

## An approximate method for calculating the resistance of a transport ship model

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**Key words:** friction resistance, total resistance, approximation of ship model resistance, autonomous ships, unmanned surface vessel (USV), ship model

### Abstract

The article presents regression formulas for calculating the friction resistance  $R_F$  and the total resistance  $R_T$  of ship models in the 2.0–10.0 m range. The method for calculating the total resistance is novel and applies to the design models of an unmanned surface vessel (USV) for experimental testing of autonomous control. For both regression models ( $R_F$  and  $R_T$ ), statistical and substantive tests were performed (the results of the calculations were compared with the experimental measurements). In both cases, convincing results were obtained, which have confirmed the possibility of their use at the preliminary design stage of unmanned ship models.

### Introduction

In recent years, research and design work on unmanned ships has begun in earnest. These vessels are to be equipped with an on-board computer with the appropriate software for autonomous control. At the same time, these ships will be equipped with a system that will enable the operator to remotely control the system – the operator (navigator) of the system can take control of the ship in case of failure or difficulties in the autonomous control. Since autonomous control is still being developed and there are no regulations permitting unmanned ships to be used, testing of this type of ships is carried out on models, as shown in Figure 1.

When designing a ship model, as in the case of a full-size ship, one of the tasks is determining the propulsion power required for a given speed. In order to determine the propulsion power and design the propulsion system (propulsors) it is necessary to know the resistance of the model on calm water. The determination of the resistance can be made by:

- measuring it in the model basin,
- numerical calculation using the CFD method.

One and the other method (full and accurate geometry of the ship's hull model is required for such tests) is not used for cost reasons at the stage of designing the initial ship model.

An alternative is to use approximation methods to calculate the ship's resistance. Such methods exist for ships (Holtrop, 1984; Hollenbach, 1998), while, for models in the 2.0–10.0 m range, there is almost nothing in the literature on this subject. The publications in the literature contain tests and calculations of the resistance of a specific model using CFD methods and then these results were compared with experiment in a basin (Lohne et al., 2011; Ebrahimi, 2012; Moctar, Shigunov & Zorn, 2012; Sukas, Kinaci & Bal, 2014; Kinaci & Gokce, 2015; Ozdemir & Barlas, 2017). This information can only be used as an estimation of the resistance for a similar model.

The dependence of the friction resistance has been presented in the literature (Molland, Turnock & Hudson, 2011) in the form of the equation:

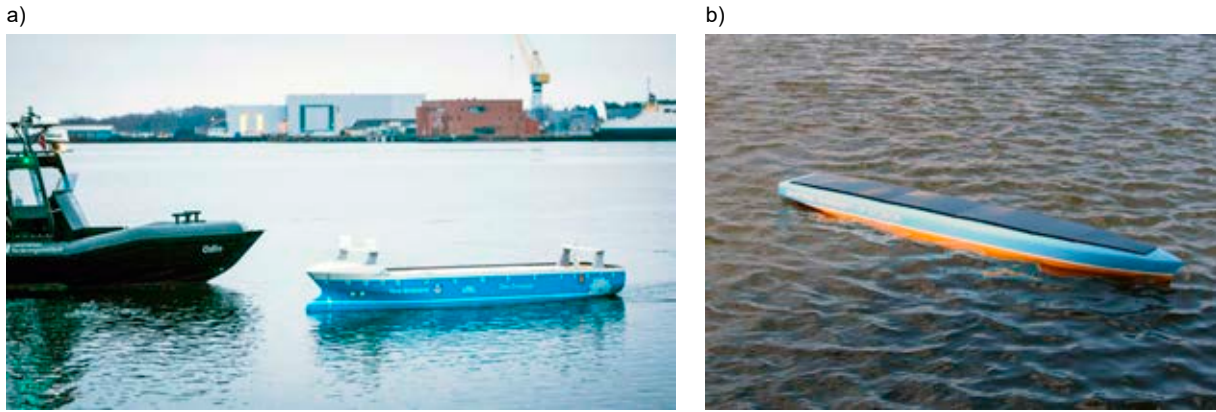


Figure 1. Models of unmanned ships: a) the model ship Yara Birkeland (Ocean News & Technology, 2017) b) model ship from the Maritime University of Szczecin

$$R_F = f \cdot S \cdot V^{1.825} \quad (1)$$

where:

- $f$  – the correction factor that depends on the length of the model,
- $S$  – the wetted surface area of the ship model,
- $V$  – model speed.

