”Research Bulletin the Jan Wyzykowski University. Studies in Technical Sciences”
2016 (5), p. 9–26

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Electronic Circuit Design by Pole and Zero Distribution Optimization via the Semi-Symbolic Transimpedance Method

Summary: In this work a new method of linear circuit design in frequency domain by poles (zeros) distribution optimization is presented. The method, which uses the relationship between poles and appropriate sums of circuit time – constants, does not need the poles determination explicitly. New relationships, allowing us to calculate the time – constants matrix for circuits having capacitors and inductors as reactive elements, have been derived. Thanks to this, the elements of this matrix can be counted in a uniform manner by the transimpedance method. In the first stage of the method the criterion function is generated in semi–symbolic form, while in the second stage the optimization process is performed. The optimization loop does not include circuit equations formulation and solution. Thanks to this fact the method proposed appears to be very efficient. The examples of optimal capacitors and inductors chosen in such a way as to reach the required transfer characteristics have also been included.

Keywords: Computer Aided Design, Electronic Circuit Optimization, Pole and Zero Approximation, Symbolic Methods.

Projektowanie układów elektronicznych poprzez optymalizację rozkładu biegunów i zer z wykorzystaniem semisymbolicznej metody transimpedancyjnej

Streszczenie: W artykule została przedstawiona nowa metoda projektowania elektronicznych układów liniowych w dziedzinie częstotliwości poprzez optymalizację rozkładu biegunów i zer. Metoda, w której wykorzystano związek pomiędzy biegunami (zerami) a stałymi czasowymi obwodu nie wymaga wyznaczania biegunów explicite. W pracy zostały wyprowadzone nowe zależności pozwalające na wyznaczenie macierzy stałych czasowych układu zawierającego zarówno kondensatory jak i cewki. Dzięki temu elementy tej macierzy są obliczane w jednakowy sposób metodą transimpedancyjną. W pierwszym etapie prezentowanej metody jest wyprowadzana funkcja kryterialna w postaci semisymbolicznej, podczas, gdy w etapie drugim jest przeprowadzany proces optymalizacyjny. Tak więc, pętla optymalizacji nie obejmuje formułowania równań układu i ich rozwiązywania. Dzięki czemu proponowana metoda okazała się bardzo efektywną pod względem wymaganego czasu obliczeń. Zostały załączone przykłady optymalnego doboru pojemności i indukcyjności w taki sposób, aby układy posiadały wymagane charakterystyki częstotliwościowe.

Keywords: Projektowanie wspomagane komputerowo, optymalizacja układów elektronicznych, aproksymacja biegunów i zer, metody symboliczne.

1. Introduction

The proper selection of electronic elements during the designing of electronic circuits in frequency domain ensures the proper shapes of network characteristics. The optimal parameter chosen in such a way as to reach the demanded transfer characteristics can be performed in different ways.

One of them is the method called the frequency domain optimization [1] in the loop of which the formulation as well as solution of the circuit equations is performed. In this approach the demanded frequency characteristics (such as magnitude, phase or delay characteristics) are approximated by actual characteristics using an optimization method operating on a determined set of parameters. This method very often suffers from weak convergence because of ill-conditioning of the hessian matrix (many local minima and deep winding valleys exist). Therefore, this is rather an extremely time consuming method and requires fast analysis and effective optimization methods.

Another approach is the optimization on the complex plane. Although several numerical methods for poles determination exist such as QR or QZ [3, 4, 15], there is no efficient numerical method for their calculation, especially for large circuits [5]. The necessity of poles determination at each step of the optimization process makes these methods rather cumbersome. Though the eigenvalue approach is more numerically stable than that basing on calculating polynomial coefficients [7], the big disadvantage of these methods is that the eigenvalue–finding stage often encounters singular matrices due to the topological structure of the network. On the other hand, the determination of poles and zeros in symbolic form is very complicated, particularly in the case of more than four poles/zeros (pole splitting method [3], state variable method [6]). Moreover, some symbolic approximated methods can suffer sometimes from their insufficient accuracy (matrix approximation method [8]). The coefficients matching method also belongs to this group [1, 9]. This method needs to have at its disposal symbolic representation of the network function, which becomes useless in the case of large circuits. The interpolative approach to symbolic analysis represents some solutions to this problem [10].

In this work a new semi-symbolic method of linear circuit design in frequency domain, which does not need the poles and zeros calculation explicitly, is presented. The main idea of the method relies on the fact that in the first stage the criterion function is generated in semi – symbolic form. In the second stage the optimization process is performed. The optimization loop does not include circuit equations formulation and their solution. Thanks to this fact the method proposed appears to be very efficient. As the criterion function, the relationship between poles (and zeros) and appropriate sums of circuit time – constants have been applied. In work [11] this method was applied to circuits having capacitors only as reactive elements. In this work, by using a modified nodal matrix description of the network as well as by generalization of transresistance and time constants matrices, this method was applied to the circuits having both type of elements – capacitors and inductors. A generalization of Haley's time –constants matrix has been developed here, which allows us to recognize whether and why the optimization task is solvable or not, as well as thanks to these relationships, the elements of the time-constant matrix can be counted in a uniform manner by the transimpedance method. This approach simplifies the whole algorithm.

In Section II the theoretical background of the method has been delivered. In Section III the optimization task has been formulated. Section IV details the algorithm and computational testing of the new method.

2. Theoretical background of the Method

2.1. Transfer functions

Let's consider a linear, time-invariant and lumped electronic circuit, which is described by the nodal (or modified nodal) equation [5]

$$YV = w \tag{1}$$

where: Y – nodal (or modified nodal) matrix, V – vector of node voltages (or node voltages and currents added for independent voltage sources and inductors), w – vector of independent currents (or independent source currents and independent source voltages).

