Kołowrocki Krzysztof
ORCID ID: 0000-0002-4836-4976

Kuligowska Ewa
ORCID ID: 0000-0002-8274-3005

Soszyńska-Budny Joanna
ORCID ID: 0000-0003-1525-9392

Maritime University, Gdynia, Poland

Critical infrastructure operation process including operating environment threats

Keywords

critical infrastructure, operation, prediction, climate-weather change

Abstract

Considering a significant influence of the critical infrastructure operating environment threats on its operation process and safety, based on semi-Markov processes theory, a convergent to reality model of the critical infrastructure operation process related to critical infrastructure operating environment threats is built. The method of defining the parameters of this operation process is presented and new procedures of their determining in the case when the critical infrastructure operating threats are not explicit separated in this process are proposed.

1. Introduction

Considering a significant influence of the critical infrastructure operating environment threats on its operation process and safety, based on semi-Markov processes theory, a convergent to reality model of the critical infrastructure operation process related to critical infrastructure operating environment threats is built. The method of defining the parameters of this operation process is presented and new procedures of their determining in the case when the critical infrastructure operating threats are not explicit separated in this process are proposed.

In the paper, the traditional semi-Markov approach to a complex technical system operation process modeling considered in [EU-CIRCLE Report D3.3-GMU3-CIOP Model1, 2016] is developed to modelling a critical infrastructure operation process including operating environment threats. The method of defining the parameters of this operation process is presented and new procedures of their determining in the case when the critical infrastructure operating threats are not explicit separated in this process are proposed.

2. Modelling critical infrastructure operation process including operating environment threats

The companies (stakeholders, operators) using different critical infrastructures often have very different organizational environments. The critical infrastructures organizational environments are composed of forces or institutions surrounding an organization that affect performance, operations and resources. They include all of the elements that exist outside of the organization's boundaries and have the potential to affect a portion or all of the organization, for instance government regulatory agencies, competitors, customers, suppliers and pressure from the public. To manage the organization effectively, managers need to properly understand the environment. It is reasonable to divide environmental factors into two parts, namely, internal and external environments. An organization's internal environment consists of the entities, conditions, events, and factors within the organization that influence choices and activities, especially in employee behaviour. It exposes the strengths and weaknesses found within the organization. Factors that are frequently considered part of the internal environment include the organization's culture, mission statement, and leadership styles. An organization's external environment consists of the entities, conditions, events, and factors surrounding the organization that influence choices and activities and determine its opportunities and threats. It is also called an operating environment. Examples of factors affecting an organization's external environment include customers, public opinion, economic conditions, government regulations, and competition. Thus, taking into account the above analysis, the critical infrastructure operating environment threat can be defined as an unnatural event that may cause the critical infrastructure damage and/or change its operation activity in the way unsafe for it and its operating environment, [EU-CIRCLE Report D1.1, 2015], [EU-CIRCLE ReportD1.4-GMU3, 2016]. For instance, the critical infrastructure unnatural threats coming from its operating environment are another critical infrastructure activity in its operating environment that can result in an accident with serious consequences for the critical infrastructure and its operating environment, a human error, an act of vandalizm and a terrorist attack changing the critical infrastructure operation process in an unsafe way.

2.1. Semi Markov model of critical infrastructure operation process including operating environment threats

We assume that the critical infrastructure operation process modelled in Section 2.1 [EU-CIRCLE Report D3.3-GMU3-CIOP Model1, 2016] can be affected by a number $\gamma,\ \gamma \in N,$ of unnatural threats coming from the critical infrastructure operating environment and mark them by

$$ut_i,\ i = 1,2,\dots,\gamma.$$

We define new operation states considering the critical nfrastructure operating environment threats as follows:

- the operation states without including operating environment threats

$$z_i' = z_i,\ i = 1,2,\dots,\nu,\ \nu \in N; \tag{1}$$

- the operation states including at least 1 and maximum w of operating environment threats

$$z_i',\ i = \nu+1, \nu+2,\dots,\nu',\ \nu' \in N. \tag{2}$$

This way, we can have:

$$-\ \nu \cdot \binom{\gamma}{0} = \nu \tag{3}$$

operation states without including operating environment threats ut_i, $i = 1,2,\dots,\gamma$;

$$-\ \nu \cdot \binom{\gamma}{1} = \nu\gamma \tag{4}$$

operation states including 1 of the operating environment threats ut_i, $i = 1,2,\dots,\gamma$;

$$-\ \nu \cdot \binom{\gamma}{2} = \nu\gamma(\gamma-1)/2 \tag{5}$$

operation states including different 2 of the operating environment threats ut_i, $i = 1,2,\dots,\gamma$;

...;

$$-\ \nu \cdot \binom{\gamma}{\gamma} = \nu \tag{6}$$

operation states including all w operating environment threats ut_i, $i = 1,2,\dots,\gamma$.

Thus, considering (1)-(6), the maximum value of the number of new operation states is

$$\nu' = \nu \cdot [\binom{\gamma}{0} + \binom{\gamma}{1} + \dots + \binom{\gamma}{\gamma}] = \nu \cdot 2^{\gamma}, \tag{7}$$

Practically most comfortable numeration of the operation states of the critical infrastructure operation process including its operating environment threats is as follows:

- the operation states without including operating environment threats by

$$z_i' = z_1 \text{ for } i = 1,\ z_i' = z_2$$
$$\text{for } i = 2^{\lambda}+1,\ \dots,\ z_i' = z_\nu$$
$$\text{for } i = (\nu-1)2^{\gamma}+1; \tag{8}$$

- the operation states including state z_1 and successively 1, 2 until γ operating environment threats ut_i, $i = 1,2,\dots,\gamma$, by

$$z_i',\ i = 2,\ \dots,\ 2^{\gamma}, \tag{9}$$

- the operation states including state z_2 and successively 1, 2 until γ operating environment threats ut_i, $i = 1,2,\dots,\gamma$, by

$$z_i^{'},\ \ i = 2^{\gamma} + 2,\ \ldots,\ 2 \cdot 2^{\gamma}, \tag{10}$$

. . . ;

- the operation states including state z_v and successively 1, 2 until w operating environment threats ut_i, $i = 1,2,...,\gamma$, by

$$z_i^{'},\ \ i = (v-1)2^{\gamma} + 2,\ \ldots,\ v \cdot 2^{\gamma}. \tag{11}$$

In the case if operating environment threats are disjoint, the number of new operation states is

$$v' = v(\gamma + 1),$$

and their numeration is as follows:
- the operation states without including operating environment threats by
- the operation states without including operating environment threats by

$$z_i^{'} = z_1 \text{ for } i = 1,\ z_i^{'} = z_2$$
$$\text{for } i = \gamma + 1,\ \ldots,\ z_i^{'} = z_v$$
$$\text{for } i = (v-1)(\gamma+1)+1; \tag{12}$$

- the operation states including state z_1 and single successive operating environment threats ut_i, $i = 1,2,...,\gamma$, by

$$z_i^{'},\ i = 2,\ \ldots,\ \gamma + 1, \tag{13}$$

- the operation states including state z_2 and single successive operating environment threats ut_i, $i = 1,2,...,\gamma$, by

$$z_i^{'},\ \ i = (\gamma+1)+2,\ \ldots,\ 2(\gamma+1), \tag{14}$$

. . . ;

- the operation states including state z_v and single successive operating environment threats ut_i, $i = 1,2,...,\gamma$, by

$$z_i^{'},\ \ i = (v-1)(\gamma+1)+2,\ \ldots,\ v(\gamma+1). \tag{15}$$

In our further considerations, we assume that, the critical infrastructure during its operation process can take v', $v' \in N$, defined above, by (3.8)-(3.11) or by (3.12)-(3.15) in a particular case of disjoint operating environment threats, different operation states

$$z_1^{'}, z_2^{'}, \ldots, z_v^{'}, z_{v+1}^{'}, \ldots, z_{v'}^{'}. \tag{16}$$

Further, we define the critical infrastructure new operation process $Z'(t)$, $t \in <0,+\infty)$, related to the critical infrastructure operating environment threats with discrete operation states from the set $\{z'_1, z'_2, \ldots, z'_{v'}\}$. Moreover, we assume that the critical infrastructure operation process $Z'(t)$ related to its operating environment threats is a semi-Markov process similar to that one considered in Section 2.1 [EU-CIRCLE Report D3.3-GMU3-CIOP Model1, 2016] with the conditional sojourn times θ'_{bl} at the operation states z'_b when its next operation state is z'_l, $b, l = 1,2,...,v'$, $b \neq l$.
Under these assumptions, the critical infrastructure operation process may be described by:
- the vector of the initial probabilities

