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UTILIZING GAUSSIAN PROCESS REGRESSION FOR NONLINEAR MAGNETIC SEPARATION PROCESS IDENTIFICATION

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***Abstract**. This paper presents a novel approach utilizing Gaussian Process Regression (GPR) to identify dynamic models with nonlinear parameters in magnetic separation processes. It aims to address the complex and dynamic nature of these processes by employing advanced modeling methods. The effectiveness of GPR is demonstrated through its application to simulated signals representing real iron ore separation processes, highlighting its potential to enhance existing models and optimize processes. Conducted within the MATLAB, this research lays the groundwork for further advancement and practical implementation. The utilization of GPR in magnetic separation offers innovative modeling of nonlinear dynamic processes, promising improved efficiency and precision in industrial applications.*

Keywords: Gaussian process regression, magnetic separation, nonlinear modeling, dynamic systems

WYKORZYSTANIE REGRESJI PROCESU GAUSSOWSKIEGO DO IDENTYFIKACJI NIELINIOWYCH PROCESÓW SEPARACJI MAGNETYCZNEJ

***Streszczenie.** Niniejsza praca prezentuje nowatorskie podejście wykorzystujące regresję procesu Gaussa (Gaussian Process Regression, GPR) do identyfikacji modeli dynamicznych z parametrami nieliniowymi w procesach separacji magnetycznej. Celem jest uwzględnienie złożonego i dynamicznego charakteru tych procesów poprzez zastosowanie zaawansowanych metod modelowania. Skuteczność GPR jest demonstrowana poprzez jego zastosowanie do symulowanych sygnałów, reprezentujących rzeczywiste procesy separacji rudy żelaza, co podkreśla jego potencjał do ulepszania istniejących modeli oraz optymalizacji procesów. Badania przeprowadzone w środowisku MATLAB stanowią podstawę do dalszego rozwoju i praktycznej implementacji. Zastosowanie GPR w separacji magnetycznej pozwala na innowacyjne modelowanie nieliniowych procesów dynamicznych, obiecując poprawę wydajności i precyzji w zastosowaniach przemysłowych.*

Słowa kluczowe: regresja procesu gaussowskiego, separacja magnetyczna, modelowanie nieliniowe, systemy dynamiczne

Introduction

With the global rise in iron ore consumption, efficient ore processing methods are necessary [32]. Magnetic separation is a common method employed for removing impurities and low-grade particles [3, 4] but faces challenges due to their instability and nonlinearity [13, 21, 28, 29]. This research introduces Gaussian Process Regression (GPR) for identifying the nonlinear dynamics in magnetic separation, offering a substantial improvement in understanding and optimizing this process. Traditional studies using linear models and static process analysis are limited in handling such complex interactions:

1. The nonlinearity of the process, where the relationship between input parameters and output indicators cannot be adequately described by linear models [30].
2. The dynamic nature of the process with time delays, inertia, and feedback loops not accounted for by static models, leading to inaccuracies in describing and predicting process behavior [23].
3. The multifactorial nature and complex interactions among numerous factors influencing the efficiency of magnetic separation, which cannot be properly addressed by linear models [24].
4. Changes in the properties of the ore entering the enrichment process affect the course of the separation process, and models built on data for a specific type of ore may be inadequate when the raw material base changes [26].
5. Limited extrapolation of linear models obtained based on data within a certain range of parameter values, beyond this range, restricting their utility for optimization and prediction [7].

Our main objective is to outline GPR's potential in identifying nonlinear aspects in magnetic separation. This research optimizes processes and advances the mining and beneficiation industry. It introduces new prospects for enhancing operating systems in the field. The novelty of this research is in its methodological progression. Using GPR provides a more accurate report of magnetic processes and the potential for real-time adaptive control systems adjusting to ore processing changes. Adopting GPR not only improves separation but also revolutionizes static models that fail to capture transient behaviors key to optimal magnetic separation. This work demonstrates the potential for the development of predictive, automated systems using kernel functions, potentially revolutionizing the industry. These advancements bring forth higher levels of automation and efficiency, proving to be a unique and valuable addition to the academic and practical realms of the industry.

1. Literature review

A comprehensive analysis of the current state of research in the field of modeling magnetic enrichment processes for iron ores reveals that significant efforts are being directed towards the development and refinement of mathematical models that capture various facets of these complex processes [16, 20].

In their seminal work, Morkun et al. [17] delve into the intricacies of modeling the influence of key technological parameters on the quality indicators of concentrate and waste. They employ advanced statistical techniques to establish robust relationships between input variables and output characteristics, providing valuable insights into process optimization. Similarly, Tron et al. [19] present a novel approach to modeling the dynamics of magnetic separation using a combination of physical principles and data-driven methods. Their model demonstrates high accuracy in predicting the behavior of the system under various operating conditions.

Furthermore, there is a growing focus on the development of automatic control systems for magnetic separation processes, which aim to optimize equipment operation modes and enhance overall enrichment efficiency [14, 15]. Morkun and Tron [14] propose an innovative control strategy based on real-time monitoring of key process parameters and adaptive adjustment of control setpoints. Their system has been successfully implemented in industrial settings, yielding significant improvements in concentrate quality and recovery. In a related study, Morkun et al. [15] explore the application of intelligent control techniques, such as fuzzy logic and neural networks, to handle the inherent nonlinearities and uncertainties in magnetic separation processes.

