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MECHANICAL INTERACTION OF THE RAIL TRANSPORT CAR AND JOINT IRREGULARITY

Summary. Mechanical models of a transport system “carriage - track” while crossing a joint irregularity are proposed. An investigation was conducted on the peculiarities of static, shock and dynamic interaction between the four-axle car and the track, considering tram wheelsets motion features over joint irregularity. A method to solve the equations of a mathematical model of static, shock and dynamic interaction is developed. Numerical analysis is used to determine deflections of the facing rail under the first sleeper for each phase of motion depending on motion phases, and car load and speed.

Keywords: railway rolling stock, four-axle car, track, ballast, joint irregularity, trailing rail, facing rail

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1. INTRODUCTION

In the current state of rail transport operating conditions, one should pay special attention to improving its design, as well as improving the reliability and operational properties of rolling stock. This is why in electric transport, scientific and technical works and research related to solving problems, devoted to the problem of ensuring the reliability of the tramcar-rail complex are particularly important at the present stage of development. Thus, scientific and technical papers and investigations that provide task solutions devoted to the problem of ensuring the reliability of the tramcar-track complex are particularly important in the electric transport area at the current stage of development.

When a tramcar moves, its components and assemblies are affected by the interaction forces that arise between the cars and the trolleys, the trolleys wheel pairs and the upper structure of the track. They change both in time and in direction. Problem solutions related to the investigation of the practical application of the mechanical interaction science of rolling stock and rail tracks significantly affects the development of electric transport and the safety of railcars, as well as on the capacity of stations, on the economy of maintenance of rolling stock and rail tracks, conditions of overall cross-country ability of rolling stock, etc. At the same time, the deformation characteristics of the ballast layer under the rail supports (parameters of elastic and residual sediment) ultimately regulate the technical resource and duration of its operation. Practice shows that the greatest sediment of the ballast layer occurs in places of butt irregularities under the first sleepers of the facing rail. This is because these places of the rails are usually subjected to the greatest dynamic load of the impact type.

Subsequently, investigations of the mechanical interaction processes in the system «tramcar - track» are becoming important, as well as establishing new regularities based on the analysis results, noting the peculiarities of their static, impact and dynamic interaction while considering the operating, design and mechanical parameters of the tramcar and upper truck structure and motion phase of the tramcar through joint irregularity.

2. LITERATURE REVIEW AND RESEARCH OBJECTIVE

Currently, significant attention is given to the development of the transport structure in many countries of the world. In addition, funds are been invested into the track and railway rolling stock. The world`s experience indicates mechanical complex`s «tramcar - track» reliability and durability indicators to depend on the characteristics of the joint operation of rolling stock and track, the type of rolling stock, the type of rails and sleepers, and the operating conditions of the mechanical system. It also depends on the ability to resist the destructive effects of the resulting shock and vibration loads.

Furthermore, it is known that the highest level of deposition of the ballast layer occurs under the first sleeper of the facing rail. This is because the first sleeper under the facing rail in these places usually experiences the greatest force interaction between the tramcar and the upper structure of the track due to their impact. Thus, the deflection of the facing rail under the first sleeper is a significant indicator that corresponds to the features of mechanical interaction of the receiving rail of the track when investigating static, impact and dynamic interaction with the upper structure of the track in places with isolated butt joints of the track.

Investigation analysis dedicated to the mechanical interaction of a four-axle car and a rail track shows [1] that the weakest link in the system is the isolated zones of the track joints. Further, it is similarly known that the highest level of deposition of the ballast layer occurs under the first sleeper of the receiving rail. This is because in these places the first sleeper under the receiving rail usually experiences the greatest force interaction between the tramcar and the upper structure of the track due to their impact. Thus, the deflection of the receiving rail under the first sleeper is a significant indicator that corresponds to the features of the mechanical interaction of the receiving rail of the track in the investigation, static, impact and dynamic interaction with the upper structure of the track in places with isolated butt joints of the track.

In addition, the expediency of considering the phases of movement through joint irregularity [2], as well as the stiffness characteristics of the ballast layer of the track is emphasised. The issues of joint operation of rolling stock and rail track are also analysed, determining the features of their static [1, 3] and dynamic interaction [4] during the passage of joints by the car.

Various factors that change during the operation of the rail track make it necessary to apply an integral model that considers various impact factors. Thus, this research, devoted to the improvement of existing models of interaction between the car and the upper structure of the track, is modern and relevant.

The load of rolling stock elements and the upper structure of the track determining the parameters of durability in operation [5], the strength and rigidity of the track [6], are conducted in the field. This together, affects the technical resource and service life. Operational experience of rail transport shows that the "car-track" indicators of reliability and durability in the mechanical complex significantly depend on the features of the interaction processes between the track and rolling stock and the operating conditions of the system under consideration. In addition, this interaction affects the ability of the system to withstand the destructive effects of the resulting shock and vibration loads [7], which are cyclically repeated.

To analyse the mechanical interaction of rolling stock and the upper structure of the track, it is necessary to solve several related problems. Much attention is focused on these issues and a sufficient amount of new research is emerging in this area.

2.1. Analysis of recent studies

Most scientific works are currently limited to the consideration of individual parameters of the electric transport operation. To ensure a reliable modelling, it is necessary to use the most generalised approach from possible ones, which considers the totality and mutual influence of various factors [8].

Improving the quality and capacity of existing services and developing new infrastructure are essential to meet the growing demand for high quality and reliable logistics of goods and people. The efficiency and reliability of the path design are crucial for a successful operation. Many modern rail track investigations focus on specific aspects of design and operation, such as fatigue [7], ballast destruction [6], ride comfort [1], noise or vibration [2].

Thus, some works [5], present the processes of mechanical interaction in the "car-track" system, where they are considered only taking into account the movement of the vehicle on non-jointed sections of the track. Other works [7] do not observe the boundary conditions for the facing rail. In practice, this does not correspond to the conditions of real mechanical loading of a tramcar, sections of the trailing and facing rails. Therefore, the values of the structural speed of rolling stock determined in the studies are not sufficiently complete and needs to be improved.

Thus, there is a need to develop a sufficient and easy-to-use model of the mechanical interaction of rolling stock and track. Additionally, an analysis technique considers a rail car as a multidimensional discrete one, and the upper structure of the track as a continuous system.

In this formulation, the characteristics of mechanical interaction in the "car-track" system in the joint area will correspond to the actual loading conditions.