Assuming that the system output is d^TV, the network transfer function H(s) for a single input w_{in} can be expressed as [3]:

$$H(s)= \det\tilde{Y}(w_{in},d)/\det Y \tag{2}$$

where: $\det Y$ – the main determinant of matrix Y,

$\tilde{Y}(w_{in},d)$ – is the Y matrix with in – th row replaced by the d^T vector.

The network poles p_i are the values of s such that $\det Y = 0$, and transfer function zeros z_i are values of s such that $\det\tilde{Y}(w_{in},d)$. Hence the network function H(s) can be expressed in the form

$$H(s,\mathbf{x}) = H_0 \frac{(z_1(\mathbf{x})+s)(z_2(\mathbf{x})+s)....(z_n(\mathbf{x})+s)}{(p_1(\mathbf{x})+s)(p_2(\mathbf{x})+s)....(p_m(\mathbf{x})+s)} \tag{3}$$

this corresponds to the rational representation:

$$H(s) = \frac{P(s)}{Q(s)} = \frac{a_0 + a_1 s + ... + a_n s^n}{b_0 + b_1 s + ... + b_m s^m} \tag{4}$$

where: $p_i(x)$, $z_j(x)$ – poles and zeros of the transmittance, respectively, $a_i = a_i(x)$, $b_i = b_i(x)$ – real coefficients, $\mathbf{x} = [x_1, x_2,..., x_M]^T$ – vector of circuit parameters. H_0 – constant.

Poles and zeros can be real or conjugate complex. Because the calculation of zeros is performed in the same way as calculation of poles by using one rank modified admittance matrix, our further considerations will concern the denominator coefficients b_i determination only.

2.2. Poles

The relationship between poles and polynomial coefficients has the well – known form of symmetric functions:

$$b_0 = 1, \tag{5a}$$

$$b_1 = \sum_{i=m}^{m} \frac{1}{p_i} \tag{5b}$$

$$b_1 = \sum_{i=1}^{m-1} \sum_{j=i+1}^{m-1} \frac{1}{p_i p_j} \tag{5c}$$

$$b_m = 1/(P_1 P_2 \ldots P_m). \tag{5d}$$

On the other hand, the characteristic equation resolved from the determinant

$$\det(Y) = \det(G+sC) = 0 \tag{6}$$

is

$$Q(\lambda) = \lambda n + \sigma_1 \lambda_{n-1} + \ldots + \sigma_{n-1} \lambda + \sigma_n = 0 \tag{7}$$

where: $\lambda = -1/s$.

Knowing the relation between the characteristic polynomial coefficients and appropriate entries of the Y matrix, we are able to express the coefficients b_i (5) as functions of these entries.

To find these relationships two cases of the parameter vector **x** will be considered.

Case of capacitance vector

$$x = [C_1, C_2, \ldots, C_{nc}]^T, \tag{8}$$

The admittance matrix $\boldsymbol{Y}$ can be formulated as a sum:

$$Y = G + sC, \tag{9}$$

where: G and C are the conductance and capacitance (nxn) matrices, respectively.

The generalized eigenvalues equation corresponding to (9), from which the poles of the circuit can be determined is:

$$(G + sC)z = 0, \tag{10}$$

where: z **–** is an eigenvector of Y.

In work [2] it was shown, that, if a circuit has n_c capacitors C_α, $\alpha = 1,..., n_c$, then the matrix C can be decomposed in the form:

$$C = A_c C' A_c^T = \sum_{\alpha=1}^{n_c} q_\alpha q_\alpha^T C_\alpha \tag{11}$$

where: A_c – is an $n \times n_c$ incidence matrix for the capacitors.

If a capacitor C_α is connected between nodes k and l, then each column α in A_c is the connection vector: $q_\alpha = e_k - e_l$. Diagonal $n_c \times n_c$ matrix C' has diagonal elements: C_α, $\alpha = 1,..., n_c$. It was further shown that if dc poles are assumed to not exist (i.e. $R = G^{-1}$ exists), then applying (11) and after some transformations the equation (10) can be formulated in the reduced form:

$$(T - \lambda I_{n_c})\, z_{n_c} = 0 \tag{12}$$

where: $z_{nc} = A^T_c z$ – transformed n_c-dimensional eigenvector, for $s \neq 0$, $\lambda = \frac{1}{s}$, (13)

I_{nc} – is n_c – dimensional identity matrix.

The entries of the $n_c \times n_c$ T matrix known as the *time-constant matrix* defined by:

$$T = A_c^T R A_c C' \tag{14}$$

are equal to

$$T_{\alpha\beta} = q_\alpha^T R q_\beta C_\beta = R_{\alpha\beta} C_\beta \tag{15}$$

where: $R_{\alpha\beta}$– is the transresitance between ports β and α.

If the capacitors are independent (e.g. capacitor loops don't exist), rank T = rank $C = n_c$, then n_c finite poles of the network are determined by the eigenvalues of the matrix T:

$$p_i = -\frac{1}{\lambda_i} \quad \text{for } i = 1,\dots n_c. \tag{16}$$

B The case of capacitance and inductance vector

Let us consider now the n – node circuit containing n_C capacitors and n_L inductors,

$$x = [C_1, C_1, ..., Cn_C, L_1, L_2, ..., Ln_L]^T = [D_1, D_2, ..., D_M]^T,\ M = n_C + n_L, \tag{17}$$

for which the modified (MNA) nodal matrix Y with part exposed inductance will be partitioned as a sum of block matrices [5,17]:

$$\tilde{Y} = \left(\begin{bmatrix} G & A_L \\ A_L^T & 0 \end{bmatrix}\right) + s\left(\begin{bmatrix} C & 0 \\ 0 & -L' \end{bmatrix}\right) \tag{18}$$

where: G and C are the conductance and capacitance n x n matrices, respectively,

$L' = diag(L)$ – diagonal inductance $n_L \times n_L$ matrix

A_L – inductance incidence $n \times n_L$ matrix.