Despite these advancements, the application of machine learning methods, particularly Gaussian processes, for modeling nonlinear dependencies in magnetic enrichment processes remains insufficiently explored. While some studies [18] have investigated the potential of these approaches in other areas of mineral processing, their systematic application to the specific domain of magnetic separation of iron ores is limited. Morkun et al. [18] demonstrate the effectiveness of Gaussian process regression in capturing complex nonlinear relationships in flotation processes, highlighting the promise of this technique for modeling other mineral processing operations.

Given the critical importance of accurate modeling and control in optimizing the efficiency of magnetic separation processes, there is a pressing need to further investigate the capabilities and benefits of Gaussian process methods in this context. By leveraging the power of machine learning and probabilistic modeling, researchers can develop more sophisticated and adaptive models that can handle the inherent complexities and variabilities of these processes. Such advancements have the potential to significantly enhance the performance of enrichment plants and pave the way for the development of intelligent, self-optimizing control systems.

In conclusion, while the current literature provides valuable insights into the modeling and control of magnetic enrichment processes for iron ores, there remain significant opportunities for further research and innovation. The application of Gaussian process methods represents a promising avenue for addressing the challenges associated with nonlinear dependencies and improving the accuracy and robustness of process models. By bridging the gap between traditional approaches and advanced machine learning techniques, researchers can unlock new possibilities for process optimization and contribute to the sustainable development of the iron ore industry.

2. Nonlinear processes in enrichment

In enrichment systems, the term 'nonlinearity' refers to complex interactions and processes that cannot be adequately described by linear models [2]. Enrichment systems encompass nonlinear material properties, parameter variability, and instability during the process [3]. Incorporating these aspects into models allows for better approximation and prediction of real enrichment conditions and outcomes. The diversity of conditions, such as temperature, pressure, substance concentration, leads to nonlinearity in the enrichment system. Accounting for nonlinearity is crucial for accurate modeling of enrichment processes for several reasons: complex component interactions, system parameter changes, process instability, non-standard influences.

Research highlighted in sources [13, 28] shows that the iron content in both the concentrate (β) and the tails (θ) varies nonlinearly based on primary control factors [3]. This variability can be modeled using different mathematical functions. Specifically, our calculations suggest that the relationship between β and θ and the drum rotation speed of the magnetic separator, denoted as 'n', can be described through polynomial and exponential functions, as illustrated in Fig. 1 (author's own calculations and [13, 28]).

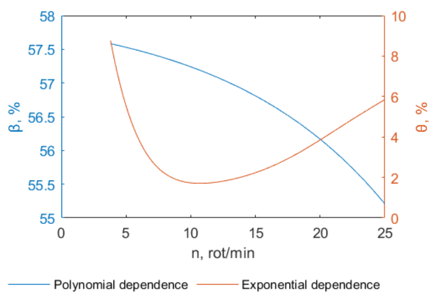


Fig. 1. Dependence of iron content in the concentrate β (polynomial dependence) and in the tails θ (exponential dependence) on the drum rotation speed n

As control influences, similar to the drum rotation speed, the pulp density δ, magnetic field intensity H, and additional water flow into the separator bath q_w are widely used, which also have nonlinear dependencies on the regulated enrichment parameters [13, 21, 28, 29].

In the study [29], an example of more complex nonlinear dependencies is provided. One of them determines the iron content in the concentrate β depending on the interaction of drum rotations ω and feed density δ. The three-dimensional visualization of this dependency is presented in Fig. 2 (author's own calculations and [2]).

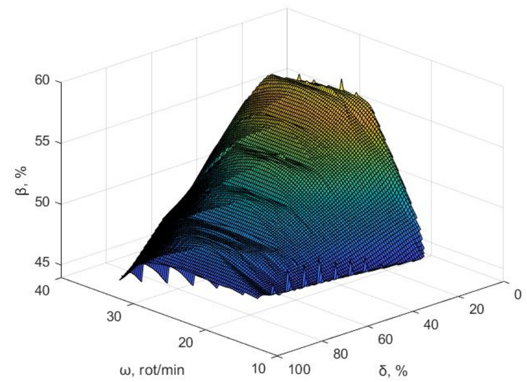


Fig. 2. Approximation of the dependence of iron content in the concentrate (β) on drum rotations (ω) and feed density (δ)

In the study [21], a stabilizing control system for the wet magnetic separation process with nonlinear elements is investigated. One of the subsystems of this scheme is represented by the dependence of iron content in the concentrate on the pulp density after the hydrocyclone, the nonlinear part of which can be expressed by equation (1):

$$\beta = -0{,}695\delta^2 + 0{,}893\delta + 0{,}712 \qquad (1)$$

where β represents the iron content in the concentrate, and δ denotes the pulp density after the hydrocyclone.

According to the complete structural diagram of the ore enrichment system described in the referenced work, the magnetic separator module consists of a nonlinear component characterized by formula (1) and a dynamic block composed of a serially connected aperiodic first-order block and a delay link, as shown in Fig. 3 (adapted [21]).

