as ***TE*** (*technical efficiency*), formed by superposition of states fitness any ST in number, at least equal to the threshold number of ST s, ($s = 1, 2, ..., I$). ***TE*** state will be taken as the desired state when its duration is not less than the established value τ. In other cases, the status of ***TE*** will be treated as a state indicating the inability to satisfy the demand for transport services at the required level; τ value is determined for each company separately.

It is important for the management of the transport company to know the expected value of the duration of the fitness period ***TE***, when the company has only k fit ST from all existing I. Let $Y_I(t)$ mean the stochastic process of the form [5, 7]:

$$Y_I(t) = \sum_{i=1}^{I} X_i(t). \tag{6}$$

For the assumptions regarding the exploitation processes of ST, an event that in the random moment ξ k of ST from among cars owned by the company is able to fitness can be written as:

$$Y_I(\xi) = k, \quad k = 0,1,2,..., \ I. \tag{7}$$

The probability of this event is expressed by formula:

$$\gamma_{I,k} = \frac{1}{k!} \frac{d^k}{dx^k} \prod_{i=1}^{I} (q_i + xp_i) \Bigg|_{x=0}, \quad k = 0,1,2,..., \ I, \tag{8}$$

at the condition

$$\sum_{k=0}^{I} \gamma_{I,k} = 1, \tag{9}$$

where p_i is expressed by equation (4), and $q_i = 1 - p_i$.

In practice, the ***TE*** state will usually be treated as desirable, if its duration is not shorter than at least a fixed value of τ, determined for each enterprise individually.

Taking into account the assumptions regarding the processes of ST exploitation, it is possible to determine the probability that the ***TE*** state obtained as a result of the k coincidence of means of transport from among I these means, owned by the company, will last no shorter than a certain value of $\tau > 0$. It is expressed as a dependence [5]:

$$\gamma_{I,k}(\tau) = \frac{1}{k!} \frac{d^k}{dx^k} \prod_{i=1}^{I} \left(Q_i(\tau) + xP_i(\tau)\right) \Bigg|_{x=0}, \quad k = 0,1,2,..., \ I, \tag{10}$$

where, taking into account (5)

$$P_i(\tau) = E\mu_i \int_\tau^\infty (x-\tau)\, f_i^\alpha(x)\,dx = E\mu_i \int_\tau^\infty dx \int_x^\infty f_i^\alpha(y)\,dy,\ i = 1,2,...,\ I,$$

$$Q_i(\tau) = E\mu_i \int_\tau^\infty (x-\tau)\, f_i^\beta(x)\,dx = E\mu_i \int_\tau^\infty dx \int_x^\infty f_i^\beta(y)\,dy,\ i = 1,2,...,\ I\,.$$

It is possible to determine the function of density of the probability distribution of the duration of the ***TE*** state created by coincidence of fitness states ***k***, (***k = 0,1,2, ..., I***) of any among ***I*** means of transport, which will last no shorter than a certain amount $\tau > 0$ [5, 7]:

$$f_{I,k}^\alpha(\tau) = \frac{1}{E\mu_{I,k}(\tau)} \frac{d^2}{d\tau^2} \gamma_{I,k}(\tau), \qquad k = 0,1,2,\ldots,I \tag{11}$$

where

$$E\mu_{I,k}(\tau) = -\frac{1}{k!} \frac{\partial^{k+1}}{\partial x^k\, \partial\tau} \prod_{i=1}^{I} \left(Q_i(\tau) + xP_i(\tau)\right)\Big|_{x=0}, \quad k = 0,1,2,...,\ I\,. \tag{12}$$

Let $E\lambda_{I,k}$ denote the expected length (duration) of the ***TE*** state created by the coincidence of fitness ***k***, (***k = 0,1,2, ..., I***) of any among ***I*** of the means of transport. It expresses the following relationship:

$$E\lambda_{I,k} = \int_0^\infty \tau f_{I,k}^\alpha(\tau)\, d\tau = \frac{\gamma_{I,k}}{E\mu_{I,k}(0)}, \qquad k = 0,1,2,\ldots,I\,. \tag{13}$$

If the company has means of transport, which exploitation processes have similar characteristics, $E\lambda_{I,k}$ expresses the relationship:

$$E\lambda_{I,k} = \frac{1}{E\mu} \frac{p(1-p)}{(I-k)\,p + k(1-p)}, \qquad k = 0,1,2,\ldots,I\,, \tag{14}$$

wherein p and $E\mu$ express dependencies (4) and (5), respectively, and are the same for each mean of transport.