Formula (1) only applies to the friction resistance  $R_F$ , and not the total resistance  $R_T$ ; in order to use it, it is necessary to know the wetted surface and the factor  $f$  for a ship model with a given length  $L$ .

The Hollenbach method (Hollenbach, 1998) can also be used to calculate a ship's total resistance, and an approximate residual resistance  $R_F$  has been developed for this method. Add frictional resistance calculated for an equivalent flat plate according to the ITTC (ITTC, 1957). However, the calculation of the total resistance in this way is too imprecise for a ship model and requires knowledge of many geometrical parameters of the hull (Hollenbach's method is used for ships with lengths of 50.20–224.80 m, and the size of the propeller's diameter is necessary for the resistance calculations).

Approximation formulas for barge models and inland ship models have been developed and described previously (Kulczyk & Słomka, 1988; Skupień & Prokopowicz, 2014). From these formulas, the total resistance  $R_T$  can be only calculated for inland watercraft models sailing on shallow water.

### Purpose of the research

Due to the lack of satisfactory methods for calculating the total resistance  $R_T$  of a ship model, a study was carried out to develop an approximate method for calculating the resistance of a ship model. It was assumed that this method should be both as simple and accurate as possible, based on the basic

geometrical parameters of a ship model, which is known at the preliminary design stage.

To elaborate on this method, the results of the resistance measurements of ship models, made in various research centers, of ships designed in the Szczecin Shipyard in 1995–2010 were used. Resistance measurements and the geometric parameters of ship models included in the literature were also used.

### The resistance of a ship model

The total resistance of a vessel  $R$  on calm water can be written as follows:

$$R = R_T + R_{AP} + R_{AA} \quad (2)$$

where:

- $R_T$  – resistance of the bare hull (without appendage parts),
- $R_{AP}$  – resistance of the appendage parts (keel, rudder, etc.),
- $R_{AA}$  – air resistance.

The biggest share in the total resistance is the resistance of the bare hull  $R_T$ , which can be written as (Figure 2):

$$R_T = R_W + R_V = R_W + (1 + k) R_{F0} \quad (3)$$

where:

- $R_W$  – wave resistance (pressure resistance),
- $R_V$  – resistance due to viscosity,
- $R_{F0}$  – frictional resistance of an equivalent flat plate,
- $k$  – form factor taking into account the spatial flow around the hull of the ship model.

During resistance tests in the basin the resistance  $R_T$  of the model is measured, usually without any appendage parts. The result of the measurement, according to the appropriate procedure is converted

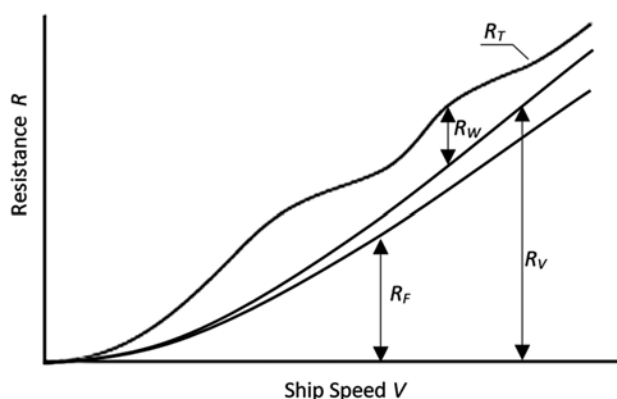


Figure 2. Resistance components  $R_T$

for the real ship. In order to be able to calculate the resistance  $R_T$ , the  $k$  factor and the wetted area  $S$  of the ship's hull must be known.

The results of the resistance measurements for a given speed range (0.5–2.8 m/s) were used to develop an approximate method for calculating the total resistance of a ship model  $R_T$ . The scope of the geometrical parameters of the ship models that were used is shown in Table 1.

