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Semi-Markov Model of the Operation and Maintenance Process of City Buses

Mirosław Szubartowski*

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Abstract

This paper presents a model of the operation and maintenance process of the technical objects within the urban transport bus system. All the investigations have been presented on the basis of the selected real operation and maintenance system of the city buses. Basing on the identification of the system under investigation and on the process being realized within it a model of the operation and maintenance process of the city buses was built, assuming that the model of this process is the homogeneous semi-Markov process $X(t)$. For this purpose, crucial states of operation and maintenance process in selected transport system were determined as well as possible transfers between those states. Based on this, an event-based model of the operation and maintenance process of the city buses was built, assuming that its model is the homogenous semi-Markov process. For operation data obtained after research conducted in an authentic transport system, values of unconditional periods of duration of process states, values of stationary distribution included in the Markov chain as well as values of probabilities of limit distribution of the semi-Markov process were determined. Based on this, an analysis of the city buses operation and maintenance process in question was performed. Presented in this article the semi-Markov model operation and maintenance process of city buses is the first stage of the creation of the availability model of the urban transport bus system.

1. Introduction

One of the transport system examples, in which the controlled processes are being carried out, are urban transport bus systems. The urban transport bus systems are such systems whose task is to satisfy transport needs as a result of the transporting performed over the specific routs. The operation and maintenance process of the city buses may be divided in general into the operational use process and the serviceability assurance process. In the operation process the basic task of transport system is obtained, at the same time generating profits due to the performance of the task. The activities performed inside the

* Wloclawek College of Enterprise and Technology, Łęgska Street 20, 87-800 Włocławek, Poland, e-mail: km@karor.com.pl

serviceability assurance process are aimed at restoring the task serviceability of the operated and maintained city buses. The realization of the process described is to bring back serviceability of the required number of the city buses within the specified period of time. The realization effectiveness of the operation and maintenance processes has influence on the possibility to accomplish a transporting task, as well as on the operation effectiveness of the system described herein. That is why it is so necessary to identify properly the system and process under investigation as well as to analyse and evaluate it in order to assure high effectiveness level of the system operation [6, 8, 9].

The analysis and evaluation of the controlled processes are performed on the basis of studying their models. The most frequently used representations of the operation and maintenance process model are maintenance graphs, including the directed graphs, whose peaks represent operation and maintenance states, while the arcs of the graph represent possible transitions between the states. Due to a random nature of the factors affecting the course and effectiveness of the operation and maintenance process being realized within a complex system, the stochastic process, with extensively applied Markov and semi-Markov processes are most frequently applied for mathematical modelling of the operation and maintenance process [1, 3, 5, 6, 7]. Implementing semi-Markov processes makes it possible to create and analyze the mathematical model of the operation and maintenance process in the case of the variables characteristic of the defined process states being distributions other than exponential. Realization of the investigations by means of the Markov and semi-Markov models of the operation and maintenance process makes it possible to analyse the detailed problems related to the operation and maintenance of a technical object on the one hand, and on the other hand it enables to analyse the interrelations between the set number of the model parameters.

It has been attempted in this paper to describe mathematically and analyse the operation and maintenance process of the city buses, being performed in a selected real transport system, assuming that the model of this process is the homogenous semi-Markov process $X(t)$. The urban bus transport operation and maintenance system forming one of the subsystem of an urban transportation system within a selected conurbation was chosen as the investigation object.

2. Event-based Model of the Operation and Maintenance Process of City Buses

The model of operation and maintenance process was created on the basis of the analysis of state space as well as operational events pertaining to city buses used in the analyzed authentic transport system. Due to the identification of the multi-state operation and maintenance process of city buses, crucial operation states of the process as well as possible transfers between the defined states were designated. Based on this, a graph was created, depicting the changes of operation and maintenance process states, shown in figure 1.

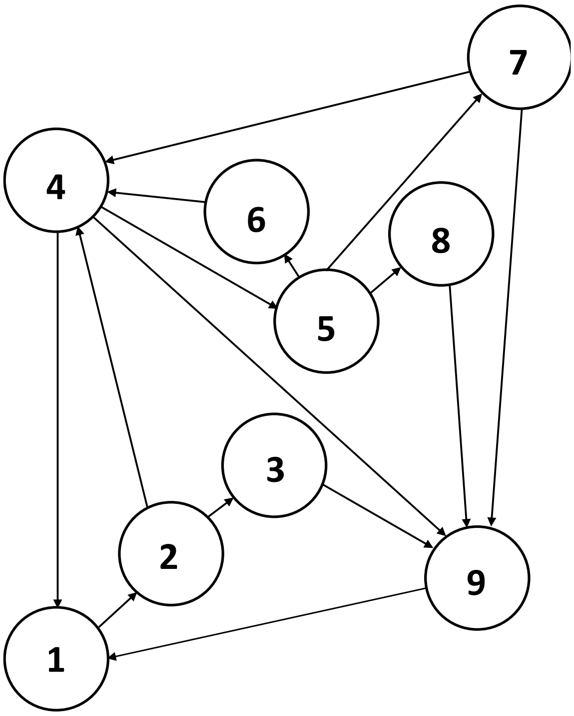

Fig. 1. Directed graph representing the operation and maintenance process of city buses
1 - stopover at the bus depot, **2 -** obtain the transport task, **3 -** damage to the bus at the bus depot, **4 -** carrying out of the transport task, **5 -** damage to the bus on route, **6 -** repair by technical support unit without losing a ride, **7 -** repair by technical support unit with losing a ride, **8 -** emergency exit, **9 -** repair at the efficiency implementation subsystem posts