3. Demand for transport services

According [6] it is assumed that the further demand for transport services can be described by means of continuous stochastic process $\boldsymbol{Z(t)}$ class CC. It is assumed also that process $\boldsymbol{Z(t)}$ is stationary, ergodic and differentiable in the mean-square sense [1, 2, 3, 4, 9, 10, 11, 12, 13, 14, 15, 16, 17]. Let m_z be the expected value of this process and $K_x(\tau)=\sigma_z^2 r(\tau)$ its correlation function, where $r(\tau)$ defines a normalized correlation function.

Managing a transport company requires knowledge about the extent to which the supply of transport services that it can offer is matched to the forecasted demand for these services. The introduction states that one measure to match the transport capacity of a transport undertaking with the demand for transport services may be the probability that the transport potential will not be outweighed by demand over the foreseeable time horizon. Therefore, the problem considered here concerns is to determine the probability of exceedance by the demand for transport services (process $\boldsymbol{Z(t)}$) of the transport potential of transport company fixed at a level $\boldsymbol{a}$. An exemplary implementation of demand for transport services as a function of time (exemplary implementation of the stochastic process $\boldsymbol{Z(t)}$) for the constant value of the transport potential is shown in Figure 2.

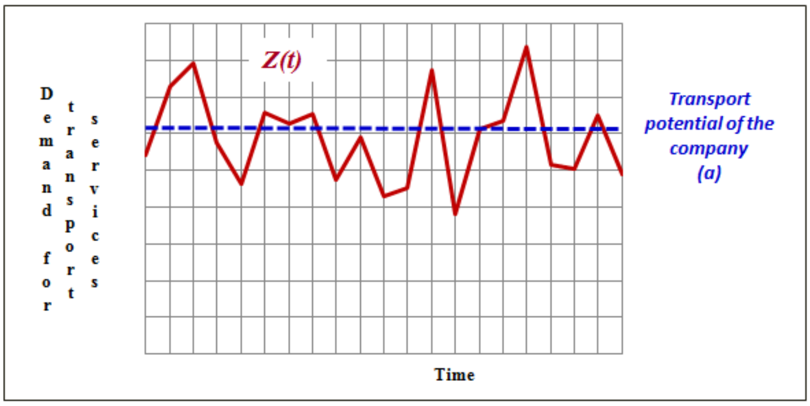

Figure 2. Exemplary implementation of the stochastic process $\boldsymbol{Z(t)}$ describing the demand for transport services in case of value of transport potential.

Calculation of this probability is difficult for the generally formulated process $\boldsymbol{Z(t)}$. Practically useful calculation formulas can be obtained relatively easily only for normal stochastic processes. This problem was discussed in detail in [6], and a useful calculation formula for the probability of not exceeding the transport potential of the transport company by the demand for transport services in the ***T***-period was obtained for the normal stochastic process in the following form:

$$P_0(a,T) = exp\left(-\frac{T}{2\pi}\cdot\sqrt{-\frac{K_z''(0)}{K_z'(0)}}\cdot exp\left(-\frac{(a-m_z)^2}{2\sigma_z^2}\right)\right). \tag{15}$$

In the case of difficulties related to the identification of the correlation function of the process ***Z(t)***, you can use practically useful estimates of this probability:

- lower estimation

$$P_0(a,T) \geq P_0^{min} = \Phi\left(\frac{a-m_z}{\sigma_z}\right) - n_0\cdot T\cdot exp\left(-\frac{(a-m_z)^2}{2\cdot\sigma_z^2}\right), \tag{16}$$