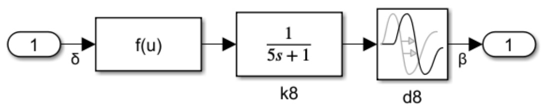


Fig. 3. Structural diagram of the magnetic separator with pulp density control

When inputting the pre-formed imitation control signal δ into the subsystem, the model output yields a response proportional to β, as shown in Fig. 4 (author’s calculations). In this case, by applying the principle of using only information regarding input and output signals in modeling, we can transition from a subsystem with specific internal content to the concept of a black box [12].

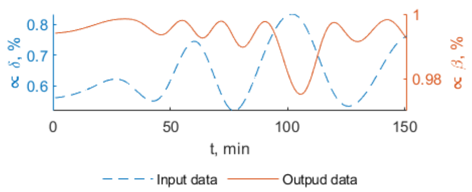


Fig. 4. Nonlinear relationship between input and output data in the magnetic separator control subsystem

3. Gaussian Process Regression

Gaussian Process Regression (GPR) is a powerful machine learning method used to model nonlinear processes [9]. Unlike traditional regression methods such as linear regression [8], GPR enables modeling complex nonlinear dependencies between input and output variables [10, 11].

GPR is based on the concept of a Gaussian process, which is a set of random variables, any finite number of which has a joint Gaussian distribution. In the context of GPR, a Gaussian process is used to determine the distribution of a function that maps input variables to outputs [6, 10].

One of the key advantages of GPR is its ability to account for uncertainty in the model. GPR provides a probabilistic estimate of output values, allowing for consideration of noise in the data and uncertainty in the modeling process [9, 10].

GPR is also a flexible method that allows for the use of different covariance functions to model various types of nonlinear dependencies. The covariance function determines the degree of dependence between input points and influences the correlation between them [10].

Advantages of GPR:

- modeling nonlinear relationships;
- handling high-dimensional data;
- optimization;
- accounting for uncertainty;
- resistance to overfitting.

Limitations of GPR:

- computational cost for large datasets;
- sensitivity to the choice of covariance function.

Overall, GPR is a powerful method for system identification and other modeling tasks where uncertainty and resistance to overfitting are important [6].

3.1. Rationale for choosing GPR

The choice of GPR for modeling magnetic enrichment systems with nonlinear parameters is justified by several factors [9]:

1. Nonlinearity: One of the main advantages of GPR is its ability to model complex nonlinear dependencies between input and output variables. In the case of magnetic enrichment, where processes may have complex and nonlinear nature, GPR can be an effective tool for capturing these characteristics.

2. Noise presence: GPR handles noise in output data well. In the case of magnetic enrichment, where various factors affecting the process may be present, accounting for noise is an important aspect of modeling.

3. Uncertainty: GPR provides the ability to account for uncertainty in the model, which can be useful in the case of complex technical systems where not all parameters can be precisely determined.

4. Flexibility: GPR allows for the use of various covariance functions, providing flexibility in modeling diverse systems and considering their unique properties.

Compared to other methods such as linear regression or neural networks, GPR has the ability to consider nonlinear dependencies without explicit definition of approximation functions. This makes it effective for modeling complex systems where nonlinearity plays an important role. Additionally, GPR may be more robust to data noise compared to other methods, making it attractive in conditions of uncertainty and mixed influences on the process.

Thus, the choice of GPR for modeling magnetic enrichment systems is justified by its ability to account for nonlinear dependencies, handle data noise, consider uncertainty, and flexibility in choosing covariance functions.

3.2. Comparative analysis with other methods of modelling nonlinear systems

In a comparative analysis of methods for modeling nonlinear systems, including GPR, several methods can be considered, such as neural networks, Support Vector Machine (SVM), decision trees, and regression methods [11].

1. Neural networks: Neural networks are powerful tools for modeling nonlinear systems, capable of capturing complex dependencies between input and output data. They can be effective in solving complex prediction and classification tasks. However, neural networks may require a large amount of data for training and tuning, as well as complex optimization processes [25].

2. Support Vector Machine (SVM): SVM is an effective method for modeling nonlinear systems, especially in cases with high-dimensional data. They can work well with limited datasets and capture complex dependencies. However, SVM may be demanding in terms of tuning hyperparameters and may have limited ability to handle large datasets [5].

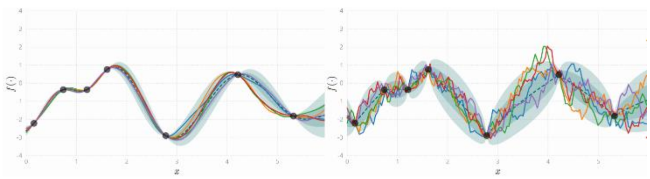
3. Regression methods: Classical regression methods, such as linear regression and polynomial regression, can be effective for modeling simple nonlinear dependencies [22]. They are simple and easily interpretable but may be limited in their ability to model complex nonlinear interactions.