2.2. Purpose and research objective

This study investigates the mechanical interaction of the car and the upper structure of the track for the improvement of the parameters of a discrete-continuous system by rationally selecting and optimising the parameters of its components. This will provide an additional impact on the reliability and durability of the system through components that depend on the parameters of dynamic interaction in the transport mechanical complex "car-rail track on a section with isolated joint irregularity".

Hereby, one can formulate the following research task:

- to create a method of sequential static-impact-dynamic calculation of deflections of the facing rail under the first sleeper in the growth phase, considering the phases of movement of the tramcar relative to the butt joint;
- to study the influence of the phases of movement of the tramcar on the height of the joint when varying the operational, structural and mechanical parameters of the tramcar and the upper structure of the track;
- to improve the dynamic model of deflections of the receiving rail under the first sleeper in the growth phase, noting the characteristics of the elastic suspension of the tramcar, the ballast layer, as well as the quantitative parameters of the reduced masses of the wheels corresponding to the phase of movement of the tramcar;
- analyse the features of the influence of operational, structural and mechanical parameters of the tramcar and the upper structure of the track on the deflections of the receiving rail under the first sleeper in the growth phase.

3. RESEARCH MODEL AND METHOD

A mechanical discrete-continuous model of a four-axle car is considered. This can be a tramcar, a passenger or a freight car of a railway transport. This takes into account the multi-factor influence of structural, operational and mechanical parameters of the "car-rail track on a section with isolated butt joint" system. These are car speed, its loading, the conditions for connecting the facing and trailing rails to each other through the butt rail cover, the rigidity of the ballast layer, and others.

The phases of the car movement during static interaction are determined as follows: in the first phase, all wheelsets are located on the trailing rail, in the second phase – three remain on it, in the third – two and in the fourth only one. At the same time, accordingly, the number of wheels on the facing and trailing rails affect their static deflections, that is, the height of the joint irregularity that is created.

The calculated mechanical scheme of the dynamic interaction processes of the facing rail with the vehicle in the joint area on the example of the third phase of the car's movement is shown in Figure 1. This scheme differs from the processes of static interaction, as well as other phases of movement of the facing rail in conditions of dynamic interaction. Currently, there is only one wheel on the trailing rail, and all three on the facing rail.

Here c_1, b_1 – suspension stiffness and damping coefficient; c_2, b_2 – ballast stiffness and damping coefficient; $c_{р.к.}$ – stiffness of the rail at its end; m_1, m_1 – reduced masses of the wheel and the car, considering the load; y_1, y_1 – displacement of reduced mass of the wheel and the car; $l_i\left(i=1\div 24\right)$, $l_{ki}\left(i=1-2\right)$ – geometrical coordinates of the elastic supports, as well as the wheels of the first and second axles of the car.

Accordingly, the number of wheels on the facing rail significantly changes its mechanical load relative to the oscillatory motion, both relative to the car and the ballast layer.

When calculating deflections of a discrete-continuous system, mechanical, geometric characteristics, static deflections, and post-impact velocity are set for the receiving rail. Deflections of the system are considered as a superposition of the first five eigenvalues of vibrations of the receiving rail as a discrete-continuous mechanical system.

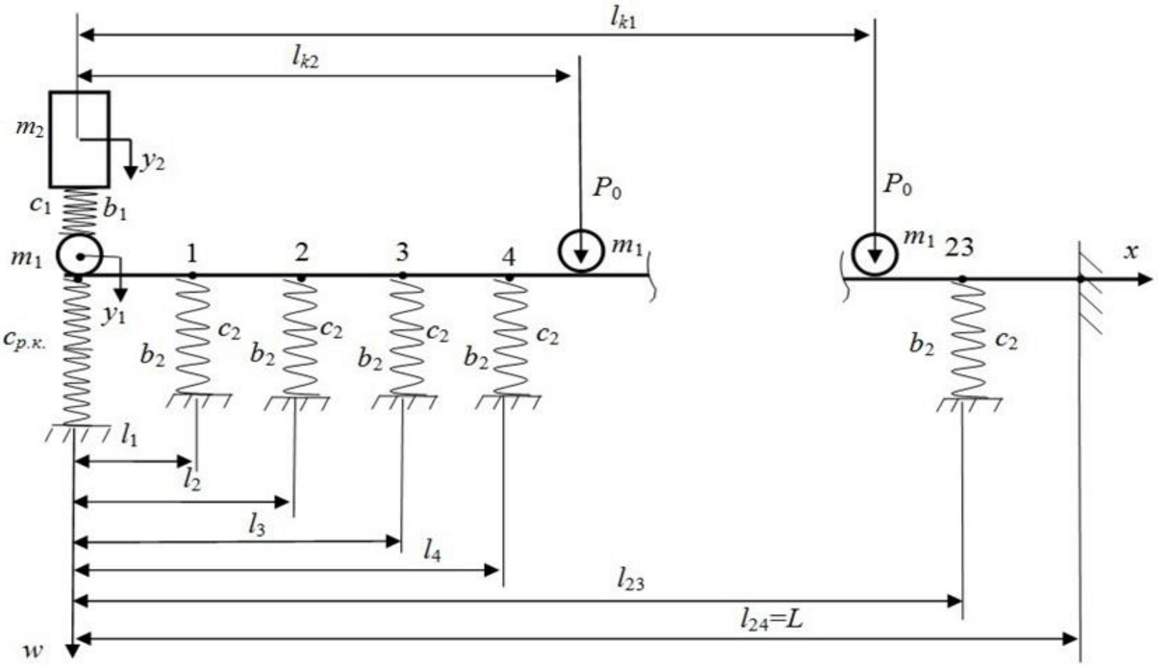

Fig. 1. Calculated mechanical scheme of dynamic interaction for the facing rail during the first phase of the car motion

3.1. Static interaction

The essence of the static interaction of the car and the rail track assumes the height determining of the joint irregularity by the initial parameters method based on elastic lines of the facing and trailing rails, which loading corresponds to all four phases of the car's movement. In the static calculation of rail deflections, a model of a multi-span beam is used on 24 elastic supports (23 sleepers and a support that simulates the connection to an adjacent rail through a working pad) and is shown in the third phase of the car's movement in Figure 2. The force factors here that ultimately determine the static deflection of the facing and trailing rails, as well as the height of the joint corresponding to the third phase of movement of the tramcar, are constant forces P_0. They correspond to the current number of wheel pairs on the rails and have coordinates: for the facing rail (Figure 2a) X_{Bj}, where $j=3,4$ – tramcar wheelset number; for the facing rail (Figure 2b) $X_{\Pi j}$ (j=1, 2).

