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MODELING OF MIMO SYSTEM OVER FAST FADING CHANNELS

Abstract

Two models of the Alamouti multiple input – multiple output (MIMO) system have been formed: the analytical and simulation one. The first model is simple and it reflects the quasi-static Rayleigh fading phenomena, while the second one is advanced and it expresses the fast fading effects. The results of analysis and simulations show that the Alamouti MIMO architecture significantly over performs the SISO systems (the SNR gain ~20 dB), but mainly within the quasi-static environment. In mobile conditions this gain and/or the overall system capacity under go a sharp decline.

1. INTRODUCTION

The new communications systems in a road traffic requires the channels of high capacity. This concerns both to the standard communications as well as to the new monitoring and safety systems and even the searching network for crimes with a help of cars. The quantity of cars is counted in dozens of millions, their speed in motorways is getting up all the time. This creates the very severe situation regarding the capacity of transmission channels.

The frequency spectrum (2-11 GHz) is fully occupied. The new perspective is joined with the new multiple input – multiple output systems (MIMO). In slow Rayleigh fading environment these systems can increase the capacity proportional to the number of antennas at one side [5-7]. For fast fading Nakagami channels these figers are not known up to the present paper. It concers to the popular Alamouti architecture [3] and it still assumes the transmission functions of the channels are known to the receiver via an adaptive process [9]

2. RAYLEIGH FADING MODEL

The two transmitting and two receiving antennae Alamouti scheme, MIMO2x2, is considered throughout this paper [3]. Hence, four individual space-time channels are formed. At the beginning it is supposed that all these channels are uncorrelated and subjected to quasi-static Rayleigh changes of the transmission functions, $h(t) \approx h(t+T)$, where T – duration of a signal symbol. Then the probability density functions of the signal-to-noise power ratio are as follows

$$f(\gamma) = \gamma_0^{-1} e^{-\gamma/\gamma_0}, \quad F(\gamma) = e^{-\gamma/\gamma_0} \tag{1}$$

where γ is the value of a current SNR and γ_o is its mean; $f(\gamma)$ and $F(\gamma)$ are the probability density distributions, *ddf* and *cdf*, respectively.

For the selection combining diversity system the resultant probability density function for the *m*-th channel, $f_m(\gamma|N)$, can be derived on the basis of the positional statistics [1]

$$f_m(\gamma \mid N) = \frac{N!\, f(\gamma)}{(m-1)!\,(N-m)!} F^{m-1}(\gamma)[1 - F(\gamma)]^{N-m} \quad (2)$$

where N is the total number of individual channels (the diversity order).

The formula for the best channel $m=N$ is then as follows

$$f_N(\gamma \mid \gamma_0) = \frac{N}{\gamma_0} e^{-\gamma/\gamma_0}(1 - e^{-\gamma/\gamma_0})^{N-1}$$

(3)

Taking a mean of an error probability $P(\gamma)$ for the given mode of keying, e.g. DPSK, $P(\gamma)=exp(-\gamma/2$, in respect to the density distribution function $f_N(\gamma|\gamma_0)$ (4), we obtain the resultant BER formula for the best channel as follows

$$P_N(\gamma_0) = \frac{N}{\gamma_0}\int_0^\infty 0.5\, e^{-\gamma} e^{-\gamma/\gamma_0}(1-e^{-\gamma/\gamma_0})^{N-1} d\gamma = \frac{N}{2}\sum_{k=0}^{N-1}(-1)^k \binom{N-1}{k}\frac{1}{\gamma_0+k+1} \quad (4)$$

These same probabilities for the second best and other channels are incomparable, so they will be neglected with regard to (5). The final results for *N=1* (SISO) and *N=4* (MIMO) are as follows, Fig.1.

$$P_1(\gamma_0) = \frac{1}{2\gamma_0 + 2}$$

$$P_4(\gamma_0) = \frac{2}{\gamma_0+1} - \frac{6}{\gamma_0+2} + \frac{6}{\gamma_0+3} - \frac{2}{\gamma_0+4} \quad (5)$$

It should be noted that the original Alamouti architecture operates under maximum ratio combining rule (MRC). However, the difference between the selection combining and MRC is small enough in comparison with the channel movement effects and other environmental factors. For example, the discrepancy between calculated $P_4(\gamma_0)$ and the corresponding simulated curve do not exceed 1 dB, Fig.1. It can be shown that the same conclusion can be drawn for the differences between keying modes, PSK and DPSK.