The generalized eigenvalue equation corresponding to (18), from which the poles of the circuit can be determined, is:

$$\left(\begin{bmatrix} G & A_L \\ A_L^T & 0 \end{bmatrix}\right) + s\left(\begin{bmatrix} C & O \\ O & -L' \end{bmatrix}\right)\tilde{z} = 0 \qquad (19)$$

where: $\tilde{z}$ is an eigenvector of $(n+n_L) \times (n+n_L)$ matrix $\tilde{Y}$.

Although the poles distribution determination basing on eq. (19) is possible, the knowledge concerning the existence of its solution or the reasons for the nonexistence of this solution is not attainable. To overcome this problem, the equation (19) will be transformed to the form similar to that like (12).Taking into account (11) the generalized eigenvalue equation (19) can be formulated as

$$\left(s^{-1} I + \begin{bmatrix} G & A_L \\ A_L^T & 0 \end{bmatrix}\right) + s\left(\begin{bmatrix} C & O \\ O & -L' \end{bmatrix}\right)\tilde{z} = 0 \qquad (20)$$

where: I – is $n + n_L$ – dimensional identity matrix, $C = dia\ g\ (C)$.

Using the block matrix inversion method and some matrix manipulations one gets

$$(\tilde{T} - \lambda I)z_M = O \qquad (21)$$

where: z_M – transformed $M = n_C + n_L$ – dimensional eigenvector, for $s \neq 0$, $\lambda = \frac{1}{s}$,
I_M – is M-dimensional identity matrix.

The details of the transformation of eq. (20) to suitable form (21) as well as determination of the entries of the generalized *time-constant M×M* matrix $\tilde{T}$ can be comprehensibly explained using the following theorem.

Theorem 1

If the conductance matrix G and transresistance matrix $R_{LL} = A_L^T G^{-1} A_L$ formulated for the circuit described by the modified nodal admittance matrix (18) are nonsingular, then the eigenvalue equation (21) will exist corresponding to this matrix with generalized *time-constant M×M* matrix $\tilde{T}$, which can be defined as a matrix partitioned:

$$\tilde{T} = \begin{bmatrix} T_I & T_{II} \\ T_{III} & T_{IV} \end{bmatrix} \qquad (22)$$

having the following parts:

$$\tilde{T}_I = [(R_{LL})] - R_{CL} R_{LL}^{-1} R_{LC})\ C' \qquad (23a)$$

$$\tilde{T}_{II} = R_{LL}^{-1} L' \qquad (23b)$$

$$\tilde{T}_{III} = R_{LL}^{-1} R_{LC}\ C' \qquad (23c)$$

$$\tilde{T}_{IV} = R_{LL}^{-1} L' \qquad (23d)$$

where:

$$R_{LL} = A_L^T R A_L \qquad (24)$$

is the transresistance $n_L \times n_L$ matrix determined between inductor ports;
$C' = diag(C)$, $L' = diag(L)$, and $R = G^{-1}$,

$$\mathrm{R}_{CC} = \mathrm{A}_L^T \mathrm{R} \mathrm{A}_C \tag{25}$$

is the transresistance $n_c \times n_c$ matrix determined between capacitor ports,

$$\mathrm{R}_{CL} = \mathrm{A}_L^T \mathrm{R} \mathrm{A}_L \tag{26}$$

is the transresistance $n_c \times n_L$ matrix determined between capacitor and inductor ports, and finally,

$$\mathrm{R}_{LC} = \mathrm{A}_L^T \mathrm{R} \mathrm{A}_C \tag{27}$$

is the transresistance $n_L \times n_c$ matrix determined between inductor and capacitor ports.

The proof of this theorem is delivered in Appendix A.

The matrix $\tilde{T}$ can be expressed as a multiplication of two matrices

$$\tilde{T} = \tilde{\mathrm{R}}\, diag(D) \tag{28}$$

where:

$$\tilde{R} = \begin{bmatrix} \tilde{R}_I & \tilde{R}_{II} \\ \tilde{R}_{III} & \tilde{R}_{IV} \end{bmatrix} \quad \begin{bmatrix} [(\mathrm{R}_{LL})] - \mathrm{R}_{CL}\mathrm{R}_{LL}^{-1}\mathrm{R}_{LC}) & \mathrm{R}_{CL}\mathrm{R}_{LL}^{-1} \\ \mathrm{R}_{LL}^{-1}\mathrm{R}_{LC} & \mathrm{R}_{LL}^{-1} \end{bmatrix} \tag{29}$$

is the generalized transresistance matrix and

$$diag(D) = \begin{bmatrix} C & 0 \\ 0 & L' \end{bmatrix} \tag{30}$$

is the extended diagonal matrix, $C' = diag(C)$, $L' = diag(L)$.

It should be noticed that if the capacitors and inductors are independent (e.g. capacitor loops or inductor cut sets do not exist), rank $\tilde{T} = M$, then M finite poles of the network will be given by the eigenvalues of the matrix $\tilde{T}$:

$$p_i = -\frac{1}{\lambda_i}, \qquad \text{for } i = 1, \ldots, M. \tag{31}$$

2.3. The relationship between poles (zeros) and time constants

Let's consider a circuit with n_c capacitors and n_L inductors for which the generalized time-constant matrix $\tilde{T}$, having eigenvalues, λ_i, $i = 1, 2, \ldots, M$, exists. The scalar invariant constraints for traces of $\tilde{T}$ matrix can be expressed in the following compact form [2]:

$$\sum_{i_1 < i_2 < \ldots < i_k} \lambda_{i_1} \lambda_{i_2} \cdot \ldots \cdot \lambda_{i_k} = T_k \tag{32}$$

where: $k = 1, 2, \ldots, r$, $\qquad r = \text{rank } \tilde{T} \le M$.