A typical regression method based on the least squares algorithm was used to develop an approximate method for calculating the total resistance  $R_T$  of a ship model. The choice of the method resulted, among others, from the assumptions that were made, i.e. the simplicity of the model. Regression dependencies can be easily used for calculations and implemented in simulation programs.

The set of results of the total resistance measurements for ship models was divided into two subsets – the main subset was used to develop the method for calculating the resistance, and the second (smaller set) was used for substantive tests of the developed method.

First, an approximate method for calculating the friction resistance  $R_F$  for ship models was developed (for ship models this component of the total resistance is decisive).

Secondly, an approximate method for calculating the total resistance  $R_T$  for ship models was developed.

When developing both methods, the algorithm that was used was as follows:

1. Determination of a set of geometrical parameters that will significantly affect the described size (resistance of a ship model).
2. Developing a set of geometrical values, velocities and resistance of ship models (a ship model database).
3. Selection of representative models for substantive verification.
4. Searching for the approximation function model.
5. Determination of the function that approximates the resistance of a ship model based on the selected parameters – estimation.
6. Statistical verification of the approximation function obtained on the basis of statistical analysis (significance tests, analysis of the variance, residual analysis, etc.).
7. Substantive verification of the approximation function; obtained on the basis of a comparison of the results obtained from the estimation with the model tests for the model ships of the reference vessels (relative and absolute error).
8. The final choice of the model – the form of the approximation function.

## The received objectives

### Approximation of the friction resistance $R_F$ for ship models

From the regression analyses that were performed, the best formula for approximating the resistance  $R_F$  has the following form:

$$R_F = a_1 \cdot \frac{S^{0.99}}{L_{WL}^{0.15}} \cdot V^{1.8} \quad (4)$$

where,  $a_1 = 2.2652197$ , and  $L_{WL}$  – length of the model on the waterline.

The measure that allows the degree of fit of the model to the empirical data to be assessed is the  $R^2$  coefficient of determination; i.e. the ratio of explained volatility to total volatility – the adjusted  $R^2$  coefficient is usually taken into account. The standard estimation error provides information about the average magnitude of the empirical deviations of the values of the dependent variable (explained) from the values that are calculated from the model.

Table 1. Range of the geometric parameters of the ship's hulls

	$L_{WL}$ [m] length on waterline	$B$ [m] breadth	$T$ [m] draught	$C_b$ [–] block coefficient	$\nabla$ [m <sup>3</sup> ] displacement	$S$ [m <sup>2</sup> ] wetted surface	$L_{WL}/B$	$B/T$
max	9.174	1.288	1.169	0.837	3.232	13.121	7.853	4.600
min	2.236	0.380	0.083	0.593	0.046	0.892	5.405	0.707

Experimental method

For the presented model, these values are at a very good level: A value of  $R^2 = 0.9999$  means that 99.99% of the total resistance variability is explained by the model, the standard error of the estimation is small and amounts to  $Se = 0.366$ .

The results of one statistical test are shown in Figure 3.

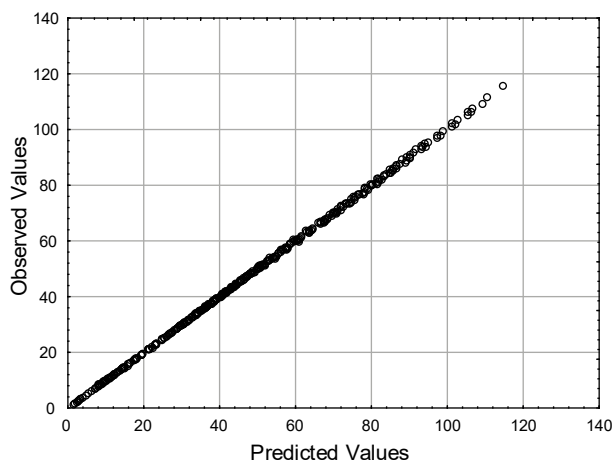


Figure 3. Chart of the observed values versus the predicted values

Substantive tests were performed for ships M1, M2 and M3, whose geometrical parameters and resistance model tests were not used to develop the formula (4). The test results for the ships M1, M2, and M3 are shown in Figures 4–6 and in Tables 2–4, respectively.