In Figure 2: $l_i\left(i=1\div 24\right)$, $l_{24}=l_{р.к.}$ – geometric coordinates of elastic supports; $P_0 = P/8$ – load on the side of a tramcar per wheel; P_0 – weight of a tramcar, including its load; $F_{пр3ф} = h_{В3ф}\cdot c_{р.к.}$ – elastic force applied in the third phase of movement of the tramcar to the end of the facing rail from the side of the trailing rail when $x = l_{р.к.} = 12{,}5$ m; $h_{В3ф}$ – deflection of the facing rail at the end in the third phase of the tramcar movement; Q_0, M_0 – cross force and bending moment at the origin of the coordinates; *c* – stiffness of the ballast layer under the sleeper of the upper track structure; $X_{В4} = 10{,}6$ m; $X_{В3} = 12{,}5$ m; $X_{П1} = 4{,}5$ m; $X_{П2} = 6{,}4$ m; $l_1 = 0{,}21$ m; $l_2 = 0{,}759$ m; $l_3 = 1{,}308$ m; $l_4 = 1{,}857$ m; $l_5 = 2{,}406$ m; $l_6 = 2{,}955$ m; $l_7 = 3{,}504$ m; $l_8 = 4{,}053$ m; $l_9 = 4{,}602$ m; $l_{10} = 5{,}151$ m; $l_{11} = 5{,}7$ m; $l_{12} = 6{,}249$ m; $l_{13} = 6{,}798$ m; $l_{14} = 7{,}347$ m; $l_{15} = 7{,}896$ m; $l_{16} = 8{,}445$ m; $l_{17} = 8{,}994$ m; $l_{18} = 9{,}543$ m; $l_{19} = 10{,}092$ m; $l_{20} = 10{,}641$ m; $l_{21} = 11{,}19$ m; $l_{22} = 11{,}739$ m; $l_{23} = 12{,}288$ m; $l_{р.к.} = 12{,}5$ m.

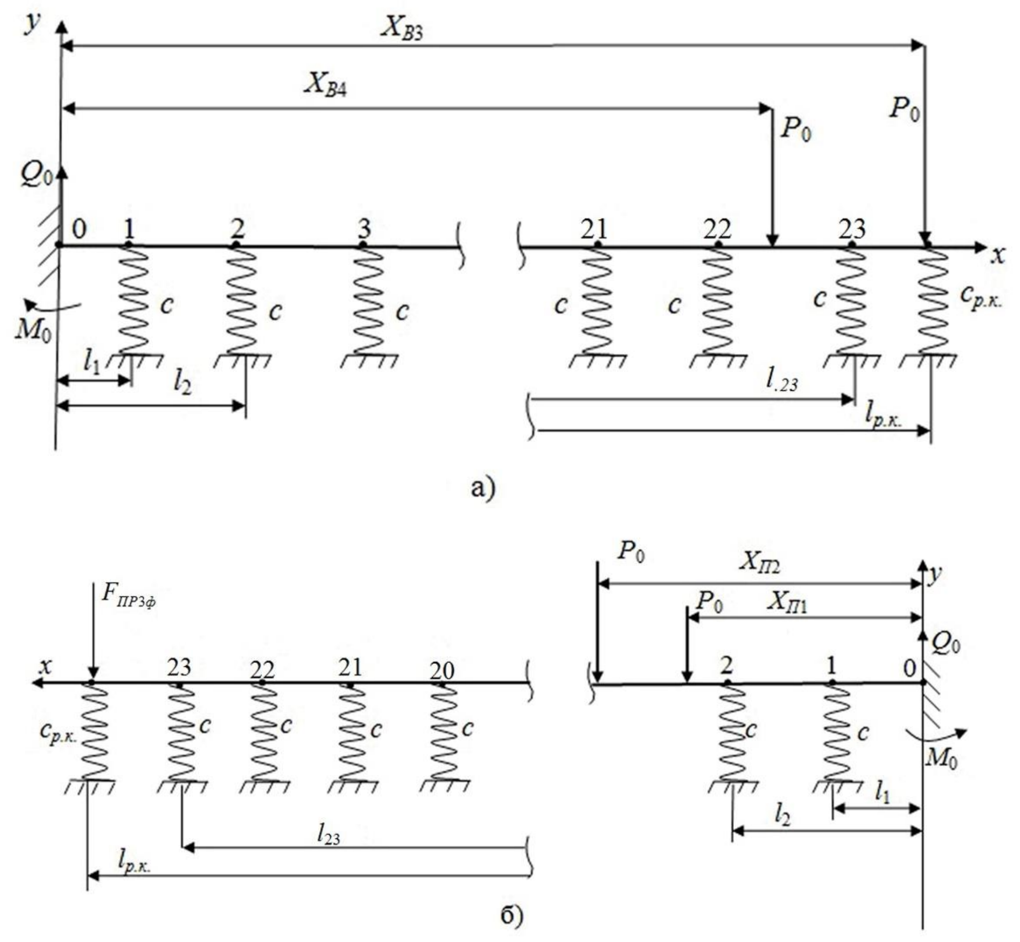

Fig. 2. Calculated mechanical schematic of static interaction during the third phase of the car motion: a) trailing and b) facing rails

The equation of the curved axis of the rails in the first phase of movement is written using the method of initial parameters, noting the conditions of their fixation $(y_0 = y'_0 = 0)$:

trailing rail:

$$y_B(x) = \frac{1}{EJ} \times \left[Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} - \sum_{j=3}^{4} P_0 \frac{(x - x_{Bj})^3}{6} + \sum_{i=1}^{23} cy_i \frac{(x - l_i)^3}{6} + c_{р.к.} h_B \frac{(x - L)^3}{6} \right], \quad (1)$$

facing rail:

$$y_П(x) = \frac{1}{EJ} \times \left[Q_0 \frac{x^3}{6} + M_0 \frac{x^2}{2} - F_{пр3ф} \frac{(x - L)^3}{6} - \sum_{k=1}^{2} P_0 \frac{(x - X_{Пk})^3}{6} + \sum_{i=1}^{23} cy_i \frac{(x - l_i)^3}{6} + c_{р.к.} h_П \frac{(x - L)^3}{6} \right]. \quad (2)$$

The elastic lines of the facing and trailing rails in the third phase of movement are shown in Figure 3, where $y_{B3}, y_{B4}, y_{П1}, y_{П2}, y_{23}$ – deflections of the rails, respectively, under the wheels of the car and the first sleeper of the receiving rail.