The advantages of GPR lie in its ability to handle nonlinearity and system dynamics, generalize to new data, and account for uncertainty and noise in the data. However, GPR may require a large amount of data for training and tuning hyperparameters, and may be limited in handling large datasets.

In summary, the choice of a method for modeling nonlinear systems depends on the specific task, the volume and nature of the data, as well as the requirements for model accuracy and generalization.

3.3. Covariance functions

The covariance function serves as a key element of the Gaussian Process Regression model, determining the degree of dependence between input points and forming the correlation structure between them. The selection of an appropriate covariance function is crucial for building an effective GPR model capable of accurately capturing complex nonlinear dependencies in the data [1]. Fig. 5 illustrates an example of the influence of different covariance functions on data prediction, generated using the Interactive Gaussian Process Visualization online tool [33].



a) b)

Fig. 5. Example of using covariance functions on the same dataset: a) Matern 5/2; b) exponential

As seen from Fig. 5, different covariance functions have varying impacts on the data prediction process. Therefore, in choosing the appropriate function, it is necessary to consider several data properties such as their type, structure, distribution, as well as the nature of relationships between features. For data with complex nonlinear dependencies, it may be advantageous to use covariance functions with high expressiveness that can adapt to different forms of variable relationships. On the other hand, for data with structured correlations, it may be beneficial to use simple covariance functions that effectively approximate local dependencies. Thus, the choice of covariance function should be made considering the specific characteristics of the dataset and the forecasting task.

There are several types of covariance functions, each with unique properties suitable for different types of data and modeling tasks. Some of the most common covariance functions include:

- Exponential covariance function, characterized by rapid correlation decay with increasing distance between input points. It is well-suited for modeling data with local dependencies.
- Squared-exponential covariance function, which is a generalization of the exponential covariance function allowing control over the smoothness of the function. It is suitable for modeling data with smoother dependencies.
- Matérn covariance function is a flexible covariance function that allows control over both the degree of smoothness and the rate of correlation decay. It is suitable for modeling data with various types of dependencies.
- Periodic covariance function is used for modeling data demonstrating periodic behavior. It is suitable for modeling seasonal data or data collected from periodic processes.

The choice of covariance function depends on the specific modeling task and the nature of the data. Experimental tuning of the hyperparameters of the covariance function is often necessary to optimize the performance of the GPR model.

Key differences between covariance functions include correlation decay rate, smoothness degree, and periodicity.

In conclusion, covariance functions are an important component of GPR models, determining the degree of dependence between input points and influencing the correlation structure between them. Choosing the right covariance function is crucial for building an effective GPR model capable of accurately capturing nonlinear dependencies in the data.

3.4. The methodology and training process GPR models

The Gaussian Process Regression (GPR) methodology is a powerful tool for modeling and predicting nonlinear processes, such as those encountered in magnetic enrichment. This study aims to explore the application of GPR in the context of magnetic separation, utilizing data generated from a specific technological scheme described in the literature [21]. By employing GPR, we seek to develop an effective model capable of accurately capturing the complex relationships between input parameters and output indicators in the magnetic enrichment process.

The methodology of training GPR models involves several key stages. Firstly, the input and output data for model training are prepared, which includes collecting and preprocessing relevant information about the magnetic enrichment process, such as pulp density, magnetic field intensity, water flow rates, and iron content in concentrate and tailings. The data used in this study is generated from a scheme representing the magnetic separator module and its nonlinear and dynamic components, as presented in the referenced literature [21]. This ensures that the training data accurately reflects the real-world process and its specific characteristics.

Next, the data is divided into training and testing sets to evaluate the model's performance on unseen data. Since the dataset is generated randomly and contains normally distributed values, a 50-50 split is employed to ensure representativeness and preserve the statistical properties of the data. This allows for a reliable assessment of the model's generalization ability and predictive performance.

Choosing the appropriate covariance function is crucial for building an effective GPR model, as it determines the dependence structure between input points. In this study, multiple covariance functions, including exponential, squared-exponential, Matérn, and periodic functions, are investigated to identify the one that yields the best performance and minimizes prediction errors.

The GPR model is then trained on the training data, establishing the relationship between input and output variables while accounting for nonlinearities and correlations. The trained model's performance is evaluated on the testing data by comparing predicted values with actual data points and calculating various metrics such as Root Mean Squared Error (RMSE), Coefficient of Determination (R-squared), Mean Squared Error (MSE), and Mean Absolute Error (MAE). These metrics provide a quantitative assessment of the model's goodness of fit and its ability to generalize to unseen examples.

In conclusion, the GPR methodology offers a promising approach for modeling and predicting nonlinear processes in magnetic enrichment. By leveraging data generated from a specific technological scheme and employing rigorous training and evaluation procedures, this study aims to develop an accurate and reliable GPR model. The methodology encompasses data preparation, covariance function selection, model training, and performance evaluation, enabling the construction of an efficient model for predicting complex relationships in magnetic separation processes. The insights gained from this study can contribute to the optimization and control of magnetic enrichment operations, ultimately enhancing the efficiency and effectiveness of the overall process.