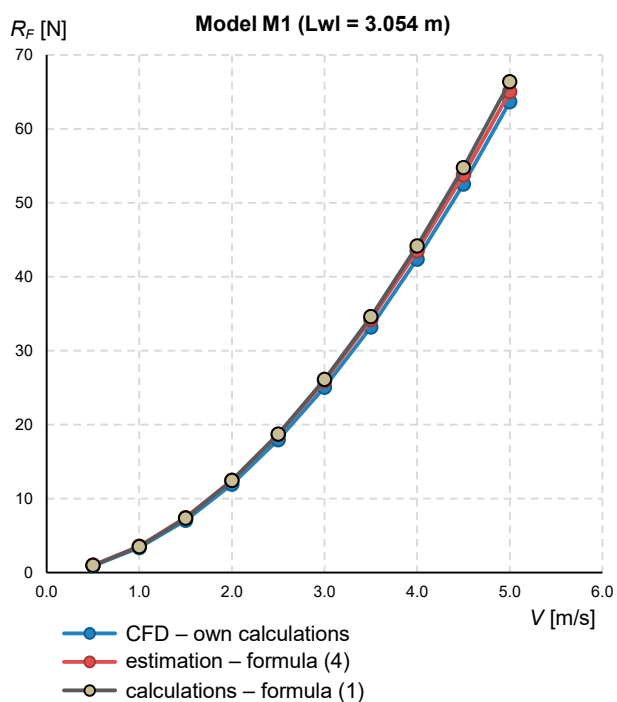


Figure 4. Substantive test of the developed method for the M1 ship model

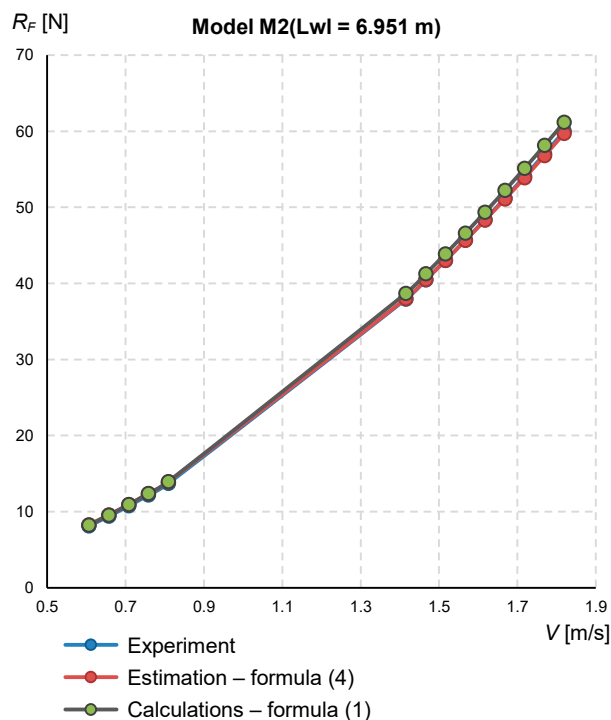


Figure 5. Substantive test of the developed method for the M2 ship model

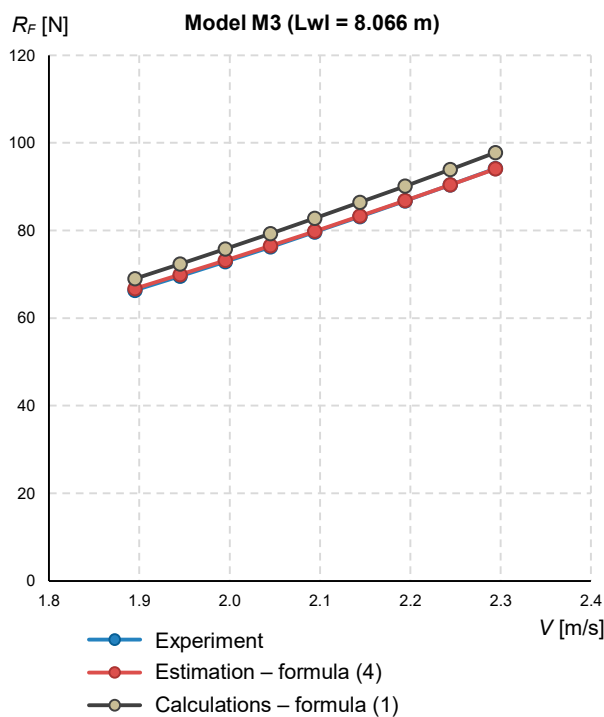


Figure 6. Substantive test of the developed method for the M3 ship model

**Table 2. Parameters of the M1 model and the results of the  $R_F$  estimation in the form of the relative error value**

Parameters of the M1 model			CFD – own calculations	Estimation (formula (4))		Calculations from formula (1)	
$L_{WL}$ [m]	$S$ [m <sup>2</sup> ]	$V$ [m/s]	$R_F$ [N]	$R_F$ [N]	error [%]	$R_F$ [N]	error [%]
3.054	1.885	0.5	0.95	1.03	-8.48%	0.99	-4.56%
		1.0	3.38	3.59	-6.17%	3.52	-4.12%
		1.5	7.08	7.45	-5.16%	7.38	-4.18%
		2.0	11.96	12.50	-4.49%	12.47	-4.26%
		2.5	17.97	18.67	-3.92%	18.74	-4.27%
		3.0	25.07	25.93	-3.42%	26.13	-4.24%
		3.5	33.21	34.22	-3.04%	34.62	-4.26%
		4.0	42.38	43.52	-2.68%	44.18	-4.24%
		4.5	52.54	53.79	-2.38%	54.77	-4.25%
		5.0	63.68	65.03	-2.11%	66.39	-4.25%

**Table 3. Parameters of the M2 model and the results of the  $R_F$  estimation in the form of the relative error value**

Parameters of the M2 model			Experiment	Estimation (formula (4))		Calculations from formula (1)	