- upper estimation

$$P_0(a,T) \leq P_0^{max} = \Phi\left(\frac{a-m_z}{\sigma_z}\right)\cdot exp\left(-n_0\cdot T\cdot exp\left(-\frac{(a-m_z)^2}{2\cdot\sigma_z^2}\right)\right), \tag{17}$$

where the function $\Phi(x)$ means the integral function of Laplace, and n_0 - the expected number of additions by the process ***Z(t)*** of its expected value in a unit of time, which can be taken as equal [12]:

$$n_0 = \frac{1}{2\sqrt{\pi}}. \tag{18}$$

Formulas (15) - (17) can be used when the following inequality is met:

$$T \leq \frac{\Phi\left(\frac{a-m_z}{\sigma_z}\right)}{n_0}\cdot exp\left(\frac{(a-m_z)^2}{2\cdot\sigma_z^2}\right) \tag{19}$$

4. Optimization problem

One of the practical optimization problems that can be formulated in the case under consideration is the problem of ensuring the maximum likelihood of not exceeding the transport company's transport potential by demand for transport services in the desired period of time. Verbally, such a problem can be formulated, for example, as follows:

specify a minimum number of usable means of transport from among the ***I*** *possessed with known operational characteristics, which will ensure maximization of the probability of not exceeding the company's transport potential (supply of transport services), described by the linear function* ***g*** *of the number of usable means of transport in a given time horizon* δ.

Using the calculation formulas given above, the above-mentioned problem can be given the following formal form:

- decision variable:

m - the number of means of transport usable,

- optimization goals:

$$m \to min \tag{20}$$

$$P_0\left(g(m),\delta\right) = exp\left(-\frac{\delta}{2\pi}\sqrt{-\frac{K_z^{''}(0)}{K_z^{'}(0)}}\,exp\left(-\frac{\left(g(m)-m_z\right)^2}{2\sigma_z^2}\right)\right) \to max \tag{21}$$

- restrictions:

$$E\lambda_{I,m} \ge \delta \tag{22}$$

$$\delta \le 2\sqrt{\pi}\cdot\Phi\left(\frac{g(m)-m_z}{\sigma_z}\right)\cdot exp\left(\frac{\left(g(m)-m_z\right)^2}{2\cdot\sigma_z^2}\right) \tag{23}$$

- boundary conditions:

$$\delta, T \ge 0 \tag{24}$$

$$m \in \{1,2,...,I\} \tag{25}$$

The formulated problem is a probabilistic non-linear task of a two-criteria optimization whose solution is a set of combinations of the number of means of transport and the probability of not exceeding the company's potential.

5. Conclusions

Point 4 of this article formulates an example of the optimization problem related to the basic problem of each transport company, which is the best match of the offered by her size of transport services to the forecasted demand for these services on the market. A particular difficulty for the company is the identification of how the demand for transport services is shaped. This paper proposes a description of demand using a normal stationary continuous stochastic process. It allowed to obtain calculation formulas enabling to carry out approximate calculations regarding the level of matching supply of transport services to the expected demand for them. Of course, the problem formulated in point 4 is not the only one possible and should be treated as an example of a formalized approach to the problem, which is usually solved using experience and intuition and statistical relations between supply and demand for transport services in the past. In case the company would be interested in minimizing the expected duration of the demand exceeded by the supply volume, the previously formulated problem should include the function of the character's purpose:

$$E\tau = \pi \sqrt{-\frac{K_z^{'}(0)}{K_z^{''}(0)}} \cdot exp\left(-\frac{\left(g(m)-m_z\right)^2}{2\sigma_z^2}\right) \cdot \left(1-\Phi\left(\frac{g(m)-m_z}{\sigma_z}\right)\right) \qquad (26)$$

with unchanged restrictions, where $E\tau$ is the expected value of the duration of the excess of demand over the supply of ***g(m)***.

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