Taking into account relationship (31), the constraints in (32) are given consecutively by

$$\sum_{i=1}^{r} p_i^{-1} = -T_1 \tag{33a}$$

$$\sum_{i=1}^{r-1} \sum_{j=i+1}^{r} (p_i p_j) \tag{33b}$$

$$\sum_{i=1}^{r-2} \sum_{j=i+1}^{r-1} \sum_{k=j+1}^{r} (p_i \tag{33c}$$

$$\sum^{r-(n-1)} \sum^{r-(n-2)} \sum^{r-(n-3)} \dots \sum^{r} (p_i p_j p_k \dots p_m p_n)^{-1} = (-1)^r T_l \tag{33d}$$

Generally, we can tell that traces are some functions of poles (zeros): .

On the other hand the traces can be estimated by time – constants:

$$\tilde{T}_1 = \sum_{\alpha=1}^{M} T_{\alpha\alpha} = \sum_{\alpha=1}^{M} R_{\alpha\alpha} D_\alpha \tag{34a}$$

$$\tilde{T}_2 = \sum_{\alpha=1}^{M-1} \sum_{\beta=\alpha+1}^{M} \det \begin{bmatrix} R_{\alpha\alpha} & R_{\alpha\beta} \\ R_{\beta\alpha} & R_{\beta\beta} \end{bmatrix} D_\alpha D_\beta \tag{34b}$$

$$\tilde{T}_3 = \sum_{\alpha=1}^{M-2} \sum_{\beta=\alpha+1}^{M-1} \sum_{\gamma=\beta+1}^{M} \det \begin{bmatrix} R_{\alpha\alpha} & R_{\alpha\beta} & R_{\alpha\gamma} \\ R_{\beta\alpha} & R_{\beta\beta} & R_{\beta\gamma} \\ R_{\gamma\alpha} & R_{\gamma\beta} & R_{\gamma\gamma} \end{bmatrix} D_\alpha D_\beta D_\gamma \tag{34c}$$

...

$$\tilde{T}_n = \sum_{\alpha=1}^{M-(n-1)} \sum_{\beta=\alpha+1}^{M-(n-2)} \sum_{\gamma=\beta+1}^{M-(n-3)} \dots \sum_{n=\psi+1}^{M} (\det \begin{bmatrix} R_{\alpha\alpha} & R_{\alpha\beta} & R_{\alpha\gamma} & \dots & R_{\alpha\psi} & R_{\alpha n} \\ R_{\beta\alpha} & R_{\beta\beta} & R_{\beta\gamma} & \dots & R_{\beta\psi} & R_{\beta n} \\ R_{\gamma\alpha} & R_{\gamma\beta} & R_{\gamma\gamma} & \dots & R_{\gamma\psi} & R_{\gamma n} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ R_{\psi\alpha} & R_{\psi\beta} & R_{\psi\gamma} & \dots & R_{\psi\psi} & R_{\psi n} \\ R_{n\alpha} & R_{n\beta} & R_{n\gamma} & \dots & R_{n\psi} & R_{nn} \end{bmatrix} D_\alpha D_\beta D_\gamma \dots D_n) \tag{34d}$$

where: D_j denotes *j –th* component of the parameter vector (17). Generally, we can tell that traces are some functions of network parameters: $\tilde{T}_j = \tilde{T}_j(X)$

2.4 Determination of two – port transresistances

In this work, the so called *transimpedance method* is proposed for the determination of transresistances [11, 12, 14]. Consider the two – port shown in Fig. 1 in which the circuit resulting after extracting all capacitors and inductors, short – circuiting all independent voltage sources and open – circuiting all independent current sources is characterized by the conductance matrix G. Port $\beta = (\beta_1, \beta_2)$ is formed by extracting element D_β and port $\alpha = (\alpha_1, \alpha_2)$ is formed by extracting element D_α. Dividing voltage V_α (measured at port α) by excitation current I_β (at port β) we get the transresistance from port β to port α which can be expressed by elements of the matrix $R = G^{-1}$:

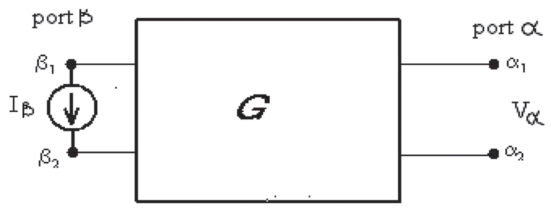

Figure 1. Explanation of the transresistance $R_{\alpha\beta}$ definition.

$$R_{\alpha\beta} = V_\alpha / I_\beta, = R(\alpha, \beta) = r_{\alpha 1 \beta 1} - r_{\alpha 1 \beta 2} - r_{\alpha 2 \beta 1} + r_{\alpha 2 \beta 2} \quad (35)$$

where: r_{ij} represents the entry from the i – th row and the j – th column of R, α_i, i = 1, 2 – pair of natural numbers representing nodes of port α. The entries of G^{-1} matrix, as it is well – known, can be determined by:

$$r_{ij} = \frac{(-1)^{i+j} M_{ji}}{\det G} = \frac{\Delta_{ji}}{\det G}, \quad (36)$$

where: $\Delta_{ji} = (-1)^{i+j} M_{ji}$ is the ji – th cofactor of G, $\det G = |G| \neq 0$ is the main determinant of G.