$$p'_b(0) = P(Z'(0) = z'_b),\ b = 1,2,...,v', \tag{17}$$

of the critical infrastructure operation process $Z'(t)$ staying at particular operation states at the moment $t = 0$

$$[p'_b(0)]_{1 \times v'} = [p'_1(0), p'_2(0), ..., p'_{v'}(0)]; \tag{18}$$

- the matrix of probabilities

$$p'_{bl},\ b, l = 1,2,...,v', \tag{19}$$

of the critical infrastructure operation process $Z'(t)$ transitions between the operation states z'_b and z'_l

$$[p'_{bl}]_{v' \times v'} = \begin{bmatrix} p'_{11} & p'_{12} & \cdots & p'_{1v'} \\ p'_{21} & p'_{22} & \cdots & p'_{2v'} \\ \cdots \\ p'_{v'1} & p'_{v'2} & \cdots & p_{v'v'} \end{bmatrix}, \tag{20}$$

where by formal agreement

$$p'_{bb} = 0 \text{ for } b = 1,2,...,v';$$

- the matrix of conditional distribution functions

$$H'_{bl}(t) = P(\theta'_{bl} < t),\ b, l = 1,2,...,v', \tag{21}$$

of the critical infrastructure operation process $Z'(t)$ conditional sojourn times θ'_{bl} at the operation states

$$[H'_{bl}(t)]_{v' \times v'} = \begin{bmatrix} H'_{11}(t)\, H'_{12}(t) \ldots H'_{1v'}(t) \\ H'_{21}(t)\, H'_{22}(t) \ldots H'_{2v'}(t) \\ \ldots \\ H'_{v'1}(t)\, H'_{v'2}(t) \ldots H'_{v'v'}(t) \end{bmatrix} \quad (22)$$

where by formal agreement

$H'_{bb}(t) = 0$ for $b = 1,2,...,v'$.

We introduce the matrix of the conditional density functions

$h'_{bl}(t), \quad b, l = 1,2,...,v'$,

of the critical infrastructure operation process $Z'(t)$ conditional sojourn times θ'_{bl} at the operation states corresponding to the conditional distribution functions $H'_{bl}(t)$

$$[h'_{bl}(t)]_{v' \times v'} = \begin{bmatrix} h'_{11}(t)\, h'_{12}(t) \ldots h'_{1v'}(t) \\ h'_{21}(t)\, h'_{22}(t) \ldots h'_{2v'}(t) \\ \ldots \\ h'_{v'1}(t)\, h'_{v'2}(t) \ldots h'_{v'v'}(t) \end{bmatrix}, \quad (23)$$

where

$h'_{bl}(t) = \frac{d}{dt}[H'_{bl}(t)]$ for $b, l = 1,2,...,v'$,

and by formal agreement

$h'_{bb}(t) = 0$ for $b = 1,2,...,v'$.

We assume that the suitable and typical distributions suitable to describe the critical infrastructure operation process $Z'(t)$ conditional sojourn times θ'_{bl} , $b, l = 1,2,...,v'$, $b \neq l$, in the particular operation states are of the same kind as that listed in Section 2.1 for the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl}, eventually with different parameters they are dependent on.

2.2. Various cases of critical infrastructure operation process including operating environment threats

In practice, to build the model from Section 2.1 the next step is to identify the unknown parameters of the critical infrastructure operation process $Z'(t)$, i.e. to identify the vector of the initial probabilities $[p'_b(0)]_{1 \times v'}$, the matrix of probabilities of transitions $[p'_{bl}]_{v' \times v'}$ and the matrix of conditional distribution functions $[H'_{bl}(t)]_{vxv'}$.