$L_{WL}$ [m]	$S$ [m <sup>2</sup> ]	$V$ [m/s]	$R_F$ [N]	$R_F$ [N]	Error [%]	$R_F$ [N]	Error [%]
6.951	12.313	0.606	8.121	8.255	-1.65%	8.229	-1.32%
		0.657	9.402	9.548	-1.55%	9.536	-1.43%
		0.708	10.767	10.923	-1.45%	10.930	-1.51%
		0.758	12.184	12.351	-1.37%	12.380	-1.61%
		0.809	13.713	13.886	-1.26%	13.942	-1.67%
		1.415	37.921	37.988	-0.18%	38.676	-1.99%
		1.516	43.005	43.007	-0.01%	43.862	-1.99%
		1.617	48.377	48.301	0.16%	49.341	-1.99%
		1.718	54.038	53.867	0.32%	55.110	-1.98%
		1.769	57.009	56.779	0.40%	58.133	-1.97%
	1.819	59.984	59.701	0.47%	61.166	-1.97%	

**Table 4. Parameters of the M3 model and the results of the  $R_F$  estimation in the form of the relative error value**

Parameters of the M3 model			Experiment	Estimation (formula (4))		Calculations from formula (1)	
$L_{WL}$ [m]	$S$ [m <sup>2</sup> ]	$V$ [m/s]	$R_F$ [N]	$R_F$ [N]	Error [%]	$R_F$ [N]	Error [%]
8.066	13.073	1.895	66.303	66.684	-0.57%	68.969	-4.02%
		1.945	69.544	69.885	-0.49%	72.326	-4.00%
		1.995	72.854	73.152	-0.41%	75.755	-3.98%
		2.045	76.238	76.485	-0.32%	79.256	-3.96%
		2.094	79.611	79.815	-0.26%	82.756	-3.95%
		2.144	83.131	83.278	-0.18%	86.398	-3.93%
		2.194	86.721	86.807	-0.10%	90.110	-3.91%
		2.244	90.380	90.400	-0.02%	93.893	-3.89%
		2.294	94.109	94.058	0.05%	97.746	-3.87%