3. Mathematical Model of the Operation and Maintenance Process of City Buses

The mathematical model of the operation and maintenance process of city buses was built with the use of the semi-Markov processes theory. The semi-Markov $X(t)$ process is one, where periods of time between the changes of consecutive process states have arbitrary probability distributions and a transfer to the consecutive state depends on the current process state. Using the semi-Markov processes in mathematical modeling of the operation and maintenance process, the following assumptions were put forward:

– the random process $X(t)$ being the mathematical model of the operation and maintenance process is a homogenous process,
– the modeled operation and maintenance process has a finite number of states $i = 1,2,\ldots,9$,
– if technological object (city bus) at moment t is in state i, then $X(t) = i$, where $i = 1,2,\ldots,9$,

– at moment $t = 0$, the process finds is in state $i = 1$, i.e. the initial distribution takes up the following form:

$$p_i(0) = \begin{cases} 1 & when \quad i = 1 \\ 0 & when \quad i \neq 1 \end{cases}, \quad i = 1,2,...,9 . \tag{1}$$

In order to assign the values of non-conditional duration periods for process states, values of limit probabilities π_i of Markov chain hidden in the process and values of limit probabilities p_i^* of semi-Markov process, based on the directed graph shown in figure 1, the following were created matrixes P of the states change probabilities and matrix Θ of conditional periods of duration of the states in process $X(t)$:

$$P = \begin{bmatrix} 0 & p_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_{23} & p_{24} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{39} \\ p_{41} & 0 & 0 & 0 & p_{45} & 0 & 0 & 0 & p_{49} \\ 0 & 0 & 0 & 0 & 0 & p_{56} & p_{57} & p_{58} & 0 \\ 0 & 0 & 0 & p_{64} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & p_{74} & 0 & 0 & 0 & 0 & p_{79} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & p_{89} \\ p_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

where:

p_{ij} – the conditional probability of transfer from state i to state j:

$$p_{ij} = \lim_{t \to \infty} p_{ij}(t) = \lim_{t \to \infty} \{P\{X(t) = j | X(0) = i\}\}; \tag{3}$$

$$T = \begin{bmatrix} 0 & \overline{T}_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \overline{T}_{23} & \overline{T}_{24} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{T}_{39} \\ \overline{T}_{41} & 0 & 0 & 0 & \overline{T}_{45} & 0 & 0 & 0 & \overline{T}_{49} \\ 0 & 0 & 0 & 0 & 0 & \overline{T}_{56} & \overline{T}_{57} & \overline{T}_{58} & 0 \\ 0 & 0 & 0 & \overline{T}_{64} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \overline{T}_{74} & 0 & 0 & 0 & 0 & \overline{T}_{79} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \overline{T}_{89} \\ \overline{T}_{91} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{4}$$

where:

$\overline{T}_{ij}$ – the period of duration of technological object in *i*-th state, under the condition that the next state will be *j*-th state.

The homogenous semi-Markov process is unequivocally defined when initial distribution (1) and its kernel $Q(t)$ are given [4, 6, 7]:

$$Q(t) = \begin{bmatrix} 0 & Q_{12}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & Q_{23}(t) & Q_{24}(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{39}(t) \\ Q_{41}(t) & 0 & 0 & 0 & Q_{45}(t) & 0 & 0 & 0 & Q_{49}(t) \\ 0 & 0 & 0 & 0 & 0 & Q_{56}(t) & Q_{57}(t) & Q_{58}(t) & 0 \\ 0 & 0 & 0 & Q_{64}(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & Q_{74}(t) & 0 & 0 & 0 & 0 & Q_{79}(t) \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & Q_{89}(t) \\ Q_{91}(t) & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{5}$$

where:

$$Q_{ij}(t) = P\{X(\tau_{n+1}) = j, \tau_{n+1} - \tau_n \le t \mid X(\tau_n) = i\}, \quad i, j = 1,2,...,9, \tag{6}$$

means that the state of semi-Markov process and the period of its duration depends solely on the previous state, and does not depend on earlier states and periods of their duration, where $\tau_1 < \tau_2 < ... < \tau_n$ are arbitrary moments in time, and:

$$Q_{ij}(t) = p_{ij} \cdot F_{ij}(t), \tag{7}$$

where:

$$F_{ij}(t) = P\{\tau_{n+1} - \tau_n \le t | X(\tau_n) = i, X(\tau_{n+1}) = j\},\ \ i, j = 1,2,...,9, \tag{8}$$

is a distribution function of random variable T_{ij} signifying period of duration of state *i*, under the condition that the next state will be state *j*.