The current height of the joint irregularity in the third phase of the tramcar movement is calculated by the expression:

$$h_{3ф} = h_{П3ф} - h_{B3ф}, \quad (3)$$

Where:

$h_{П3ф}, h_{B3ф}$ – deflections of the facing and trailing rails at the ends, respectively, when $x = 12{,}5$ m, that is $h_{П3ф} = y_П(l_{р.к.})$, $h_{B3ф} = y_B(l_{р.к.})$.

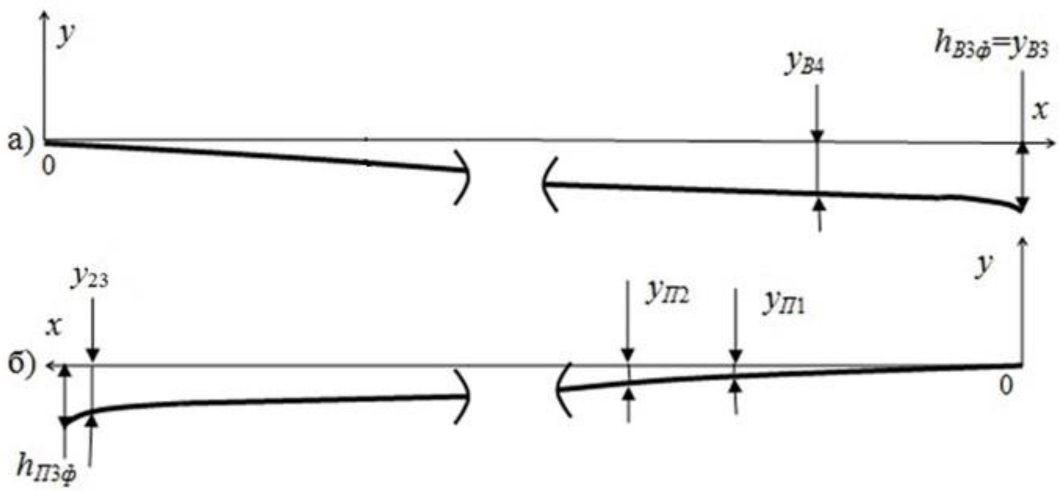

Fig. 3. Elastic lines of the trailing a) and facing b) rails in the third phase of the tramcar movement

Considering the fact that expressions (1) and (2) contain summands on the right side, which in turn depends on deflections, the solution of these equations is performed numerically according to the work algorithm. The calculations were performed following the mechanical scheme of the tramcar and the rail track, as shown in Figure 1, as well as geometric and mechanical characteristics of the P-65 rail and T-3 tram [9]: elastic modulus of the rail material – $E = 2{,}6 \cdot 10^{11}$ N/m^2; moment of inertia of the rail section relative to the neutral axis – $J = 3573$ cm^4; path ballast layer stiffness – $c = 0{,}5 \cdot 10^{8}$ N/m. Mass of a tramcar reduced to one wheel (empty) $m = 2125$ kg, when maximum (with 193 passengers) uploaded – $m = 3814$ kg.

The established values of the joint stage value in the third phase of the tramcar movement ($h_{3ф} = 1{,}3 \div 1{,}95$ mm) makes it possible (using mass, load and reduced wheel mass) to determine the value after the impact speed of the facing rail at the end at the stage of impact interaction.

3.2. Impact in reaction

This work mainly discusses the peculiarities, assumptions, and expressions for determining the post-impact vertical velocity of the end face of the facing rail. The results obtained are used further at the stage of dynamic interaction in the form of initial data when calculating deflections of the facing track rail under the first sleeper during its growth phase. It is established that in the third phase of movement of the tramcar, the post-impact speed changes in the range of $V_1 = [2{,}745 \div 8{,}234]$ m/s. The results obtained are used as initial data when calculating the deflections of the facing track rail under the first sleeper during its growth phase.

3.3. Dynamic interaction

The solution of the formulated problem is carried out by the method of sequential static-impact-dynamic calculation. It is based on the principle of superposition of the action of forces with respect to elastic linear systems that deform [10].

Structurally, the method consists of the following three steps.

Stage 1. Here we consider the problem of free vibrations of the mechanical system “tramcar-rail track”, which is reduced to a superposition of eigenforms. This does not consider the components associated with the phenomena of dissipation, as well as the impact of static load from the tramcar on the facing rail. In a generalised form for all phases of movement of the tramcar, the deflections of the receiving rail considering the geometric coordinates and sprung mass of the tramcar are determined by the expressions:

$$w(t,x) = \sum_{s=1}^{5} z^s(x)(D_s \sin\omega_s t + M_s \cos\omega_s t),$$

$$y_1(t,l^*) = w(t,x=l^*) = \sum_{s=1}^{5} z^s(l^*)(D_s \sin\omega_s t + M_s \cos\omega_s t) = \sum_{s=1}^{5}(\lambda_1^s D_s \sin\omega_s t + \lambda_1^s M_s \cos\omega_s t), \quad (4)$$

$$y_2(t) = \sum_{s=1}^{5}(\lambda_2^s D_s \sin\omega_s t + \lambda_2^s M_s \cos\omega_s t),$$

where D_s, M_s – constant integrations, coefficients found from the conditions of orthogonality of oscillation forms, considering the pre-impact speed of the wheel of the third wheelset (defined in Section 3.2 of this paper) and the post-impact speed of the rail compatible with the wheel; $z^s(x)$, ω_s – eigenvalues and eigen frequencies of vibrations of the facing rail as a system with distributed parameters; l^* – linear mass coordinate m_1.

Stage 2. Here, an exponential component is superimposed on the solution of expression (4):

$$w_{Em2}(t,x)=\sum_{s=1}^{5} z^s(x)e^{-h_s t}(D_s \sin\omega_s t + M_s \cos\omega_s t), \quad (5)$$

where h_s – damping coefficient of the corresponding waveform.

In addition, the dissipation factor is determined by the expression $h_s = b_s / 2m_{зв.р.}$, where b_s – coefficient of resistance relative to the corresponding waveform; $m_{зв.р.}$ – reduced weight of the rail. In calculations, it is $m_{зв.р.} = 150$ kg, $b_s = 0{,}6 \cdot 10^5$ N·s/m (independent from s [10]).