The sufficiently accurate evaluation of these parameters can be performed according to the methods and procedures presented in [Kołowrocki, Soszyńska-Budny, 2011] under the conditions that there is the possibilty of the statistical data collection coming from empirical realizations of the operation process with the separated operation states including the operating environment threats. In the case these operation process realizations are not available, the less accurate evaluations of the uknown parameters can be performed in the analogous way either applying the procedures included in [Kołowrocki, Soszyńska-Budny, 2011] and using approximate necesssary data coming from experts or to ask them for direct approximate evaluation of the unknown parameters of the vector $[p'_b(0)]_{1 \times v'}$, the matrix $[p'_{bl}]_{v' \times v'}$ and the matrix of the mean values $[M'_{bl}(t)]_{v'xv'}$ of the critical infrastructure operation process $Z'(t)$ conditional sojourn times θ'_{bl} $b, l = 1,2,...,v'$, $b \neq l$, at the operation states instead of the matrix of their distributions $[H'_{bl}(t)]_{v'xv'}$.

Another case that can be met in practice is that we have in disposal the statistical evaluations of the parameters of the vector $[p_b(0)]_{1 \times v'}$ including operating environment threats, the matrix $[p_{bl}]_{v \times v}$ and either the matrix of the mean values $[M_{bl}]_{vxv}$ of the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl} $b, l = 1,2,...,v$, $b \neq l$, at the operation states or the matrix of their distributions $[H_{bl}(t)]_{vxv}$ without of separation the operation states including the operating environment threats. In this case, to get the evaluations of the unknown parameters of the vector $[p'_b(0)]_{1 \times v'}$, the matrix $[p'_{bl}]_{v' \times v'}$ and the matrix of the mean values $[M'_{bl}]_{v'xv'}$ of the conditional sojourn times θ'_{bl}, $b, l = 1,2,...,v'$, $b \neq l$, at the operation states (instead of the matrix of their distributions $[H'_{bl}]_{v'xv'}$) of the critical infrastructure operation process $Z'(t)$ with included and separated operating threats, we proceed as follows.

Since according to Section 2.1, the critical infrastructure operation process can be affected by a number $\gamma, \gamma \in N$, of unnatural threats ut_i, $i = 1,2,...,\gamma$, coming from the critical infrastructure operating environment, we assume that they are random and we mark the probability of the operating environment threat ut_i, $i = 1,2,...,\gamma$, appearance at the operation state z_b, $b = 1,2,...,\nu$, (they can be different for various operation states) by

$$P_b(ut_i),\ i = 1,2,...,\gamma,\ b = 1,2,...,\nu. \tag{24}$$

Further, to get the initial probabilities of the vector $[p'_b(0)]$ of the operation process $Z'(t)$ with separated operation states including the operating environment threats, under the assumption that the threats are disjoint (they do not appear simultaneously), we distribute the initial probabilities of the vector $[p_b(0)]$ in the following way:

- if $p_b(0) \neq 0$, $b = 1,2,...,\nu$,

we replace it by

$$p'_{(\gamma+1)(b-1)+1}(0) = p_b(0) - [P_b(ut_1) + P_b(ut_2) + \ldots + P_b(ut_\gamma)], \tag{25}$$

$$p'_{(\gamma+1)(b-1)+1+i}(0) = P_b(ut_i),\ i = 1,2,...,\gamma, \tag{26}$$

for $b = 1,2,...,\nu$,

- if $p_b(0) = 0$, $b = 1,2,...,\nu$,

we replace it by

$$p'_{(\gamma+1)(b-1)+1}(0) = 0, \tag{27}$$

$$p'_{(\gamma+1)(b-1)+1+i}(0) = 0,\ i = 1,2,...,\gamma, \tag{28}$$

for $b = 1,2,...,\nu$.