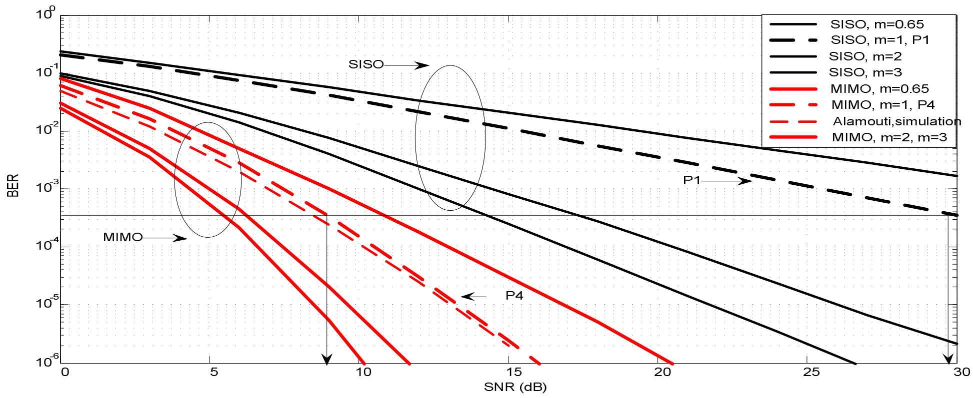

Fig.1. Calculated (P_1,P_4) and simulated BER curves vs. SNR for SISO and MIMO systems for quasi-static environment, F_DT_S=0.01

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3. ALAMOUTI MODEL OVER FAST FADING EFFECTS

3.1. Simulation procedure

The Monte Carlo simulation model has been developed in a similar manner as it is done in the literature for the Rayleigh case [4]. The Exact Doppler Shift technique (EDS) has been used as a basic operation [5]. The two normalized fading bandwidth were chosen, $F_D T_S$= 0.01 and $F_D T_S$=0.1, where F_D is maximum Doppler shift and T_S – time duration of the signal symbol. The state $F_D T_S$=0.01 corresponds to the very slow movement of objects in the signal path, while the state $F_D T_S$=0.1 - to the fast movement [6]. The fast movement causes multiplicative interference.

Instead of commonly used Rayleigh distribution of fades, the more general Nakagami-*m* model has been adapted and applied [5]. This stems from the fact that numerous papers report significant departures of the physical processes from the Rayleigh distribution [6], [7].
The values of the fading depth parameter *m* and the cross-correlation coefficient *r* are as follows: *m* = [0.65, 1, 2, 3] and *r* = [0, 0.5, 0.7]. The details are explained in [2].

3.2. Quasi-static characteristics

In the first step, the conditions close-to-ideal were assumed, i.e. the cross-correlation was completely excluded and the extremely slow movement of objects within the channels were admitted by the restriction of *r*=0 and $F_D T_S$=0.01.

The BER characteristics, both for SISO and MIMO are shown in Fig.1 (black and red lines, respectively). The Rayleigh case for *m* = 1 is also singled out by dashed lines (eq. 6) . **We can see that MIMO in quasi-static environment over performs SISO by more than 20dB for BER≤10^{-3}**. It is also evident that the depth of fading - parameter *m* - affects considerably the BER characteristics, especially for SISO group. This confirms the earlier objections on the unsuitability of the Rayleigh model for fading analysis.

3.3. Movement effects

The next group of characteristics refers to the fast movement scenario and it is shown in Fig.2. The normalized fading bandwidth has been set to $F_D T_S$=0.1. The other parameters remain unchanged.

One can see, that characteristics have changed much in reference to Fig.1. A new phenomenon appeared: the irreducible error zone (floor effect). This is a result of multiplicative interference caused by the moving objects within the channels (cars, trees, persons). In a consequence, the phase of signal is changed during the symbol frame, which causes additional errors, independently on noise.

It is characteristic that MIMO system is more sensitive to the movement parameters than SISO. **MIMO over performs SISO only for the low range of SNR.** Above some threshold (SNR≈20dB) the SISO offers lower error rate than MIMO, especially for non deep fading (*m* ≥ 2).

The floor effect is a serious harmful phenomenon. There is no cheap and effective countermeasure against it, except for the reduction of the transmission rate and the overall capacity. Sometimes additional antennas or the more advanced strategies of reception are proposed, e.g. minimum mean square error (MMSE) or maximum likelihood space time (MLST) [6], [9], [10]. According to the authors the third antenna gives the best results.

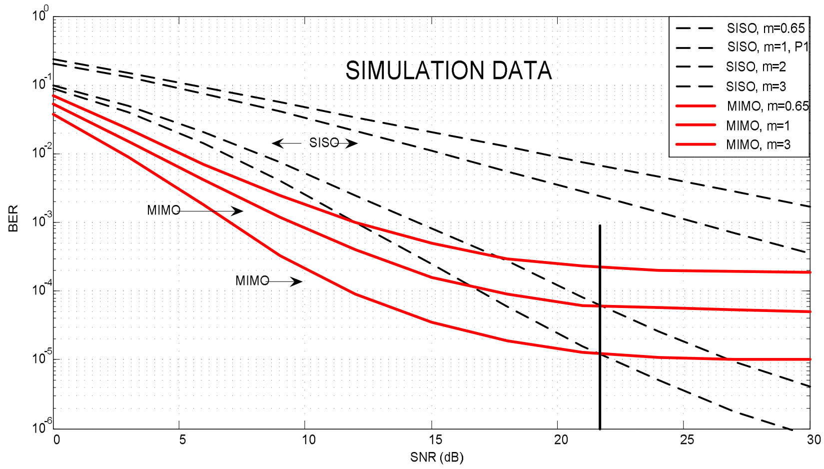

Fig.2. BER curves for SISO and MIMO systems in mobile environment, F_DT_S=0.1, r=0,