Application of the LU – factorization method speeds up the whole process of transresistance calculation significantly. It should be pointed out that these transresistances can be obtained also in symbolic form, if needed [12]. Consider a two – port shown in Fig. 2. Denote its input terminals as a pair $i = (i_1, i_2)$ and its output terminals as $o = (o_1, o_2)$. Let the elements extracted D_α be connected to pairs of nodes $\alpha = (\alpha_1, \alpha_2)$, α = 1, 2... M. All kind of transresistances (24) – (27) can be determined using the method delivered above. Although zeros can be determined in the same way as poles for modified admittance matrix $\tilde{Y}(w_{in}, d)$ they can be also obtained by the method delivered above with only small modification of the time – constants matrix T [2,3,14].

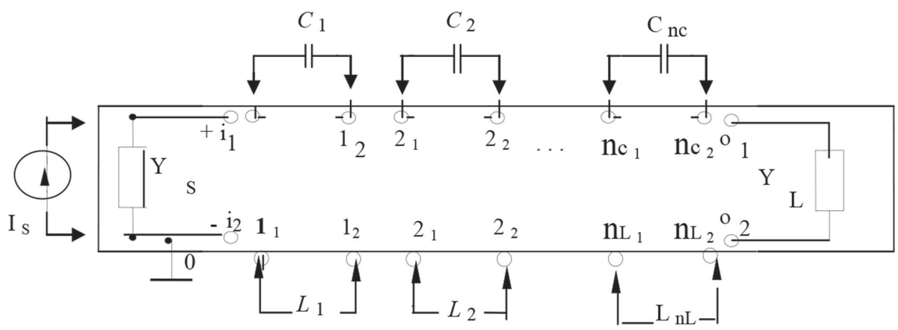

Figure 2. Network prepared for time – constant matrix calculation.

3. Optimization Task

The goal of the optimization task is to find such circuit parameters, for which the network function will meet the requirement shape. The optimization task is formulated in the following way:

$$\text{mim } [F\{x)/\ x \in X(x)\} \tag{37a}$$

where:

$$F = \sum_{j=1}^{jmax} \left(\text{EMBED Equation.3 } (\mathbf{x}) - T_j(\mathbf{p}) \right)^2 \tag{37b}$$

The feasible solutions are constrained by the following set of parameters:

$$X(x_1) = \{x_1 | x_{min_i} \le x_1 \le x_{max_i}\},\ i = 1, 2, \ldots, M. \tag{37c}$$

When all finite poles are taken into account in this task, the number of equations should be equal to the number of parameters (variables): $r = M = jmax$ and the following relation should be fulfilled:

$$\tilde{T}_j = T_j,\ j = 1,2,\ldots, M. \tag{38a}$$

In this case the absolute minimum should reach value 0, but in cases when some insignificant poles are neglected:

$$M \ge r = jmax \text{ and } \tilde{T}_j = T_j,\ j = 1,2,\ldots, r. \tag{38b}$$

To avoid the unstable solutions, the not important poles should be moved to infinity (to very high frequencies). The set of parameters should be restricted appropriately, in accordance with the demands of their range of work. To avoid operating large numbers during optimization process, the criterion function (37b) can be normalized:

$$F = \sum_{j=1}^{jmax} \left(1 - \text{EMBED Equation.3 } \frac{\mathbf{x}}{T_j(\mathbf{p})} \right)^2 . \tag{39}$$

Basing on the method presented the universal computer program *SemiSymPolOpt* was written in MS Visual Basic computer language. The main computational effort of this program is devoted to the derivation of the functions (34) in semi symbolic form and the generalized time constants matrix (22). Derived relationships allow us to identify what causes the task to be unsolvable. The computer program monitors these cases (when matrices G and/or R_{LL} are singular).

As the optimization method, the Rosenbrock procedure with constraints [13] was applied. As the stopping criterion the minimal value of criterion function was applied: $F < \varepsilon$. If this condition is not met, then the program will continue calculations until the number of function evaluations will reach the required number: $L < L_{max}$ and then the Rosenbrock procedure can be followed by the Fletcher-Reeves procedure [13]. For the Fletcher-Reeves procedure the computer program generates gradients in semi-symbolic form, too. Special attention should be paid to the choice of starting points. In this matter an important role is played by the engineer's knowledge. The computer program is supplied with the automatic (random) choice of starting points as an option. The computer program was supplied with QR procedure [15], which can be used, optionally, once if the user wants to verify the results of optimization.

4. Algorithm and Computational Experiments

The Algorithm

Basing on the theoretical background delivered above, the following algorithm was formulated:

Step 1. Determine poles (zeros), which have to be realized basing on requirements concerning the shape of the network characteristics demanded.

Step 2. Identify capacitor and inductor ports: $\alpha = (\alpha_1, \alpha_2)$, $\alpha = 1, 2, \ldots, M$.

Step 3. Extract all capacitors and inductors: D_α, $\alpha = 1, 2, \ldots M$. and short-circuit independent voltage sources, and open-circuit independent current sources.

Step 4. Formulate the conductance matrix G.

Step 5. Determine the transresistances needed R_{ij}.

Step 6. Calculate the appropriate elements of the generalized time – constant matrix: $T_{\alpha\beta} = R_{\alpha\beta} . D_\beta$

Step 7. Calculate the appropriate cofactors of the matrix traces: $T_1, \ldots, T_M$ and write in semi-symbolic form (34) and form equations (38).

Step 8. Form the criterion function and solve the task (37).

Derived relationships (theorem1 and (34) and (35)) allow us to identify what causes the task to be unsolvable and, thanks to these relationships, the elements of the time-constant matrix can be counted in a uniform manner by the transimpedance method. This approach simplifies the whole algorithm.

B. Examples: Example1

Let's consider the two stage common emitter broad-band amplifier, of which small signal PSPICE model schematic is shown in Fig. 3.