### Approximation of the total resistance $R_T$ for the ship models

From the regression analyses that were performed, the best formula approximating the resistance  $R_T$  has the following form:  $R_T = f(L_{WL}, B, T, C_B, V)$ :

$$R_T = a_1 L_{WL} + a_2 B^{13} + a_3 T^{0.67} + a_4 C_B + a_5 V^{-2.85} + a_6 L_{WL}^{0.98} B^{0.02} + a_7 L_{WL}^{0.6} T + a_8 (\ln(L_{WL}))^3 C_B^{4.65} + a_9 \left(\frac{1}{4}\right)^{L_{WL}} V^{0.04} + a_{10} B^{0.99} T^{0.73} + a_{11} B^{10} C_B^{15} +$$

$$+ a_{12} \frac{(\ln(V))^2}{B^{0.05}} + a_{13} (e^T)^2 e^{C_B} + a_{14} T^{5.15} V^{15} + a_{15} (\ln(C_B))^7 V^{11} + a_{16} \frac{T^{1.45}}{L_{WL}^{11} \cdot B^9} + a_{17} B^{4.1} T^{0.75} C_B^{0.89} + a_{18} T^{2.6} C_B^{1.35} V^6 + a_{19} \frac{L_{WL}^{14} (e^T)^3}{B^9 C_B^{2.8}} + a_{20} \frac{T^{2.15} (e^{C_B})^2 V^{10}}{B^{1.15}} + a_{21} L_{WL}^{0.35} B^{1.4} T^{0.7} (e^{C_B})^3 V^{2.75} \quad (5)$$

where, the values of the coefficients  $a_1$ – $a_{21}$  are given in Table 5.

The developed regression model includes cases where the individual ship models differ only in the value of one of the parameters that describe its geometry, e.g. the  $C_B$  coefficient; therefore, the pattern that was obtained is complex.

For the presented model, the values of the determination coefficient at the level  $R^2 = 0.9994$  and the standard estimation error  $Se = 1.885$  are satisfactory. Correcting the obtained model, e.g. by reduction of some of the elements, caused an increase in the standard estimation error to a large extent. Student's t-statistic with the significance level  $p$  (Table 5) indicates that all the explanatory variables are significant.

Table 5. Values of the estimation coefficients for the regression dependency (5) and results of the statistical tests

	Beta	Std Err. of Beta	Value t df = 408	p-level
a1	-4143.03	153.98	0,00 00E-01	
a2	-4,54	0,20	0,00 00E-01	
a3	-95,31	23,65	0,00 00E-01	
a4	593,71	27,19	0,00 00E-01	
a5	-4,34	0,27	0,00 00E-01	
a6	4362,74	161,97	0,00 00E-01	
a7	404,77	16,17	0,00 00E-01	
a8	-72,13	3,45	0,00 00E-01	
a9	-4903,43	252,11	0,00 00E-01	
a10	-2298,66	81,42	0,00 00E-01	
a11	-21,80	2,96	0,00 00E-01	
a12	33,77	1,97	0,00 00E-01	
a13	-334,61	13,16	0,00 00E-01	
a14	-0,00	0,00	0,00 00E-01	
a15	0,42	0,01	0,00 00E-01	
a16	11215,23	494,00	0,00 00E-01	
a17	448,03	17,03	0,00 00E-01	
a18	-4,24	0,08	0,00 00E-01	
a19	0,00	0,00	0,00 00E-01	
a20	0,03	0,00	0,00 00E-01	
a21	0,86	0,02	0,00 00E-01	

The statistical test for dependence (5) is shown in Figure 7.