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3.4. Joint action of two factors

The cross-correlation generally makes a little effect on BER in the quasi-static environment. The correlation of *r*=0.5 at *m*=3 causes losses - in reference to the uncorrelated case - of the order of 1 dB at the level of BER=10^{-6}, while the correlation of *r*=0.7 increases these losses to ~2 dB [8]. The joint effect of the cross-correlation and mobility (F_DT_S =0.1) is, however, severe, see fig. 3.

The BER for *m*=3 and *r*=0 is the same as in Fig.2 (the lowest curve). However, as far as the cross-correlation or the depth of fading or both are increasing, the probability of error rate increases fast. In the worst case of fading ($F_DT_S \geq 0.1$, $r \geq 0.7$, $m \leq 0.65$), the $BER_{min} \approx 10^{-3}$ independently on SNR.

This is a very poor result. Fortunately, in the real world the transmission rate can be adopted to get F_DT_S <0.1, so the BER may be hold better, but the overall system capacity will be proportionally decreased. it is worth to note, that a coincidence of two or more severe circumstances can seriously deteriorate the Alamouti MIMO system performance [9].

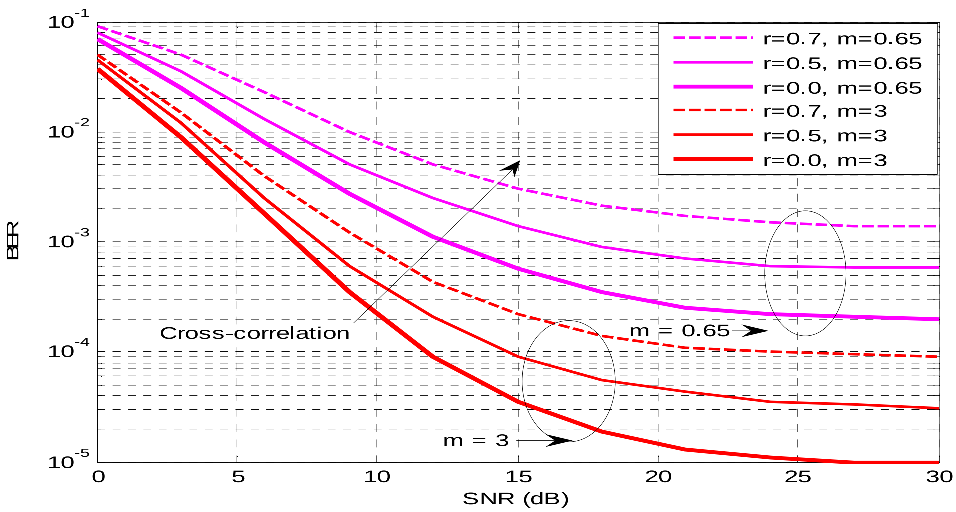

Fig.3. BER curves for MIMO system in presence of cross-correlation and movement, 0 ≤ r ≤0,7, F_DT_S=0.1.

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CONCLUDING REMARKS

In the first part of the paper an extra simple model of the Alamouti MIMO system composed of two transmitting and two receiving antennas and operating under the slow Rayleigh fading was proposed. It is based on the theory of positional statistics and it defines the coarse MIMO characteristics for the quasi-static environment. In the second part of paper the fast fading conditions are modeled in the process of Monte Carlo simulation.

It was found, basing on these two models, that the Alamouti system is highly sensitive to fast fading conditions. Its large steady-state gain (~20 dB) starts to drop at the normalized fading bandwidth of the order of F_DT_S=0.1. If the depth parameter of fading $m \leq 1$ and the coefficient of cross-correlation $r \geq 0.7$, the lowest possible error rate can reach merely 10^{-3}, independently on SNR.

Any increase of the normalized bandwidth up to F_DT_s=0.01 and/or any increase of the cross-correlation over r=0.5 causes a substantial decline of the SNR gain and/or the system capacity. One of the effective countermeasure against this decline is the third receiving antenna, [10].

MODELOWANIE SYSTEMÓW MIMO W ŚRODOWISKU ZANIKÓW SZYBKICH

Streszczenie

W referacie poddano eksperymentom symulacyjnym dwa modele MIMO Almaoutiego. Model funkcjonujący w warunkach quasi-statycznych zaników Rayleigha oraz w warunkach zaników szybkich Nakagami'ego. Dokonano oszacowania jaki jest rzeczywisty zysk zastosowania MIMO względem systemu jednokanałowego SISO. W modelowaniu wykorzystano Metodę Monte Carlo.

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