4. Model identification

4.1. Transition to the "black box" model

In the study, we transition from the structural scheme of the magnetic separator (gray box) used in the work [21] to a "black box" model with a single input and a single output (SISO). In this process, we do not employ any physical understanding but construct the model solely based on experimental data using learning systems and the selected model structure. Modeling the black box can also be referred to as empirical modeling [25]. This transition allows us to focus on the system's external behavior without delving into the intricate details of its internal structure.

To achieve this, we apply the principle of aggregating internal elements to the subsystem of the magnetic separator, as depicted in Fig. 3. Additionally, we consider assumptions regarding the limited availability of information regarding the system's structure.

To provide the necessary signals at the input of the subsystem, we utilize a signal δ proportional to the pulp density, which is a crucial parameter in the magnetic separator control process. At the output, we obtain a signal β proportional to the iron content in the concentrate, which is a key result of the separation operation. The separation model can be presented as a black box, as illustrated in Fig. 6 (adapted from [27]).

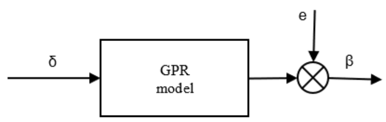


Fig. 6. SISO black box model obtained from the structural scheme of the pulp density-controlled magnetic separator

The general representation of the system in a black box corresponds to the formula of the system's behavior [25]:

$$y(t) = f(\varphi(t)) + e(t)$$

where $y(t)$ represents the iron content in the concentrate β; $f(*)$ denotes the modeled function; $\phi(t)$ stands for the parameter vector, in our case, the pulp density after the hydrocyclone δ; $e(t)$ represents the influence noise.

This approach confirms the effectiveness and adaptability of the black-box model in dealing with complex systems where accurately modeling internal processes is challenging. The black-box model allows us to focus on the external behavior of the system and utilize system identification methods to build a model capable of predicting output variables based on input variables.

4.2. Data preparation

To develop and evaluate the Gaussian Process Regression (GPR) model, a smoothed signal of 500 data points was employed as the input vector X. This signal was fed into a black-box dynamic model to generate the corresponding output vector Y, as illustrated in Fig. 4. The resulting signal underwent preprocessing to ensure its suitability for subsequent modeling and analysis, with an adequate number of data points for both model training and validation.

In the context of optimal separator control, the study focuses on maximizing the iron content in the concentrate, denoted as β, by examining function (1). The optimal operating conditions, which determine the values of vector X, were identified at points where the iron content in the concentrate reached its maximum, particularly in the second and third quadrants.

An additional objective of the study involves assessing the identification capabilities of GPR-trained models in the presence of noisy signals. This approach of incorporating noise during GPR modeling has been effectively applied in prior research [27].

In the magnetic ore enrichment process, several disruptive factors, or "noises," can be considered:

- Iron content in the pulp (α).
- Degree of material liberation (R).
- Granulometric composition of the pulp (C).
- Pulp flow rate (q).

The study aims to investigate the robustness of identification methods when input data is subject to noise and to develop GPR-based models that can adapt to such conditions. To achieve this, an additional vector of noisy data was generated based on the output data, with the noise level limited to a maximum of 0.1% of the output signal's maximum amplitude.

For training purposes, the input vectors X, Y, and Y' were partitioned into two separate sets: a training set and a testing set, each containing 250 data points, as depicted in Fig. 7.

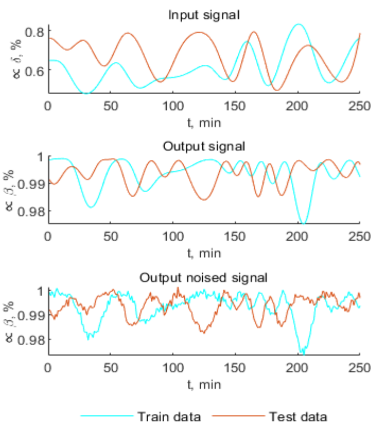


Fig. 7. Training and testing sets of input and output data

4.3. Model training

Training of the GPR model and prediction of the output signal were conducted using MATLAB software in accordance with the general methodology [31].

```
% creation of training arrays including input, output, and output arrays with noise:
x_train=x(1:250,:);
y_train=y(1:250,:);
y_trainnoised=y_train + 0.001*randn(size(x_train));
```

```
% creating a test data array similar to the training one:
x_test=x(251:500,:);
y_test=y(251:500,:);
y_testnoised=y_test + 0.001*randn(size(x_test));
```

```
% training the fitrgp model on noise-free data using different covariance functions: squared exponential, exponential, Matérn32, Matérn52, rational quadratic.
gprMdlsq = fitrgp(x_train,y_train, 'KernelFunction','squaredexponential', 'OptimizeHyperparameters','auto');
gprMdlexp = fitrgp(x_train,y_train, 'KernelFunction','exponential', 'OptimizeHyperparameters','auto');
gprMdl32 = fitrgp(x_train,y_train, 'KernelFunction','matern32', 'OptimizeHyperparameters','auto');
gprMdl52 = fitrgp(x_train,y_train, 'KernelFunction','matern52', 'OptimizeHyperparameters','auto');
gprMdlrq = fitrgp(x_train,y_train, 'KernelFunction','rationalquadratic', 'OptimizeHyperparameters','auto');
```

```
% predicting test data using models trained on noise-free data:
[ypred1_sq,~, ~] = predict(gprMdlsq,x_test);
[ypred1_exp,~, ~] = predict(gprMdlexp,x_test);
[ypred1_32,~, ~] = predict(gprMdl32,x_test);
[ypred1_52,~, ~] = predict(gprMdl52,x_test);
[ypred1_rq,~, ~] = predict(gprMdlrq,x_test);
```

```
% evaluation of the forecast quality using the Mean Squared Error metric:
msesq=immse(ypred1_sq, y_test);
mseexp=immse(ypred1_exp, y_test);
mse32=immse(ypred1_32, y_test);
mse52=immse(ypred1_52, y_test);
mserq=immse(ypred1_rq, y_test);
```