Step 3. On the elastic line $y(x)$ of the facing rail, defined for the third phase of movement in Section 3.1, according to the superposition principle, the deflection of the facing rail is superimposed by the addition method, which is calculated in step 2 by formula (5), also see the diagram in Figure 4, where:

$$w(t,x) = y(x) + w_{Em2}(t,x)$$

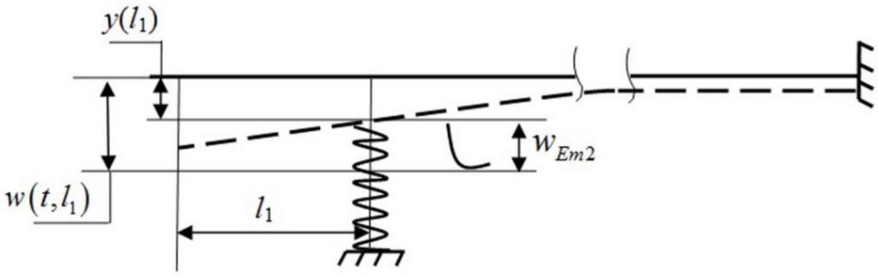

Fig. 4. Deflection diagram of the facing rail under the first sleeper

This method provides determination of the deflection under the first sleeper of the facing rail in dynamics, depending on the phase of movement of the tramcar due to joint irregularity, structural and mechanical characteristics of the tramcar and the rail track. A flowchart of the method for determining the parameters of the mechanical interaction of the tramcar and the rail track in the area of the track junction, providing determination of deflections of the facing rail track under its first sleeper in the growth phase from the loading parameters and speed of the tramcar is shown in Figure 5, where 1, 2, and 3 are blocks for calculating static, impact, and dynamic parameters of the mechanical interaction of the tramcar with the rail track, respectively; $X_{11} = \left[h_{1ф}, h_{2ф}, h_{3ф}, h_{4ф}\right]'$ – vector of joint height parameters corresponding to the car's movement phases (section 3.1); $X_{12} = \left[c_{р.к.}, y(x = l_1)\right]'$ – vector of rail stiffness at the end and static deflections of the facing rail under the first sleeper (Section 3.1); $X_2 = \left[V_{1,1ф}, V_{1,2ф}, V_{1,3ф}, V_{1,4ф}\right]'$ – post-impact velocity vector of the facing rail (Section 3.2);

$X_3 = \left[w(t, x = l_1)_{\max,\phi 1}, w(t, x = l_1)_{\max,\phi 2}, w(t, x = l_1)_{\max,\phi 3}, w(t, x = l_1)_{\max,\phi 4} \right]'$ – vector of parameters of maximum deflections of the facing rail under the first sleeper (Section 3.3).

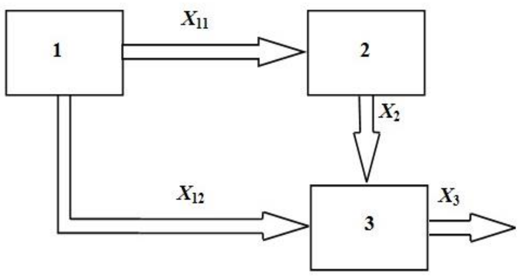

Fig. 5. Flowchart of the method

To calculate the deflections of the facing rail under the first sleeper in the zone of butt unevenness, the dynamic interaction of the tramcar with the rail track in the joint irregularity is considered with the following factors. The height of the joint irregularity corresponds to [3] the parameters of static interaction of the car, its loading, structural and operational factors of the mechanical system "tramcar – rail track". Dynamic characteristics after impact interaction at the growth phase of deflections of the facing rail under the first support are determined noting the parameters of a real object in the form of a tramcar and a rail track in a zone of isolated joint irregularity of the "gap – step up" type.

To achieve the goal, the following tasks were solved:

- using modelling methods, the mechanical system was represented in the form of a multi-span beam on elastic supports;
- the butt plate is an equivalent elastic element at the end of the facing and trailing rails of the joint with a stiffness coefficient that is determined considering its flat stress state.

With the impact interaction of the wheel with the end face of the facing rail, a tramcar interacting with the facing rail is considered as a sprung reduced mass. In addition, the current step height is determined taking into account the static interaction of the tram with the rail track, which corresponds to the phase of movement of the tramcar due to the joint irregularity.

When analyzing the processes of dynamic interaction in the system «two-dimensional discrete elastic – dissipative system – continuous system in the form of an indistinguishable multi-span beam on 24 elastic supports», the following assumptions were made. When the wheel hits the end face of the facing rail, it does not break off, as well as its sliding relative to the rail; vibrations of the wheelset and the rail head occur in continuous mode (since $m_2 >> m_1$, the assumption is acceptable); deflections of the rail are realised without violating the integrity of the ballast layer (since the deformation characteristics of the ballast layer under the first sleeper are considered during the growth phase of deflections of the receiving rail only downwards, the assumption is also acceptable).

According to the design mechanical scheme of the facing rail in the third phase of the tramcar movement in Figure 6, differential equations of oscillations at $w(0,0) = 0, \dot{w}(0,0) = V_{1,3\phi}$ are as follows:

$$\begin{cases} \dfrac{\partial^4 w(t,x)}{\partial x^4} + \dfrac{\rho F}{EJ}\cdot\dfrac{\partial^2 w(t,x)}{\partial t^2} = \dfrac{c_1(y_2 - 0{,}5w(t,0))\delta(0)}{EJ} - \dfrac{c_{р.к.}w(t,0)\delta(0)}{EJ} - \\ -\sum\limits_{i=1}^{23}\dfrac{c_2 w(t,l_i)\delta(x-l_i)}{EJ} - \sum\limits_{i=1}^{3}\dfrac{m_1}{EJ}\cdot\dfrac{\partial^2 w(t,l_{ki})\delta(x-l_{ki})}{\partial t^2} + \sum\limits_{i=1}^{2}\dfrac{P_0\delta(x-l_{ki})}{EJ} - \\ -\sum\limits_{i=1}^{23}\dfrac{b_2}{EJ}\cdot\dfrac{\partial w(t,l_i)}{\partial t}\cdot\delta(x-l_i) + \dfrac{b_1}{EJ}\cdot\left(\dfrac{\partial y_2}{\partial t} - 0{,}5\dfrac{\partial w(t,l_{k3})}{\partial t}\right)\cdot\delta(x-l_{k3}); \\ m_2\cdot\dfrac{\partial^2 y_2}{\partial t^2} + c_1(y_2 - 0{,}5w(t,l_{k3})) + b_1\cdot\left(\dfrac{\partial y_2}{\partial t} - 0{,}5\dfrac{\partial w(t,l_{k3})}{\partial t}\right) = 0, \end{cases} \quad (6)$$

where $l_{k1} = 6{,}4\ m; l_{k2} = 4{,}5\ m, l_{k3} = 0.$

The solution of the equations system (6) is constructed based on the proposed three-stage method.