<u>The limit theorem for semi-Markov processes [4, 7]:</u>

If hidden Markov chain in semi-Markov process with finite state *S* set and continuous type kernel contains one class of positive returning states such that for each state $i \in S$, $f_{ij} = 1$ and positive expected values $E(T_i), i \in S$ are finite, limit probabilities:

$$P_i^* = \lim_{t \to \infty} P_i(t) = \frac{p_i^* \cdot E(T_i)}{\sum_{i \in S} p_i^* \cdot E(T_i)}, \tag{9}$$

where probabilities $p_i^*, i \in S$ constitute a stationary distribution of a hidden Markov chain, which fulfills the simultaneous linear equations:

$$\sum_{i \in S} p_i^* \cdot p_{ij} = p_j^*,\ \ j \in S,\ \ \sum_{i \in S} p_i^* = 1. \tag{10}$$

4. Examples of Test Results

Based on source data obtained from operation research performed in authentic municipal bus transport system, the values of the elements of matrix *P* (2) as well as matrix *T* (4) were estimated. Then matrices *P* and *T* are thus defined:

$$P = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.0556 & 0.9444 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0.2907 & 0 & 0 & 0 & 0.0891 & 0 & 0 & 0 & 0.6202 \\ 0 & 0 & 0 & 0 & 0 & 0.6522 & 0.2609 & 0.0869 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0.8333 & 0 & 0 & 0 & 0 & 0.1667 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \tag{11}$$

$$T = \begin{bmatrix} 0 & 5.788 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0.199 & 0.145 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.197 \\ 3.154 & 0 & 0 & 0 & 3.790 & 0 & 0 & 0 & 12.027 \\ 0 & 0 & 0 & 0 & 0 & 0.049 & 0.297 & 0.444 & 0 \\ 0 & 0 & 0 & 0.092 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1.290 & 0 & 0 & 0 & 0 & 1.626 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.169 \\ 3.153 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad (12)$$

where the values of conditional duration periods of process $\bar{T}_{ij}$ states were shown in [h].

Then, with the use of the MATHEMATICA software, for the data shown in matrices (11) and (12) values of non-conditional duration periods for process states, values of stationary distribution hidden in Markov chain process as well as values of limit distribution of semi-Markov process. Results were shown in tables 1 to 3.

Tab. 1. Values of probabilities p_i^* of stationary distribution of hidden Markov chain

$p_1^* = 0.2523$	$p_2^* = 0.2523$	$p_3^* = 0.0140$
$p_4^* = 0.2582$	$p_5^* = 0.0230$	$p_6^* = 0.0150$
$p_7^* = 0.0060$	$p_8^* = 0.0020$	$p_9^* = 0.1772$

Tab. 2. Values of non-conditional duration periods $\bar{T}_i$ [h] of staying in the states of process *X(t)*

$\bar{T}_1 = 5.788$	$\bar{T}_2 = 0.148$	$\bar{T}_3 = 0.197$
$\bar{T}_4 = 8.714$	$\bar{T}_5 = 0.148$	$\bar{T}_6 = 0.092$
$\bar{T}_7 = 1.346$	$\bar{T}_8 = 1.169$	$\bar{T}_9 = 3.153$

Tab. 3. Values of probabilities P_i^* of limit distribution of the semi-Markov process

$P_1^* = 0.3377$	$P_2^* = 0.0086$	$P_3^* = 0.0006$
$P_4^* = 0.5204$	$P_5^* = 0.0008$	$P_6^* = 0.0003$
$P_7^* = 0.0019$	$P_8^* = 0.0005$	$P_9^* = 0.1292$

5. Summary

Based on the analysis of the values of matrix P (2) and defined values of limit probabilities p_i^* of the hidden Markov chain it may be determined, that almost 9% of city buses implementing the task undergo damage en route. At the same time it may be stated that most of them are diagnosed and repaired (91.31%) on the rout by the technical service units. The remaining city buses, damaged during the implementation of the transport task, are moved to the bus depot (emergency exit on their own or towed). Out of the repairs carried out by the technical service units, more than 71% are so called minor repairs, mainly such as adjustment and replacement of small parts, performed without loss of a ride. The remaining repairs (28.57%) are done over a longer period of time than breaks between rides making it necessary to substitute city buses damaged during the performance of the transport task by reserve city buses (about 3% of the city buses realizing the task). Almost 5.5% city buses awaiting the start of transport task undergo damage at the bus depot parking space before the performance of the transport task.

Based on the non-conditional value of time periods $\overline{T}_i$ as well as limit probabilities P_i^* defined for the semi-Markov process one may conclude that the states of the process in which a statistical city bus stays the longest are states:

- **4** - carrying out of the transport task (more than 52% of the operation and maintenance time),
- **1** - stopover at the bus depot (about 33.8% of the operation and maintenance time),
- **9** - repair at the efficiency implementation subsystem posts (almost 13% of the operation and maintenance time).

The overall percentage of time spent on repairing city buses by technical emergency units in the total time of operation and maintenance amounts to only a little more than 0.3%. However, it is significant because of the possibility to realize the task of the transport system as well as of the significant costs suffered when realizing these processes.

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