The accuracy assessment results of data forecasting using models trained based on different covariance functions indicate high model quality: all of them showed practically the same level of MSE quality indicator for the utilized data (table 1).

Table 1. Comparison of the MSE metric for different datasets

Model	gprMdlsq	gprMdlexp	gprMdl32	gprMdl52	gprMdlrq
MSE	8.959e-06	8.888e-06	8.928e-06	8.871e-06	8.888e-06

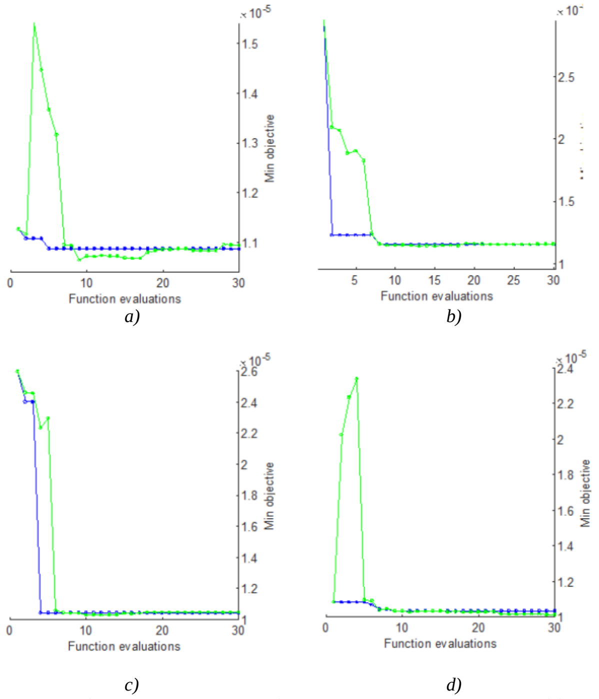


Fig. 8. Plots of the minimum objective function value versus the number of function evaluations during hyperparameter optimization of GPR models using different covariance functions: a) squared exponential; b) exponential; c) Matern 3/2 function; d) Matern 5/2 function

As illustrated in Fig. 8, the decrease in the minimum objective function value with an increasing number of evaluations indicates a successful hyperparameter optimization process for all considered covariance functions. According to the training results, all models, despite using different covariance functions, achieve optimal parameter values in approximately the same number of steps, ranging from 5 to 6.

Based on the provided information and the conducted analysis, it can be confidently concluded that the squared exponential covariance function is the most reliable choice for the given Gaussian Process Regression (GPR) model. This assertion is supported by the fact that the range of the hyperparameter σ values, for which the expected improvement (objective function) is minimized, is an order of magnitude larger when using the squared exponential function compared to the other covariance functions, all other conditions being equal (Fig. 9).

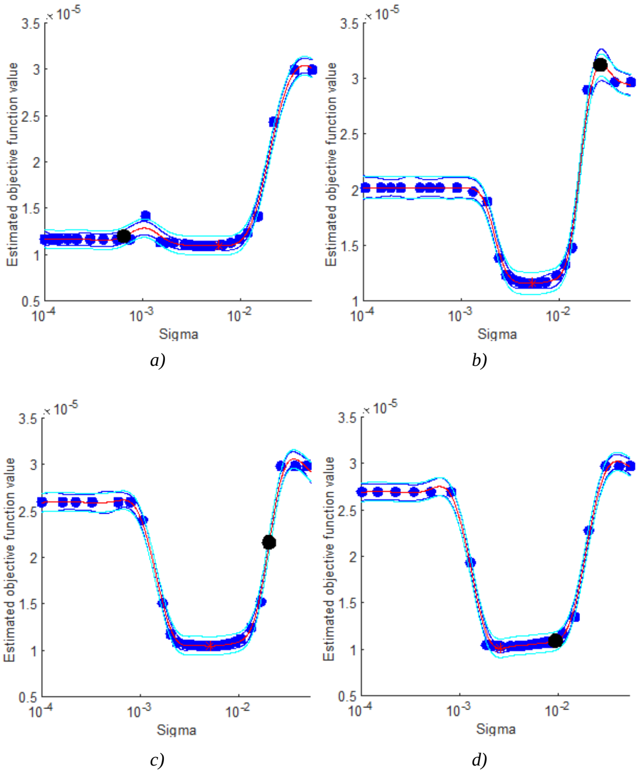


Fig. 9. Optimization of GPR model hyperparameters with different covariance functions: a) squared exponential; b) exponential; c) Matérn 3/2 function; d) Matérn 5/2 function

This wider range of optimal σ values demonstrates the robustness and flexibility of the squared exponential function in capturing the underlying patterns in the data. Consequently, a GPR model utilizing this covariance function is expected to exhibit superior generalization capabilities and be less susceptible to overfitting, making it a more dependable tool for modeling and prediction tasks.