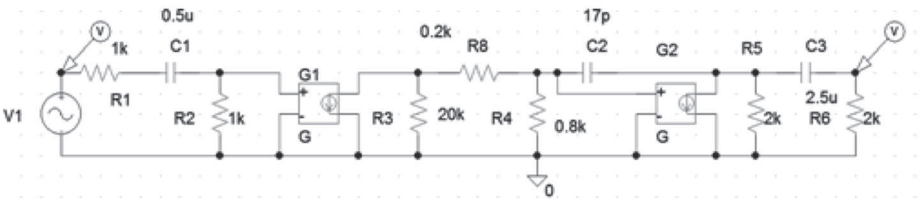

Figure 3. Small-signal equivalent of a two – stage amplifier prepared for poles optimization.

The transconductances G1 and G2 are equal to 21mS. The following schedule of poles was accepted: p1 = –1e2 rad/s, p2 = –1e3 rad/s, p3 = –3.26e6 rad/s (Fig. 4.), which corresponds to the following cutting frequencies: f_1 = 15.9 Hz, f_2 = 159 Hz, f_3 = 519 kHz.

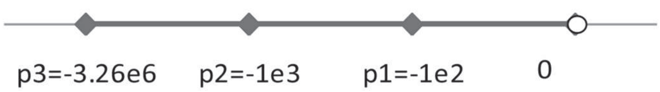

Figure 4. Demanded distribution of poles: ◆

The ranges of changeability of parameters were limited as follows: C_{min1} = C_{min3} = 1e3 pF, C_{min2} = 1e-3 pF, C_{max1} = C_{max2} = C_{max3} = 1e7 pF. The SemiSymPolOpt computer program basing on the input data concerning the circuit structure and element values generated the equations (38a):

– 1.10E7 = – 2.00E0*C1 – 3.51E1*C2 – 4.00E0*C3

1.00E13=+7.02E1*C1*C2+8.00E0*C1*C3+ 7.17E1*C2*C3

– 3.07E15 = – 1.43E2*C1*C2*C3;

and the goal function in semi – symbolic forms (computer printouts):

F = (1 – (– 2.00E0*X(1) – 3.51E1*X(2) – 4.00E0*X(3)) / (– 1.10E7))^2 + (1 – (+ 7.02E1*X(1)*X(2) + 8.00E0*X(1)*X(3) + 7.17E1*X(2)*X(3)) / (+ 1.00E13))^2 + (1 – (- 1.43E2*X(1)*X(2)*X(3)) / (– 3.07E15))^2.

Next, starting from the point: $x^0 = [1e5, 1e3, 1e5]^T$ (pF), the optimization procedure after 2111 function evaluations reached the minimum at $x^* = [0.499285e6, 17.11668e1, 2.502392e6]^T$ (pF), which corresponds to the following capacitors: C1 ≈ 500 nF, C2 ≈ 17 pF and C3 ≈ 2.5 μF.

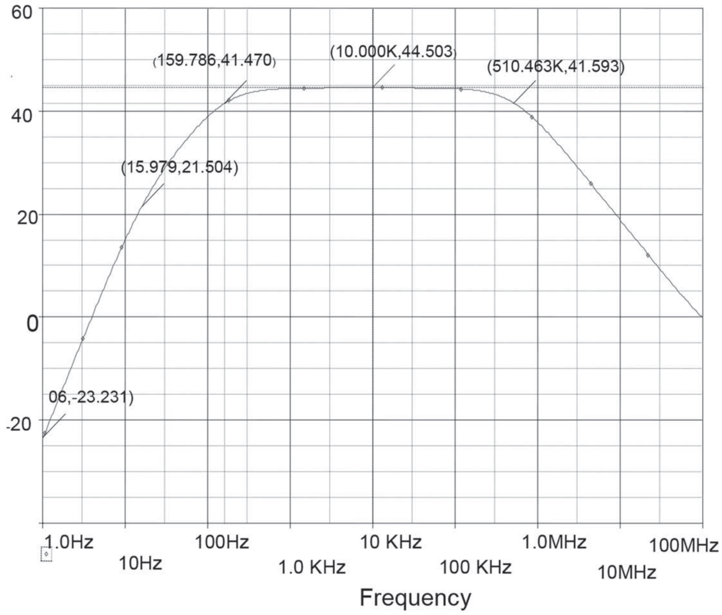

Figure 5. Magnitude of voltage transmittance.

The function value at the optimal point is equal to 0.00000078, which fulfils the global solution. Next, taking resulted capacitor values, the considered circuit was analysed by the PSPICE computer program [16]. The obtained transfer characteristic: dB|(V_{out}/V_{in})| (see Fig. 5) confirmed the demanded shape – its exact examination shows accuracy of calculation.

Example 2

Let's consider the one stage common emitter broad-band amplifier, of which the small signal PSPICE model schematic is shown in Fig. 6.

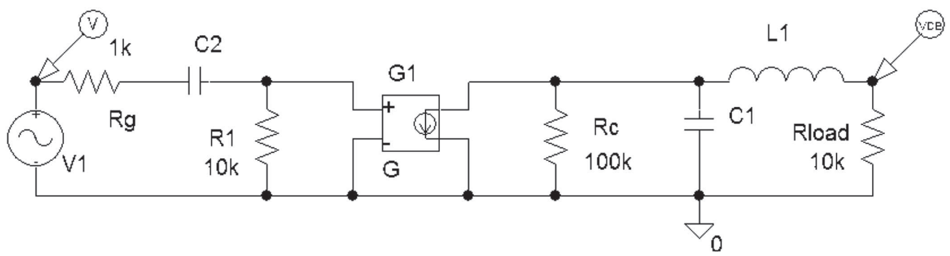

Figure 6. Small-signal equivalent circuit of one – stage amplifier prepared for poles optimization.