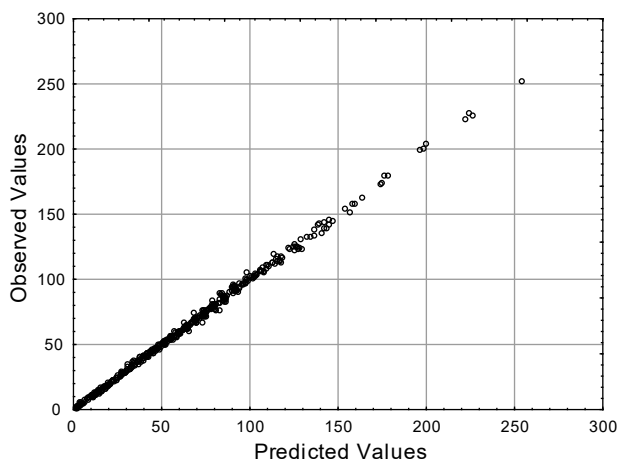


Figure 7. Chart of the observed values versus the predicted values

Substantive tests for the M2 and M3 models are shown in Figures 8 and 9 and in Tables 6 and 7 respectively.

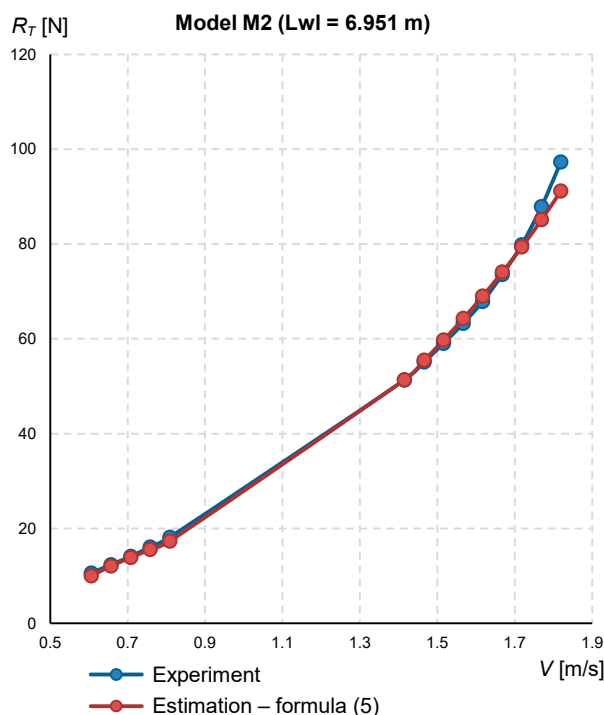


Figure 8. Substantive test of the developed method for the M2 ship model

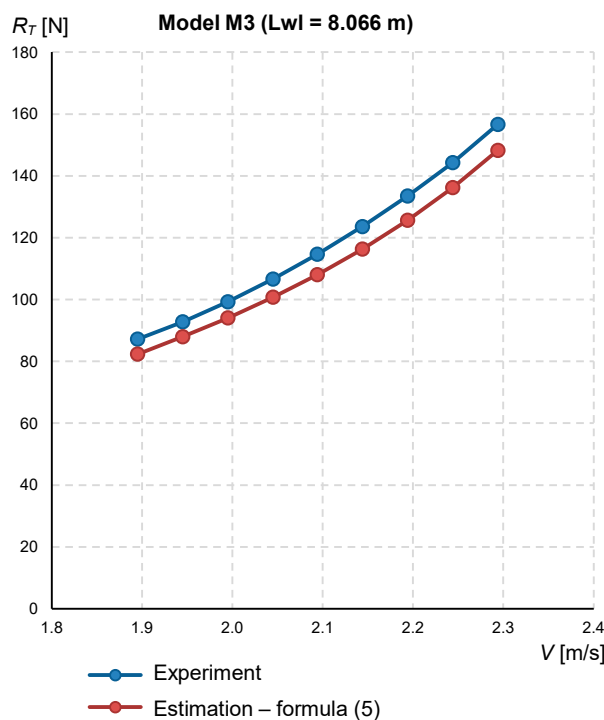


Figure 9. Substantive test of the developed method for the M3 ship model

**Table 6. Parameters of the M2 model and the results of the  $R_T$  estimation in the form of the relative error value**