In light of these findings, it is strongly recommended to employ the squared exponential covariance function in the GPR model for this particular application. By leveraging its advantageous properties, the model is likely to yield more accurate and reliable results, ultimately leading to improved decision-making and enhanced performance in the given context.

As previously mentioned, the analysis involved training fitrgp models with different covariance functions, such as 'squared exponential', 'exponential', 'matern32', 'matern52', and 'rational quadratic', on noise-free data. Subsequently, these models were utilized to predict test data. The models' forecasting quality was evaluated using the Mean Squared Error (MSE) metric.

The results demonstrated high model accuracy. All the models exhibited similar levels of MSE for the applied data, with values in table 1 approximately ranging from 8.88e-06 to 8.96e-06. This indicates that the choice of covariance function in the Gaussian Process Regression (GPR) model does not significantly influence the prediction accuracy for the tested data.

However, this result might be specific to the data used in this study, and different results might be obtained with different datasets or under different conditions. Additional tests with various data types and various noise levels may be necessary to understand the impact of the covariance function on the GPR model's accuracy better.

In this regard, further model training was conducted in fully automatic mode:

```
% automatic training of the fitrgp model on regular gprMdl1 and noisy gprMdl2 output data:
gprMdl1 = fitrgp(x_train,y_train);
gprMdl2 = fitrgp(x_train,y_trainnoised);

% forecasting test data using trained models:
[ypred1_ts,~,yint1_ts] = predict(gprMdl1,x_test);
[ypred2_ts,~,yint2_ts] = predict(gprMdl2,x_test);
```

4.4. Processing and interpretation of the results

During the forecasting process using the trained GPR models (gprMdlx), two key variables were obtained: 'ypredx_ts', representing the predicted values of the output data, and 'yintx_ts', denoting the 95% confidence interval, which reflects the quality of the output parameter prediction.

Visualizing the test data output array alongside the predicted values and confidence interval offers valuable insights into the learning outcome, particularly the accuracy of data prediction using GPR methods (Fig. 10).

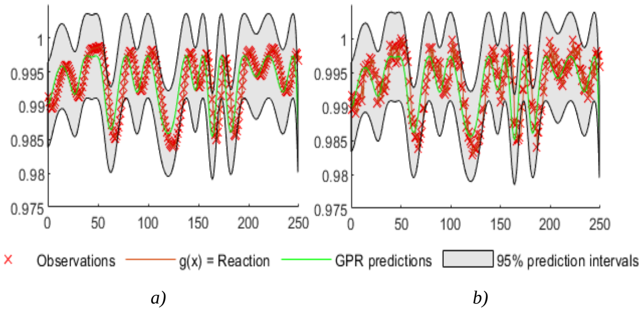


Fig. 10. Accuracy of Gaussian Process Regression training in the context of magnetic separation process identification: a) test data without noise; b) noisy test data

In analyzing the data represented in Fig. 10, the accurate forecasting of both sets of test data is apparent. The high accuracy is endorsed by the close alignment of the Gaussian Process Regression (GPR) model's predictions with the actual test data. Significantly, these predictions fall within the 95% confidence interval throughout the entire range of testing, demonstrating the model's robust performance in predicting the intricacies of the magnetic separation process.

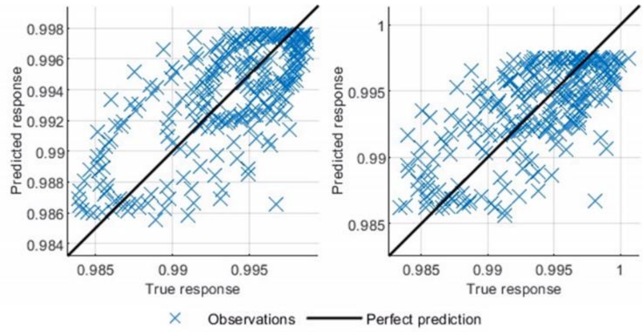
The graphical comparison of predicted versus actual test values, depicted in Fig. 11, elucidates the appropriateness of the Gaussian Process Regression technique in accurately predicting the nonlinear dependencies inherent in magnetic mineral separation. The exhibited symmetry along the line of perfect correlation implies high predictive precision of the adopted model. The coherency between actual and predicted values, even when dealing with noisy data, affirms the robustness of the Gaussian Process Regression method in tackling the complexities of iron ore separation processes.

Fig. 12 presents the leave-one-out residuals (loores) for the Gaussian Process Regression models applied to the magnetic separation process data. The analysis of these residual plots reveals several key characteristics that are common to both the clean and noisy data models:

1. Randomness: The residuals are randomly scattered around zero, suggesting the absence of systematic errors in the GPR models. This indicates that the models have effectively captured the underlying patterns in the data without introducing bias.