The transconductance G2 is equal to 10mS. The schedule of demanded poles was the following: lower dominant pole p1 = – 0.909e2 rad/s, double upper poles p2 = p3 = – 4e7 rad/s, which correspond to the following cutting frequencies: f_1 = 14.47 Hz, f_2 = f_3 = 6.36 MHz. It should be noticed that first and second upper poles are not split e.g. the inequality p2 << p3 is not fulfilled. As a starting point the following parameter vector was taken: x^0 = $[C1, C2, L1]^T$ = $[1\ pF, 0.1\ \mu F, 100\ \mu H]^T$. The set of feasible parameters was restricted by the following constraints:

0.1 pF < C1 < 100 pF

0.001 μF < C2 < 100 μF

0.1 μH < L1 < 1000 μH

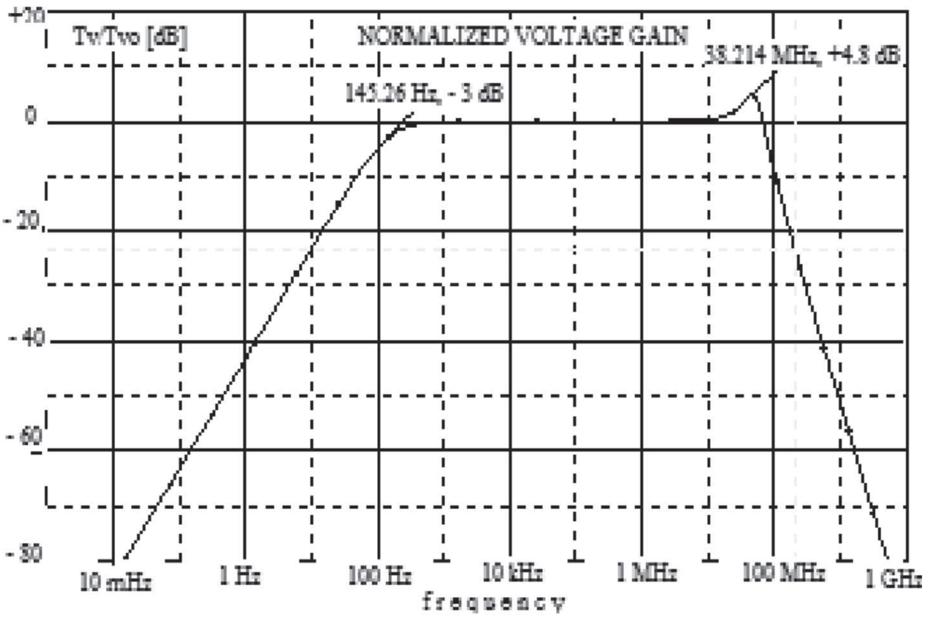

Figure 7. Magnitude of voltage transmittance at the starting point.

The magnitude of the voltage transmittance at the starting point is shown in Fig. 7 (Tvo = 38.344 dB). As we see, the lower pole is nearby 145,26 Hz, while both upper poles constitute the complex conjugate pair, which appears in the form of resonant shape with $\mathbf{f}_0$ = 48.214 MHz and peak value + 4.8 dB. The SemiSymPolOpt computer program basing on the input data concerning the circuit structure and element values generated the equations (38a) in semi – symbolic form (computer printouts):

- 1.100115011001E7 = -9.090909090909E0*C1 – 1.100000000000E1*C2 – 9.090909090909E-3*L1; +5.500556305006E8 = + 1.000000000000E2*C1*C2 + 9.090909090909E-1*C1*L1+1.000000000000E-1*C2*L1; -6.875687568757E9 = – 1.000000000000E1*C1*C2*L1

and the goal function:

F1=(1-(-9.090909090909E0*X(1)-1.100000000000E1 *X(2) -9.090909090909E-3*X(3))/(- 1.100115011001E7))^2 + (1 – (+ 1.000000000000E2*X(1)*X(2) + 9.090909090909E-1*X(1)*X(3) + 1.000000000000E-1*X(2)*X(3)) / (+ 5.500556305006E8))^2 + (1-(-1.00000000000E1 *X(1)*X(2) *X(3)) /(-6.875687568757E9))^2

Next, starting from the point: $\mathbf{x}^0$ the optimization procedure after 3735 function evaluations reached the minimum at $\mathbf{x}^*$ = [5.37202017 pF, 1.0001005 μF, 127.977836 μH]^T, which corresponds to the following values of capacitors and inductors: C1 ≈ 5.4 pF, C2 ≈ 1μF and L ≈ 127 μH. The goal function at the optimal point reached value 3.9 e-13 which fulfils the stopping criterion (global solution). Next, taking resulting element values, the considered circuit was analysed by