Then, in step 1, the motion differential equations of a mechanical system will have the form:

$$\begin{cases} \dfrac{\partial^4 w(t,x)}{\partial x^4} + \dfrac{\rho F}{EJ}\cdot\dfrac{\partial^2 w(t,x)}{\partial t^2} = \dfrac{c_1(y_2 - 0{,}5w(t,0))\delta(0)}{EJ} - \dfrac{c_{р.к.}w(t,0)\delta(0)}{EJ} - \\ -\sum\limits_{i=1}^{23}\dfrac{c_2 w(t,l_i)\delta(x-l_i)}{EJ} - \sum\limits_{i=1}^{3}\dfrac{m_1}{EJ}\cdot\dfrac{\partial^2 w(t,l_{ki})\delta(x-l_{ki})}{\partial t^2}; \\ m_2\cdot\dfrac{\partial^2 y_2}{\partial t^2} + c_1(y_2 - 0{,}5w(t,0)) = 0. \end{cases} \quad (7)$$

Equations (7) for eigenforms will be reduced to the form:

$$\begin{cases} z^{s^{IV}}(x) - k_1^{s^4} z^s(x) - k_2^{s^4}\sum\limits_{i=1}^{3} z^s(l_{ki})\delta(x-l_{ki}) + \dfrac{c_{р.к.}z^s(l_{k3})\delta(x-l_{k3})}{EJ} + \\ +\dfrac{c_2}{EJ}\sum\limits_{i=1}^{23} z^s(l_i)\delta(x-l_i) - \dfrac{c_1}{EJ}(\lambda_2^s - 0{,}5\lambda_1^s)\delta(x-l_{k3}) = 0, \\ -m_2\lambda_2^s\omega_s^2 + c_1(\lambda_2^s - 0{,}5\lambda_1^s) = 0. \end{cases} \quad (8)$$

where $k_1^{s^4} = \dfrac{\rho F}{EI}\omega_s^2$; $k_2^{s^4} = \dfrac{m_1}{EJ}\omega_s^2$; $\lambda_1^s = z(l^*)$; λ_2^s – eigen shape for the coordinate y_2 in Figure 6.

The solution of the system`s first equation is performed using the Laplace-Carson transformation:

$$\eta(p)=\frac{p^4}{p^4-k_1^4}z(0)+\frac{p^3}{p^4-k_1^4}\frac{k_1}{k_1}z'(0)+\frac{p^2}{p^4-k_1^4}\frac{k_1^2}{k_1^2}z''(0)+\frac{p}{p^4-k_1^4}\frac{k_1^3}{k_1^3}z'''(0)+$$
$$+k_2^4\sum_{i=1}^{3}z(l_{ki})\frac{pe^{-pl_{ki}}}{p^4-k_1^4}+\frac{c_1}{EJ}(\lambda_2-0,5\lambda_1)\frac{pe^{-pl^*}}{p^4-k_1^4}- \tag{9}$$
$$-\frac{c_2}{EJ}\sum_{i=1}^{23}z(l_i)\frac{pe^{-pl_i}}{p^4-k_1^4}-\frac{c_{p.\kappa.}z(l^*)}{EJ}\frac{pe^{-pl^*}}{p^4-k_1^4}.$$

Let us further move in (9) to the original using Krylov functions [10]:

$$z(\mathrm{x}) = \rho(k_1\mathrm{x})z(0)+\frac{\varphi(k_1x)}{k_1}z'(0)+\frac{\psi(k_1x)}{k_1^2}z''(0)+\frac{\gamma(k_1x)}{k_1^3}z'''(0)+$$
$$+\frac{k_2^4}{k_1^3}\sum_{i=1}^{3}z(l_{ki})\gamma[k_1(x-l_{ki})]\Big|_{x\ge l_{ki}}+\frac{c_1}{EJk_1^3}(\lambda_2-0,5\lambda_1)\gamma[k_1(x-l^*)]\Big|_{x\ge l^*}- \tag{10}$$
$$-\frac{c_2}{EJk_1^3}\sum_{i=1}^{23}z(l_i)\gamma[k_1(x-l_i)]\Big|_{x\ge l_i}-\frac{c_{p.\kappa.}}{EJk_1^3}z(l^*)\gamma[k_1(x-l^*)]\Big|_{x\ge l^*}.$$

The diagram of sections following the structure of the facing rail is shown in Figure 6.

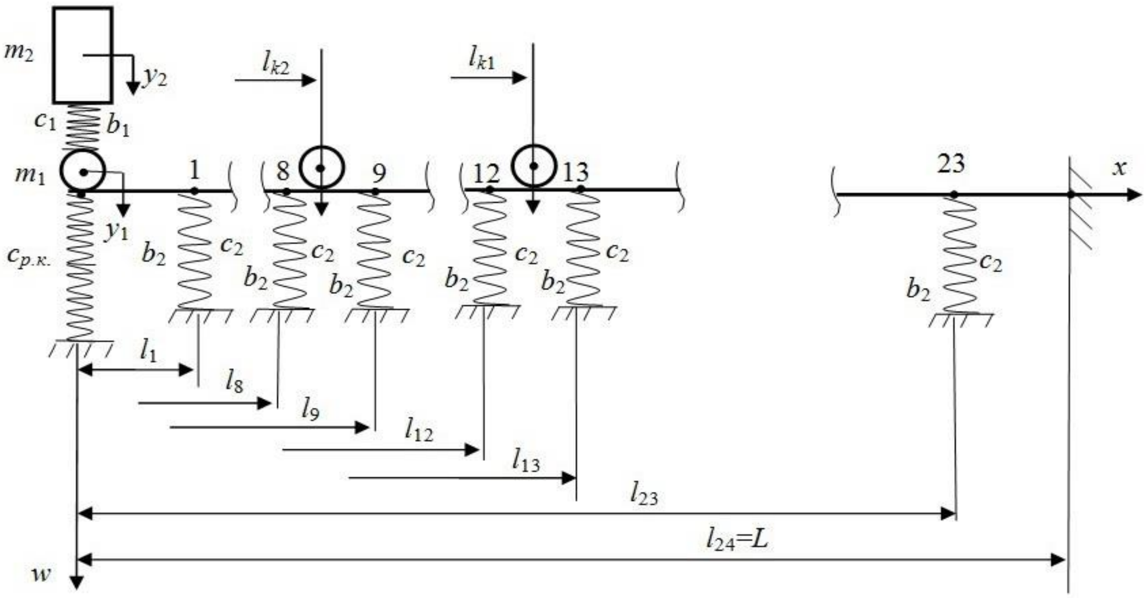

Fig. 6. Diagram of rail sections in the third phase of movement

Considering equations (8) and (10), as well as the diagram in Figure 6, we obtain:
at a section $0 \le x \le l_1$:

$$z_1(\mathrm{x}) = \rho(\kappa_1 \mathrm{x})z(0) + \frac{\varphi(\kappa_1 x)}{\kappa_1}z'(0) + \frac{\psi(\kappa_1 x)}{\kappa_1^2}z''(0) + \frac{\gamma(\kappa_1 x)}{\kappa_1^3}z'''(0) +$$
$$+\frac{\kappa_2^4 z(l^*)}{\kappa_1^3}\gamma[\kappa_1(x-l^*)]\Big|_{x\ge l^*} + \frac{c_1}{EJ\kappa_1^3}(\lambda_2 - 0,5\lambda_1)\gamma[\kappa_1(x-l^*)]|_{x\ge l^*} -$$
$$-\frac{c_{p.\kappa.}}{EJ\kappa_1^3}z(l^*)\gamma[\kappa_1(x-l^*)]|_{x\ge l^*};$$

at a section $l_i \le x \le l_{i+1}$ $(1 \le i \le 8)$ i $l_8 \le x \le l_{k2}$

$$z(\mathrm{x}) = \rho(\kappa_1 \mathrm{x})z(0) + \frac{\varphi(\kappa_1 x)}{\kappa_1}z'(0) + \frac{\psi(\kappa_1 x)}{\kappa_1^2}z''(0) + \frac{\gamma(\kappa_1 x)}{\kappa_1^3}z'''(0) +$$
$$+\frac{\kappa_2^4 z(l^*)}{\kappa_1^3}\gamma[\kappa_1(x-l^*)]\Big|_{x\ge l^*} + \frac{c_1}{EJ\kappa_1^3}(\lambda_2 - 0,5\lambda_1)\gamma[\kappa_1(x-l^*)]|_{x\ge l^*} -$$
$$-\frac{c_2}{EJ\kappa_1^3}\sum_{i=1}^{23} z(l_i)\gamma[\kappa_1(x-l_i)]|_{x\ge l_i} - \frac{c_{p.\kappa.}}{EJ\kappa_1^3}z(l^*)\gamma[\kappa_1(x-l^*)]|_{x\ge l^*}.$$

at a section $l_{k2} \le x \le l_i$ $(9 \le i \le 12)$ i $l_{12} \le x \le l_{k1}$

$$z_i(x) = \rho(k_1 x)z(0) + \frac{\varphi(k_1 x)}{k_1}z'(0) + \frac{\psi(k_1 x)}{k_1^2}z''(0) + \frac{\gamma(k_1 x)}{k_1^3}z'''(0) +$$
$$+\frac{k_2^4}{k_1^3}(z(l_{k2})\gamma[k_1(x-l_{k2})]|_{x\ge l_{k2}} + z(l_{k3})\gamma[k_1(x-l_{k3})]|_{x\ge l_{k3}}) + \tag{11}$$
$$+\frac{c_1}{EJk_1^3}(\lambda_2 - 0,5\lambda_1)\gamma[k_1(x-l^*)]|_{x\ge l^*} - \frac{c_2}{EJk_1^3}\sum_{i=1}^{12} z(l_i)\gamma[k_1(x-l_i)]|_{x\ge l_i} -$$
$$-\frac{c_{p.\kappa.}}{EJk_1^3}z(l^*)\gamma[k_1(x-l^*)]|_{x\ge l^*}.$$

at a section $l_{k1} \le x \le l_{13}$ i $l_i \le x \le l_{i+1}$ $(13 \le i \le 23)$

$$z_i(x) = \rho(k_1 x)z(0) + \frac{\varphi(k_1 x)}{k_1}z'(0) + \frac{\psi(k_1 x)}{k_1^2}z''(0) + \frac{\gamma(k_1 x)}{k_1^3}z'''(0) +$$
$$+\frac{k_2^4}{k_1^3}\sum_{i=1}^{3} z(l_{ki})\gamma[k_1(x-l_{ki})]\Big|_{x\ge l_{ki}} + \frac{c_1}{EJk_1^3}(\lambda_2 - 0,5\lambda_1)\gamma[k_1(x-l^*)]|_{x\ge l^*} - \tag{12}$$
$$-\frac{c_2}{EJk_1^3}\sum_{i=1}^{23} z(l_i)\gamma[k_1(x-l_i)]|_{x\ge l_i} - \frac{c_{p.\kappa.}}{EJk_1^3}z(l^*)\gamma[k_1(x-l^*)]|_{x\ge l^*}.$$

In this case, we will have 31 variables: $z(0)$, $z'(0)$, $z''(0)$, $z'''(0)$, $z_i(x)$ $(i = 1 \div 23)$, $z(l_{k1})$, $z(l_{k2})$, $\lambda_1 = z_1(l^*)$, λ_2.

Next, we determine the eigenfrequencies ω_s with respect to solutions (6) and the initial conditions of the mechanical system in Figure 6, which we will additionally include equalities in:

$$w(0,l_{k1}) = \Phi_1(l_{k1}) = \sum_{j=1}^{5} M_j z^j(l_{k1}); \Big| \cdot m_1 z^s(l_{k1});$$
$$w(0,l_{k2}) = \Phi_1(l_{k2}) = \sum_{j=1}^{5} M_j z^j(l_{k2}); \Big| \cdot m_1 z^j(l_{k2});$$
$$\dot{w}(0,l_{k1}) = \Phi_2(l_{k1}) = \sum_{j=1}^{5} D_j \omega_j z^j(l_{k1}); \Big| \cdot m_1 z^s(l_{k1});$$
$$\dot{w}(0,l_{k2}) = \Phi_2(l_{k2}) = \sum_{j=1}^{5} D_j \omega_j z^j(l_{k2}); \Big| \cdot m_1 z^s(l_{k2}) \cdot$$