To get the probabilities of transitions between the operation states of the matrix $[p'_{bl}]$ of the operation process $Z'(t)$ with separated operation states including the operating environment threats, we distribute the probabilities of transitions between the operation states of the matrix $[p_{bl}]$ in the following way:

- if $p_{bl}(0) \neq 0$, $b = 1,2,...,\nu$,

we replace it by

$$p'_{(\gamma+1)(b-1)+1\ (\gamma+1)(l-1)+1} = p_{bl} - [1+p_{bl}][P_b(ut_1) + P_b(ut_2) + \ldots + P_b(ut_\gamma)], \tag{29}$$

$$p'_{(\gamma+1)(b-1)+1\ (\gamma+1)(l-1)+1+j} = P_b(ut_j),\ j = 1,2,...,\gamma, \tag{30}$$

for $b,l = 1,2,...,\nu$,

and we additionally assume that

$$p'_{(\gamma+1)(b-1)+1+i\ (\gamma+1)(b-1)+1} = 1,\ i = 1,2,...,\gamma, \tag{31}$$

$$p'_{(\gamma+1)(b-1)+1+i\, j} = 0,\ i = 1,2,...,\gamma,\ j = 1,2,...,\nu(\gamma+1), \text{ and } j \neq (\gamma+1)(b-1)+1; \tag{32}$$

- if $p_{bl} = 0$, $b,l = 1,2,...,\nu$,

we replace it by

$$p'_{(\gamma+1)(b-1)+1\ (\gamma+1)(l-1)+1} = 0, \tag{33}$$

$$p'_{(\gamma+1)(b-1)+1\ (\gamma+1)(l-1)+1+i} = = P_b(ut_i),\ i = 1,2,...,\gamma, \tag{34}$$

for $b,l = 1,2,...,\nu$.

The conditions (27)-(28) mean that the transitions from the operation states including the operating environment threats is possible only to the corresponding operation states without the operating envitonment threats.

Finally, as the transformation of the matrix $[H_{bl}(t)]_{\nu\times\nu}$ of the critical infrastructure operation process $Z(t)$ conditional sojourn times θ_{bl}, $b,l = 1,2,...,\nu$, at the operation states without of separation the operation states including the operating environment threats into the matrix $[H'_{bl}(t)]_{\nu'\times\nu'}$ of the distributions of the conditional sojourn times θ'_{bl}, $b,l = 1,2,...,\nu'$, at the operation states of the critical infrastructure operation process $Z'(t)$ with included and separated operating threats on the basis of expert opinions is practically not possible, we transform the corresponding matrix $[M_{bl}(t)]_{\nu\times\nu}$ of the mean values of the conditional sojourn times θ_{bl}, $b,l = 1,2,...,\nu$, at the operation states into the matrix $[M'_{bl}(t)]_{\nu'\times\nu'}$ of the mean values of the conditional sojourn times θ'_{bl}, $b,l = 1,2,...,\nu'$.

Since according to Section 2.1, the critical infrastructure operation process can be affected by a number γ, $\gamma \in N$, of unnatural threats ut_i, $i = 1,2,...,\gamma$, coming from the critical infrastructure operating environment, we assume that mean lifetimes to eliminate the operating environment threat ut_i, $i = 1,2,...,\gamma$, at the operation state z_b, $b = 1,2,...,\nu$, (they can be different for various operation states) respectively are

$$M_b(ut_i),\ i = 1,2,...,\gamma,\ b = 1,2,...,\nu. \tag{35}$$

Further, to get the mean values of the conditional sojourn times θ'_{bl}, $b,l = 1,2,...,\nu'$, of the matrix

$[M'_{bl}(t)]_{\nu' \times \nu'}$ of the operation process $Z'(t)$ with separated operation states including the operating environment threats, we distribute the the mean values of the conditional sojourn times θ_{bl}, $b,l = 1,2,\ldots,\nu$, of the matrix $[M_{bl}(t)]_{\nu \times \nu}$ in the following way:

- if $M_{bl} \neq 0$, $b,l = 1,2,\ldots,\nu$,

we replace it by

$$M'_{(\gamma+1)(b-1)+1\ (\gamma+1)(l-1)+1} = M_{bl} - [M_b(ut_1) + M_b(ut_2) + \ldots + M_b(ut_\gamma)], \quad (36)$$

$$M'_{(\gamma+1)(b-1)+1\ (\gamma+1)(l-1)+1+j} = M_{bl} - [M_b(ut_1) + M_b(ut_2) + \ldots + M_b(ut_\gamma)],\ j = 1,2,\ldots,\gamma, \quad (37)$$

for $b,l = 1,2,\ldots,\nu$,

and we additionally assume that

$$M'_{(\gamma+1)(b-1)+1+i\ (\gamma+1)(b-1)+1} = M_b(ut_i),\ i = 1,2,\ldots,\gamma, \quad (3.38)$$

$$M'_{(\gamma+1)(b-1)+1+i\,j} = 0,\ i = 1,2,\ldots,\gamma,\ j = 1,2,\ldots,\nu(\gamma+1), \text{ and } j \neq (\gamma+1)(b-1)+1; \quad (39)$$