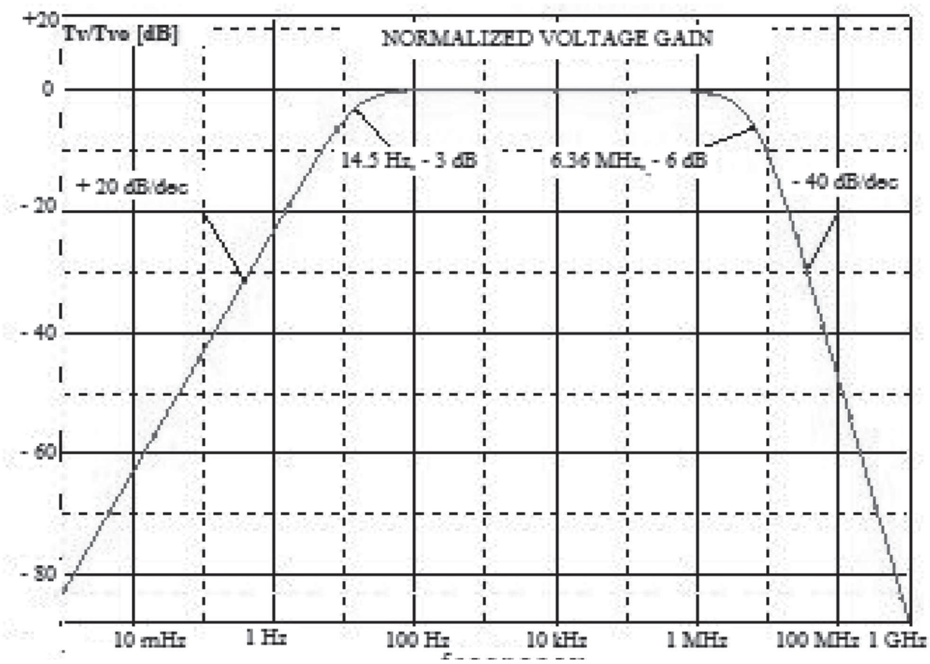

Figure 8. Magnitude of voltage transmittance in optimum

the PSPICE computer program. The obtained transfer characteristic: dB│ (V_{out}/V_{in}) │ (see Fig. 8) confirmed demanded shape. Its exact examination shows accuracy of calculation. As we see, the method can handle the tasks consisting of multiple poles.

Conclusions

In this work a new method of linear circuit design in frequency domain by poles (zeros) distribution optimization is proposed. The optimization procedure uses criterion function in semi – symbolic form, which is derived only once, basing on semi – symbolic representation of the time – constant matrix and its traces expressed via the appropriate transresistances. Moreover, it does not require calculating poles in an optimization loop repeatedly. Thank to these facts, this method appeared to be very efficient. Many computational tests confirmed this observation as well as its accuracy. Furthermore, the poles need not be separated. It should be pointed out that the method has been extended easily to inductances, by determining a few additional transresistances. Derived relationships allow us to identify what causes the task to be unsolvable and, thanks to these relationships, the elements of the time-constant matrix can be counted in a uniform manner by the transimpedance method. The proposed method can be successfully used for precise tuning of the electronic circuit in frequency domain.

Appendix A: Proof of Theorem 1

It was shown that taking into account relationship (11) the generalized eigenvalue equation (19)

$$\left(s^{-1}I + \begin{bmatrix} G & A_L \\ A_L^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} A_C C' A_C^T & 0 \\ 0 & -L' \end{bmatrix}\right.$$

can be formulated as

where: I – is $n + n_L$ – dimensional identity matrix.

Using block matrix inversion method one gets

$$\begin{bmatrix} G & A_L \\ A_L^T & 0 \end{bmatrix}^{-1} = \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}, \quad \text{(A1)}$$

where:

$$K_{22} = -\left(A_L^T R A_L\right)^{-1} = -R_{LL}^{-1} \quad \text{(A2)}$$

$$K_{12} = R A_L R_{LL}^{-1} \quad \text{(A3)}$$

$$K_{21} = R_{LL}^{-1} A_L^T R \quad \text{(A4)}$$

$$K_{11} = R(I - A_L K_{21}) = R\left(I - A_L R_{LL}^{-1} A_L^T R\right) \quad \text{(A5)}$$

where:

$$R = G^{-1}. \quad \text{(A6)}$$

The existence of K_{ij} matrices results from the nonsingularity of matrices G and R_{LL} that was assumed in the theorem. Substituting (A2 – A5) to (A1) and multiplying the block matrices we get

$$\left(s^{-1}I + \begin{bmatrix} K_{11}A_C C' A_C^T & -K_{12}L' \\ K_{21}A_C C' A_C^T & -K_{22}L' \end{bmatrix}\right)\tilde{z} = \mathbf{0} \quad \text{(A7)}$$

After expanding K_{ij} matrices the equation (A7) takes the following form

$$\left\{s^{-1}I + \begin{bmatrix} RA_C C' A_C^T - RA_L R_{LL}^{-1} A_L^T RA_C C' A_C^T & -RA_L R_{LL}^{-1} L' \\ R_{LL}^{-1} A_L^T RA_C C' A_C^T & R_{LL}^{-1} L' \end{bmatrix}\right\}\tilde{z} = \mathbf{0} \quad \text{(A8)}$$

Multiplying left side eq. (A8) by matrix $H = \begin{bmatrix} A_C^T & 0 \\ 0 & I \end{bmatrix}$, and excluding this matrix on the right side, we get

$$\left\{s^{-1}H + H\begin{bmatrix} RA_C C' A_C^T - RA_L R_{LL}^{-1} A_L^T RA_C C' A_C^T & -RA_L R_{LL}^{-1} L' \\ R_{LL}^{-1} A_L^T RA_C C' A_C^T & R_{LL}^{-1} L' \end{bmatrix}\right\}\tilde{z} = \mathbf{0} \quad \text{(A9)}$$

$$\left\{s^{-1}I_M + \begin{bmatrix} A_C^T (RA)_C - RA_L R_{LL}^{-1} A_L^T RA_C (\square) C' & -A_C^T RA_L R_{LL}^{-1} L' \\ R_{LL}^{-1} A_L^T RA_C C' & R_{LL}^{-1} L' \end{bmatrix}\right\} z_M = \mathbf{0} \quad \text{(A10)}$$

where: $z_M = Hz$ – transformed M-dimensional eigenvector, I_M – is M-dimensional identity matrix.

Introducing definitions of new transresistances (24), (25), (26) and (27) one gets the matrix

$$\left(s^{-1}I + \begin{bmatrix} (R)_{CC} - R_{CL} R_{LL}^{-1} R_{LC} (\square) C' & -R_{CL} R_{LL}^{-1} L' \\ R_{LL}^{-1} R_{LC} C' & R_{LL}^{-1} L' \end{bmatrix}\right) z_M = \mathbf{0} \quad \text{(A11)}$$

(A11)

that fulfills the theorem demands.

q.e.d.

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