The orthogonality conditions of the system in the third phase of motion then takes the following form:

$$\rho F \int_0^L z^s(x) z^j(x) dx + m_1(\lambda_1^s \lambda_1^j + z^s(l_{k1}) z^j(l_{k1}) + z^s(l_{k2}) z^j(l_{k2})) + m_2 \lambda_2^j \lambda_2^s =$$
$$= \delta^{js} \left(\rho F \int_0^L z^{s^2}(x) dx + m_1(\lambda_1^{s2} + z^{s2}(l_{k1}) + z^{s2}(l_{k2})) + m_2 \lambda_2^{s2} \right). \tag{13}$$

Consequently, we obtain the following expressions for determining the constants M_s, D_s of motion equations (when $x \neq 0$ and $j = s$):

$$D_j = \frac{m_1 \lambda_1^j V_{1,3\phi}}{\omega_j \left[\rho F \int_0^L z^{j2}(x) dx + m_1(\lambda_1^{j2} + z^{j2}(l_{k1}) + z^{j2}(l_{k2})) + m_2 \lambda_2^{j2} \right]}; \tag{14}$$

where $x = 0$ and $j = s$, when $\Phi_2(0) = V_{1,3\phi} \cdot \delta(0)$, $M_j = 0$,

$$D_j = \frac{\rho F V_{1,3\phi} z^j(0) + m_1 \lambda_1^j V_{1,3\phi}}{\omega_j \left[\rho F \int_0^L z^{j^2}(0) dx + m_1(\lambda_1^{j^2} + z^{j^2}(l_{k1}) + z^{j2}(l_{k2})) + m_2 \lambda_2^{j^2} \right]}. \tag{15}$$

Finally, the deflection of the facing rail in the third phase of the tramcar’s movement will be determined by the expression:

$$w(t,x) = \sum_{j=1}^{5} z^j(x) D_j \sin \omega_j t \cdot \tag{16}$$

At stage 2 of the method, the actions of static P_0 and inertial m_1 loads with coordinates l_{k1}, l_{k2} are additionally considered.

At stage 3 of the method, the deflection of the receiving rail under the first sleeper in the third phase of the tramcar movement will be determined by the formula:

$$w(t, x = l_1)_{\max,\phi 3} = y(x = l_1) + w_{Em3}(t, x = l_1)_{\max}. \quad (17)$$

Here $y(x = l_1)$, it is determined by the data of Section 3.1 of this article.

Analysis of these calculations shows that:

1) change in the tramcar load when $m = [2125 \div 3814]$ kg with motion speed $V_{1k} = 15$ m/s leads to a change in the dynamic deflection of the facing rail under the first sleeper in the range $w(t, x = l_1)_{\max,\phi 3} = [4{,}84 \div 9{,}214]$ mm, that is, to its growth of 1.9 times;
2) it is determined that the increase in the speed of a tramcar in the range of $V_{1k} = [5 \div 15]$ m/s when loading a tramcar $m = 3814$ kg leads to the amount change in the deflection in the range $w(t, x = l_1)_{\max,\phi 3} = [4{,}904 \div 9{,}214]$ mm, that is, to its growth of 1.89 times.

The results obtained are used when solving the issue of improving the upper structure of the rail track, as well as in determining the safe operating modes of tramcars.

4. RESEARCH RESULTS

In this paper, based on the improved dynamic model that describes the deflections of the facing rail, considering the characteristics of rigidity and damping of the suspension of the tramcar and the ballast layer, the speed, loading and phase of movement of the tramcar through the current number of reduced masses of wheels on the rail, it was possible to calculate the parameters of post-impact interaction in the mechanical system "tramcar-rail track" at the phase of growth of deflections under the first sleeper.

A method of sequential static-impact-dynamic calculation of deflections of the facing rail under the first sleeper in the growth phase was developed. This made it possible to note the phases of movement of the tramcar relative to the joint irregularity, its loading and speed, as well as the boundary conditions for fixing the facing and trailing rails, geometric and mechanical characteristics of rails and butt linings, sleepers and ballast layer. The method provides opportunities for studying the processes of mechanical interaction of a tramcar and a rail track aimed at improving the operating modes of the vehicle, as well as the upper structure of the track.

The dependences of static, impact and generalised components of the height of joint irregularity at the growth stage on the phases of movement of the tramcar are obtained. This was done taking into account the orthogonality of the vibrations forms of the facing rail, the operational, structural and mechanical characteristics of the tramcar and the upper structure of the track. Thus, allowing determining whether the maximum deflections of the receiving rail under the first sleeper belong to the range of acceptable values of elastic deformations, which are set by the regulatory document. This paper suggests using the mechanical interaction of a tramcar with a rail track in the joint zone as a normative factor – the deflection of the receiving rail under the first sleeper. Loads on the ballast layer of the upper track structure are caused by deflections of the track rails. They are affected by the operational, mechanical and geometric parameters of the vehicle, rails, butt linings, sleepers and ballast layer. Following the standards of operation of electric transport, the pressure transmitted from the sleeper to the ballast layer should not exceed the maximum permissible value, which, for example, in summer is equal to $[\sigma] = 19.62$ N/cm^2. In real operating conditions, an informative parameter that will characterise the mechanical interaction of the car with the rail track in the joint area, which is derived from the pressure factor on the ballast layer and can be measured by external controls, is the deflection of the sleeper. The generalised deflection of the rail, which corresponds to the standard maximum permissible pressure, depends on the specific area and weight of the sleeper, the specific weight of the rail and the stiffness of the ballast layer. It is known that the most important generalised deflection occurs under the first sleeper of the receiving rail. In general, the value of the specified deflection must satisfy irregularities that do not contradict the standard condition for not exceeding the maximum permissible pressure.

5. CONCLUSIONS

The numerical calculation results for the parameters of the mechanical railway rolling stock – track interaction in the area of joints are given. The analysis is based on the integral model, which considers operational, mechanical, and geometric factors. This approach allows establishing new patterns of interaction between the four-axle car and the track while passing over joint irregularity, as well as to improve the operational parameters, car characteristics and the upper structure of the track by rational selection and optimisation.

The model proposed can be used in the development of design solutions for improving the rail track joint, determining the operating modes of tramcars depending on the type of rail track, creating an experimental and theoretical complex for studying, calculating and improving the parameters of rail transport knots. All of this makes it possible to take a significant step in the development of electric transport towards a more environmentally friendly, safe, convenient and economical way to transport passengers and cargo.

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