- if $M_{bl} = 0$, $b,l = 1,2,\ldots,\nu$,

we replace it by

$$M'_{(\gamma+1)(b-1)+1\ (\gamma+1)(b-1)+1} = 0, \quad (40)$$

$$M'_{(\gamma+1)(b-1)+1\ (\gamma+1)(b-1)+1+j} = M_b - [M_b(ut_1) + M_b(ut_2) + \ldots + M_b(ut_\gamma)],\ j = 1,2,\ldots,\gamma, \quad (41)$$

for $b = 1,2,\ldots,\nu$, $b = l$, where

$$M_b = \sum_{l=1}^{\nu} p_{bl} M_{bl},\ b = 1,2,\ldots,\nu, \quad (42)$$

and by

$$M'_{(\gamma+1)(b-1)+1\ (\gamma+1)(l-1)+1} = 0, \quad (43)$$

$$M'_{(\gamma+1)(b-1)+1\ (\gamma+1)(l-1)+1+j} = 0,\ j = 1,2,\ldots,\gamma, \quad (44)$$

for $b,l = 1,2,\ldots,\nu$, $b \neq l$.

The distribution of the initial probabilities of the vector $[p_b(0)]_{1 \times \nu}$, the probabilities of transitions between the operation states of the matrix $[p_{bl}]_{\nu \times \nu}$ and the mean values of the conditional sojourn times θ_{bl} at the operation states of the matrix $[M_{bl}(t)]_{\nu \times \nu}$ of the operation process $Z(t)$, respectivety into the initial probabilities of the vector $[p'_b(0)]_{1 \times \nu'}$, the probabilities of transitions between the operation states of the matrix $[p'_{bl}]$ and mean values of the conditional sojourn times θ'_{bl} at the operation states of the matrix $[M'_{bl}(t)]_{\nu' \times \nu'}$ of the operation process $Z'(t)$ with separated operation states including the operating environment threats, using the pprocedures defined by (25)-(44) , was done under the assumption that the operarting environment threats are disjoint (they do not appear simultaneously). It means that the new operation states of the operation process $Z'(t)$ with separated operation states either do not include the operating environment threats or include one of the operating environment threats only. The procedure of this distribution in the case the operating environment threats are disjoint have to be constructed individually for each specific case.

3. Critical infrastructure operation process including operating environment threats prediction

3.1. Characteristics of critical infrastructure operation process including operating environment threats

Assuming that we have identified the unknown parameters of the critical infrastructure operation process semi-Markov model:
– the initial probabilities $p'_b(0)$, $b = 1,2,\ldots,\nu'$, of the critical infrastructure operation process staying at the particular state z'_b at the moment $t = 0$;
– the probabilities p'_{bl}, $b,l = 1,2,\ldots,\nu'$, $b \neq l$, of the critical infrastructure operation process transitions from the operation state z'_b into the climate-weather state z'_l;
– the distributions of the critical infrastructure operation process conditional sojourn times θ'_{bl}, $b,l = 1,2,\ldots,\nu'$, $b \neq l$, at the particular climate-weather states and their mean values $M'_{bl} = E[\theta'_{bl}]$, $b,l = 1,2,\ldots,\nu'$, $b \neq l$;
we can predict this process basic characteristics.
As the mean values of the conditional sojourn times θ'_{bl}, are given by

$$M'_{bl} = E[\theta'_{bl}] = \int_0^\infty t\,dH'_{bl}(t) = \int_0^\infty t h'_{bl}(t) \quad (45)$$

$b,l = 1,2,\ldots,\nu'$, $b \neq l$,

then for the distinguished distributions (2.5)-(2.11) [EU-CIRCLE Report D2.1-GMU2, 2016], the mean values of the system operation process $Z'(t)$ conditional sojourn times θ'_{bl}, $b,l = 1,2,...,\nu'$, $b \neq l$, at the particular operation states can be found similarly as in Section …..