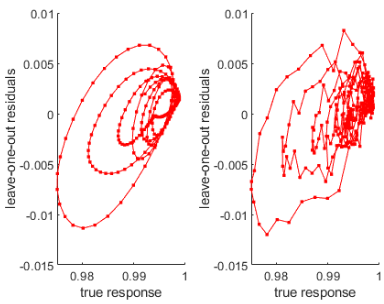
2. Normality: The residuals exhibit an approximately symmetric distribution, which is consistent with the assumption of normality. This suggests that the GPR models have adequately accounted for the stochastic nature of the magnetic separation process.

3. Homoscedasticity: The residuals display roughly equal variance across all values of the true response, indicating the absence of heteroscedasticity. This implies that the GPR models have successfully captured the variability in the data, regardless of the magnitude of the true response.



a) b)

Fig. 11. Predicted vs Actual plot for the test data set in the magnetic separation process: a) Data without noise, showcasing the model's precision; b) noisy data, demonstrating the model's robustness in handling real-world process variations



a) b)

Fig. 12. Leave-one-out residuals for the GPR models in the context of magnetic separation process identification: a) clean data, showcasing the model's performance under ideal conditions; b) noisy data, demonstrating the model's ability to handle real-world process variations

However, there are notable differences in the shape and distribution of data points between the two models. In Fig. 11a, which represents the clean data model, the data points are more horizontally distributed, suggesting a weaker correlation between the true response and the predicted value. Conversely, in Fig. 11b, which represents the noisy data model, the data points exhibit a more elliptical distribution, indicating a stronger correlation between the true response and the predicted value.

These findings suggest that the GPR models have effectively captured the complex nonlinear relationships present in the magnetic separation process data, even in the presence of noise. The models' ability to maintain randomness, normality, and homoscedasticity of residuals, despite the differences in data quality, highlights the robustness and adaptability of the GPR approach in modeling the intricacies of mineral processing systems. This underscores the potential of GPR as a powerful tool for process optimization and control in the context of magnetic separation and other mineral processing applications.

In assessing the accuracy and efficacy of the developed Gaussian Process Regression (GPR) models, various error metrics were employed. These metrics include root mean squared error (RMSE), coefficient of determination (R-Squared), mean squared error (MSE), mean absolute error (MAE). These measures were applied to both training and testing datasets, enabling a comprehensive evaluation of the models' performance and reliability across different stages of training. The outcome of these metrics is presented in table 2.

Table 2. Resulting metric values

Metrics	gprMdl1		gprMdl2	
	Validation	Testing	Validation	Testing
RMSE	0.0031717	0.0029822	0.0033064	0.0031931
R-Squared	0.66	0.47	0.64	0.43
MSE	1.00666e-05	8.89376-6	1.0932e-5	1.0196e-5
MAE	0.0024058	0.0024419	0.0025659	0.0026021

The metric results suggest that the gprMdl1 model, developed on noise-free data, exhibits superior predictive accuracy during both validation and testing stages, as indicated by its lower RMSE, R-Squared, MSE, and MAE values compared to gprMdl2 model.

In conclusion, while noise-free data contribute to enhancing the model's prediction accuracy, the model trained on data with noise (gprMdl2) remains useful when tackling uncertainties or variations inherent in the output data. Therefore, the selection between noise-free and noisy data should be context-dependent and aligned with the specific accuracy demands in predicting magnetic separation processes of iron ores.

5. Conclusions and future work

This study investigated the application of Gaussian Process Regression (GPR) for identifying and modeling nonlinear processes in the magnetic separation of iron ores. The results confirmed the effectiveness of GPR in modeling the complex nonlinear dependencies inherent in these processes, both at the input and output, which poses significant challenges for other modeling approaches. GPR demonstrated high prediction accuracy in capturing these nonlinear relationships.

Traditional approaches to modeling enrichment processes often face difficulties in accounting for the nonlinear interactions between input parameters (pulp density, magnetic field intensity, water flow rates, etc.) and the resulting indicators (iron content in concentrate, tailings, etc.). The application of GPR allows overcoming these limitations due to its ability to effectively approximate complex nonlinear dependencies based on experimental data.

While the research focused on the magnetic separation of iron ores, the findings can be adapted to other types of ores and enrichment processes where nonlinear relationships are present. However, further studies are necessary to validate the effectiveness of GPR in these cases and to address the limitations of the current study, such as the limited experimental dataset and the exclusion of certain factors like particle size distribution, mineralogical composition, and moisture content, which may influence the separation process.

The use of GPR in automatic control systems for magnetic separators also demonstrated significant potential for optimizing technological processes.

Based on the obtained results, future work aims to conduct the following research:

1. Detailed analysis of the impact of different covariance functions on the accuracy of the GPR model.

2. Development and testing of prototypes of automatic control systems based on GPR models for magnetic separators.

3. Expansion of research to systems with multiple inputs and outputs to study their influence on the process of iron ore separation.

4. Implementation of machine learning mechanisms for automatic adaptation of GPR models to changing operating conditions.

5. Evaluation of the economic efficiency of implementing GPR-based systems in practice and identifying ways to optimize them to maximize benefits for the enterprise.

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