Parameters of the M2 model					Experiment $R_T$ [N]	Estimation (formula (5)) $R_T$ [N]	Error [%]
$L_{WL}$ [m]	$B$ [m]	$T$ [m]	$C_B$ [-]	$V$ [m/s]			
6.951	1.197	0.463	0.7880	0.606	10.591	10.0106	5.48%
				0.657	12.356	12.08202	2.22%
				0.708	14.121	13.88313	1.68%
				0.758	16.083	15.57349	3.17%
				0.809	18.142	17.33425	4.45%
				1.415	51.288	51.35704	-0.13%
				1.466	55.113	55.48478	-0.67%
				1.516	59.035	59.73804	-1.19%
				1.567	63.252	64.30059	-1.66%
				1.617	67.861	69.01283	-1.70%
				1.668	73.549	74.09097	-0.74%
				1.718	79.825	79.37231	0.57%
				1.769	87.867	85.11746	3.13%
				1.819	97.281	91.16327	6.29%

**Table 7. Parameters of the M3 model and the results of the  $R_T$  estimation in the form of the relative error value**

Parameters of the M3 model					Experiment $R_T$ [N]	Estimation (formula (5)) $R_T$ [N]	Error [%]
$L_{WL}$ [m]	$B$ [m]	$T$ [m]	$C_B$ [-]	$V$ [m/s]			
8.066	1.213	0.451	0.6740	1.895	87.180	82.34676	5.54%
				1.945	92.770	87.94258	5.20%
				1.995	99.242	94.04162	5.24%
				2.045	106.597	100.745	5.49%
				2.094	114.639	108.0179	5.78%
				2.144	123.563	116.2998	5.88%
				2.194	133.467	125.6214	5.88%
				2.244	144.254	136.1838	5.59%
				2.294	156.611	148.2213	5.36%

## Discussion of the received results

The formula obtained to approximate the friction resistance  $R_F$  (4) of ship models is simple and is the product of the model's velocity  $V$ , two geometrical parameters ( $L_{WL}$ ,  $S$ ) and a constant coefficient ( $a_1$ ); this is due to the fact that  $R_F(V)$  is a parabolic function. The determination coefficient ( $R^2$ ) is very high, with a value of  $R^2 = 0.9999$ , and the standard error is small ( $Se = 0.366$ ). The comparison of the resistance values that were obtained shows that the approximation (4) is definitely better for most model speeds than in the case of approximation (1) from the literature (Molland, Turnock & Hudson, 2011). Only for low speeds was the approximation (4) slightly worse than approximation (1) – it should be noted, however, that the relationship (1) produces different values of the factor  $f$  depending on the ship model's length. However, the dependence (4) is the same for the entire assumed length range of the ship models.

The formula approximating the total resistance  $R_T$  (5) for ship models is more complex and consists

of 21 elements that encompass various geometric parameters of the ship models; this is due to the fact that the course  $R_T(V)$  (Figure 2) is not a parabolic function. The tests that were carried out showed that the calculated resistance  $R_T$  differs by only a few percent (maximum of 6%) from the value of the total resistance  $R_T$  measured in the model pool. Therefore, it can be concluded that the  $R_T$  approximation that was obtained will be useful in the preliminary design stage for calculating the resistance and power of a ship model's propulsion.

## Conclusions

The paper presents two approximation functions – one, the friction resistance  $R_F$ , and the other the total resistance  $R_T$  of ship models.

The approximation of the friction resistance  $R_F$  that was obtained produces better calculation results than the approximation presented in the literature (Molland, Turnock & Hudson, 2011), although it is a family of approximation formulas that are used for ship models of different lengths.

The approximated total resistance  $R_T$  that was obtained is more complex than in the case of  $R_F$ . This is due to the fact that the wave resistance  $R_W$  (Figure 2) changes within a large range (rising or falling) depending on the speed of the model. Such “wave” changes are difficult to approximate using a simple function for the full range of the length and speed of ship models, especially when considering the possibility of changing only one geometric parameter of the hull.

The tests carried out showed that both approximation functions ( $R_F$  and  $R_T$ ) are sufficiently accurate and that they may be useful for the design of experimental ship models, including unmanned ships.

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