From the formula for total probability, it follows that the unconditional distribution functions of the sojourn times θ'_b, $b = 1,2,...,\nu'$, of the system operation process $Z'(t)$ at the operation states z'_b, $b = 1,2,...,\nu'$, are given by [Kolowrocki, Soszyńska-Budny, 2011]

$$H'_b(t) = \sum_{l=1}^{\nu'} p'_{bl} H'_{bl}(t), \; b = 1,2,...,\nu', \tag{46}$$

Hence, the mean values $E[\theta'_b]$ of the system operation process $Z'(t)$ unconditional sojourn times θ'_b, $b = 1,2,...,\nu'$, at the operation states are given by

$$M'_b = E[\theta'_b] = \sum_{l=1}^{\nu'} p'_{bl} M'_{bl}, \; b = 1,2,...,\nu', \tag{47}$$

where M'_{bl} are defined by the formula (45) in a case of any distribution of sojourn times θ_{bl} and by the formulae (2.6)-(2.12) in the cases of particular defined respectively by (2.5)-(2.11) [Kolowrocki, Soszyńska-Budny, 2011] distributions of these sojourn times.

The limit values of the system operation process $Z(t)$ transient probabilities at the particular operation states

$$p'_b(t) = P(Z'(t) = z'_b), \; t \in <0,+\infty), \; b = 1,2,...,\nu', \tag{48}$$

are given by

$$p'_b = \lim_{t\to\infty} p'_b(t) = \frac{\pi_b M'_b}{\sum_{l=1}^{\nu'} \pi_l M'_l}, \; b = 1,2,...,\nu', \tag{49}$$

where M'_b, $b = 1,2,...,\nu'$, are given by (47), while the steady probabilities π_b of the vector $[\pi_b]_{1x\nu}$ satisfy the system of equations

$$\begin{cases} [\pi_b] = [\pi_b][p_{bl}] \\ \sum_{l=1}^{\nu} \pi_l = 1. \end{cases} \tag{50}$$

In the case of a periodic system operation process, the limit transient probabilities p'_b, $b = 1,2,...,\nu'$, at the operation states defined by (49), are the long term proportions of the system operation process $Z'(t)$ sojourn times at the particular operation states z'_b, $b = 1,2,...,\nu'$.

Other interesting characteristics of the system operation process $Z'(t)$ possible to obtain are its total sojourn times $\hat{\theta}'_b$ at the particular operation states z'_b, z_b, $b = 1,2,...,\nu'$, during the fixed system opetation time. It is well known [Kolowrocki, Soszyńska-Budny, 2011] that the system operation process total sojourn times $\hat{\theta}'_b$ at the particular operation states z'_b, for sufficiently large operation time θ, have approximately normal distributions with the expected value given by

$$\hat{M}'_b = E[\hat{\theta}'_b] = p'_b \theta, \; b = 1,2,...,\nu', \tag{51}$$

where p'_b are given by (49).

4. Simplified approach to critical infrastructure operation process including operating environment threats limit transient probabilities determination

According to Section 2.1, we assume that the critical infrastructure operation process can be affected by a number γ, $\gamma \in N$, of unnatural threats ut_i, $i = 1,2,...,\gamma$, coming from the critical infrastructure operating environment, we assume that they are random and we mark the probability of the operating environment threat ut_i, $i = 1,2,...,\gamma$, appearance at the operation state z_b, $b = 1,2,...,\nu$, (they can be different for various operation states) by

$$P_b(ut_i), \; i = 1,2,...,\gamma, \; b = 1,2,...,\nu. \tag{52}$$

Further, to get the approximate evaluations of the transient initial probabilities of the vector $[p'_b]_{1x\nu\gamma}$ of the operation process $Z'(t)$ with separated operation states including the operating environment threats, under the assumption that the threats are disjoint (they do not appear simultaneously), we distribute the limit transient probabilities of the vector $[p'_b]_{1x\nu}$ of the operation process $Z(t)$ without separated operation states including the operating environment threats in the following way:

- if $p_b \neq 0$, $b = 1,2,...,\nu$,

we replace it by

$$p'_{(\gamma+1)(b-1)+1} = p_b - [P_b(ut_1) + P_b(ut_2) + \ldots + P_b(ut_\gamma)], \quad (53)$$

$$p'_{(\gamma+1)(b-1)+1+i} = P_b(ut_i), \; i = 1,2,\ldots,\gamma, \quad (54)$$

for $b = 1,2,\ldots,v$,

- if $p_b = 0$, $b = 1,2,\ldots,v$,

we replace it by

$$p'_{(\gamma+1)(b-1)+1} = 0, \quad (55)$$

$$p'_{(\gamma+1)(b-1)+1+i} = 0, \; i = 1,2,\ldots,\gamma, \quad (56)$$

for $b = 1,2,\ldots,v$.

5. Conclusions

The probabilistic model of the critical infrastructure operation process presented in this report is the basis for further considerations in particular tasks of the EU-CIRCLE project. First, this model will be developed in order to construct the integrated model of critical infrastructure Safety (IMCIS) Including Operating Environment Threats (OET) – IMCIS Model 2.

Acknowledgements

The paper presents the results developed in the scope of the EU-CIRCLE project titled "A pan – European framework for strengthening Critical Infrastructure resilience to climate change" that has received funding from the European Union's Horizon 2020 research and innovation programme under grant agreement No 653824. http://www.eu-circle.eu/

References

EU-CIRCLE Report D3.3-GMU3, Modelling inside and outside dependences influence on safety of complex multistate ageing systems (critical infrastructures) – Integrated Model of Critical Infrastructure Safety (IMCIS) related to its operation process including operating environment threats (with other critical infrastructures influence, without climate-weather change influence), 2016

Ferreira F., Pacheco A., Comparison of level-crossing times for Markov and semi-Markov processes. Statistics and Probability Letters, Vol. 7, No 2, 151-157, 2007

Glynn P.W., Haas P.J., Laws of large numbers and functional central limit theorems for generalized semi
Markov processes. Stochastic Models, Vol. 22, No 2, 201-231, 2006

Grabski F., (2002) Semi-Markov Models of Systems Reliability and Operations Analysis. System Research Institute, Polish Academy of Science, 2002 (*in Polish*)

Kołowrocki K., Reliability of Large and Complex Systems, Amsterdam, Boston, Heidelberd, London, New York, Oxford, Paris, San Diego, San Francisco, Singapore, Sidney, Tokyo, Elsevier, 2014b

Kołowrocki K., Soszyńska J., A general model of industrial systems operation processes related to their environment and infrastructure. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 2, Vol. 2, 223-226, 2008

Kołowrocki K., Soszyńska J., Methods and algorithms for evaluating unknown parameters of operation processes of complex technical systems. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 3, Vol. 1, 2, 211-222, 2009d

Kołowrocki K., Soszyńska J., Statistical identification and prediction of the port oil pipeline system's operation process and its reliability and risk evaluation. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 4, Vol. 2, 241-250, 2009e

Kolowrocki K., Soszynska J., Reliability modeling of a port oil transportation system's operation processes. International Journal of Performability Engineering, Vol. 6, No. 1, 77-87, 2010a

Kolowrocki K., Soszynska J., Testing uniformity of statistical data sets coming from complex systems operation processes. Summer Safety & Reliability Seminars. Journal of Polish Safety and Reliability Association, Issue 4, Vol. 1,123-132, 2010b

Limnios N., Oprisan G., Semi-Markov Processes and Reliability. Birkhauser, Boston, 2005

Limnios N., Ouhbi B., Sadek A., Empirical estimator of stationary distribution for semi-Markov processes. Communications in Statistics-Theory and Methods, Vol. 34, No. 4, 987-995 12, 2005

Macci C., Large deviations for empirical estimators of the stationary distribution of a semi-Markov process with finite state space. Communications in Statistics-Theory and Methods, Vol. 37, No. 19,3077-3089, 2008

Mercier S., Numerical bounds for semi-Markovian quantities and application to reliability. Methodology and Computing in Applied Probability, Vol. 10, No. 2, 179-198, 2008