CONSTRUCTION OF PARABOLA

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Abstract: In the present article, the author gives an interesting and relatively simple construction of parabola determined by a vertex, an axis and its random point. The construction of next points of parabola was deduced from the properties of circle transformation by the pencil of concentric circles.

Keywords: parabola, pencil of parabolas, circle, pencil of concentric circles

1 Introduction

In the present paper the author gives an original and relatively simple construction of parabola determined by its given elements: a vertex, an axis and its random point. The construction of successive points of this parabola was deduced from the properties of circle transformation by a pencil of concentric circles.

The first part of paper (point 2) contains the definition of transformation and the proof of the theorem declaring that the image of circle in this transformation is the pencil of parabolas about the common axis and vertex belonging/no to the considered (transformed) circle. The proof of this theorem was put forth by the method of analytic geometry.

The second part of the paper (point 3) presents the algorithm of construction of successive points of parabola, determined by its given elements: a vertex, an axis and a random point.

2 Transformation of circle

Let's assume an optional circle k with center M and radius r≠0 on the plane and center O of the pencil of concentric circles, different from point M. Moreover, let us assume that point O is at the same time the beginning of the orthogonal system of reference about the axis x which includes point M (Fig. 1). In this coordinate system O(x,y), the equations of circles k(M, r) and $o_i(O, R_i)$ are in shape of:

$$(x - a)^2 + y^2 = r^2 \tag{1}$$

and

$$x^2 + y^2 = R_i^2. \tag{2}$$

The common points X_i and X_i^s of the circles (1) and (2) belong to lines s_i parallel to axis y, for which the axis of abscissae x is the axis of symmetry. To the points $X_{i,}$, $X_i^s(x,y)$ respectively assigned points X_i, $X_i'^s(x',y')$ which belong to lines s_i and whose abscissae x'= x (in the same coordinate system). The coordinates y' of these points can be equal to

ISSN 1644-9363 / PLN 15.00

the arbitrary values of radii R_i $(0<y'\geq R_i)$. In the definite transformation we assume $y'=\pm\frac{R_i}{n}$, or $y'=\pm n\cdot R_i$ $(n=1,2,3\ldots)$. And because $R_i^2=x'^2+y^2$ so $y'=\pm\frac{1}{n}\sqrt{x'^2+y^2}$ or $y'=\pm n\cdot\sqrt{x'^2+y^2}$.
The following formulas of this transformation result from these determinations:

$$x=x' \text{ and } y'=\pm\sqrt{-x'^2+y^2n^2} \text{ or } y'=\pm\sqrt{-x'^2+\frac{y^2}{n^2}}. \qquad (3)$$

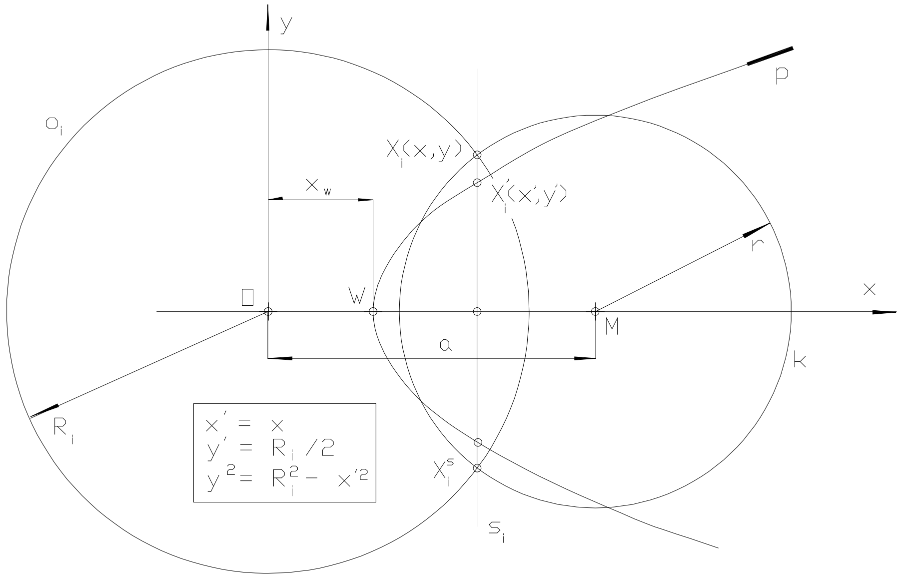

Fig. 1

Let us take these formulas (3) and put them instead of x and y into the equation (1) of the circle k(M, r), after rearrangement we obtain relations:

$$y'^2=2x'a/n^2+(r^2-a^2)/n^2 \qquad (4)$$

and

$$y'^2=2x'a\,n^2+(r^2-a^2)\,n^2, \qquad (4a)$$

which are the equations of the pencils of parabolas with the common axis x and vertex W. From equation (4) ((4a)), for $y'=0$ we obtain a formula for its abscissa:

$$x'=(a^2-r^2)/2a, \quad (\text{and for } a=r,\ x'=0) \qquad (5)$$

For $a=r$, the equations (4) and (4a) take the following form:

$$y'^2=2x'r/n^2 \qquad (6)$$

and

$$y'^2=2x'r\,n^2 \qquad (6a)$$

and are the equations of the pencils of parabolas with the axis x and the common vertex W which belong to circle k(M, r).

Moreover, if we assume that $n=1$, then the relations (6) and (6a) take shape of the vertical equation of parabola:

$$y'^2=2x'r. \qquad (7)$$

It proves the following statement: the image of a circle in a transformation determined in this way is a pencil of parabolas with a common axis and vertex, which belongs (doesn't belong) to a transformed circle.

Thus defined transformation of degenerated conic to the pair of straight lines was used in paper [1] for the construction of equilateral triangles whose vertices belong to three given straight lines.

3 Construction of a parabola

The proposed method for the construction of points of a parabola determined by a vertex, an axis and its random point comes out directly from the discussion of the statement proved (in point 2), for the case in which the vertex of the pencil of parabolas belongs to the transformed circle.

Assuming the vertex of parabola W on a plane of design (sketch), the axis d and its arbitrary point A_p, the algorithm of construction of its points is as follows (Fig. 2):

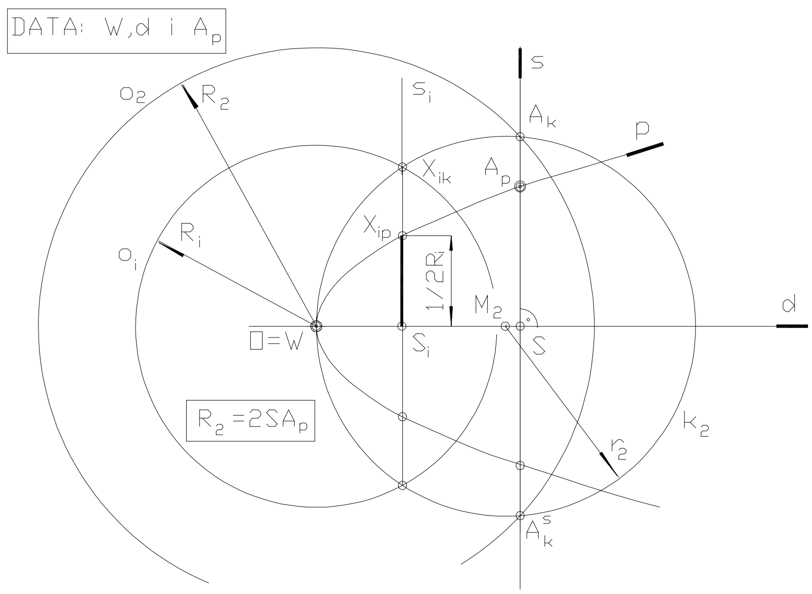

Fig. 2

– W = O – the center pencil of concentric circles;
– $A_p \in s \perp d$;
– $s \cap d = \{S\}$;
– $o_2(O, R_2 = 2SA_p) \cap s = \{A_k, A_k^s\}$, $(R_2 \geq SW)$;
– $\{W, A_k, A_k^s\} \subset k_2(M_2, r_2)$;
– $o_i(O, R_i) \cap k_2(M_2, r_2) = \{X_{ik}, X_{ik}^s\}$, $(0 \leq R_i \leq 2r_2)$;
– $s_i(X_{ik}, X_{ik}^s) \cap d = \{S_i\}$;
– $\{X_{ip}, X_{ip}^s\} \subset s_i$, these that $S_i X_{ip} = S_i X_{ip}^s = \frac{R_i}{2}$ are the points of parabola $p(W,d,A_p)$.

ISSN 1644-9363 / PLN 15.00

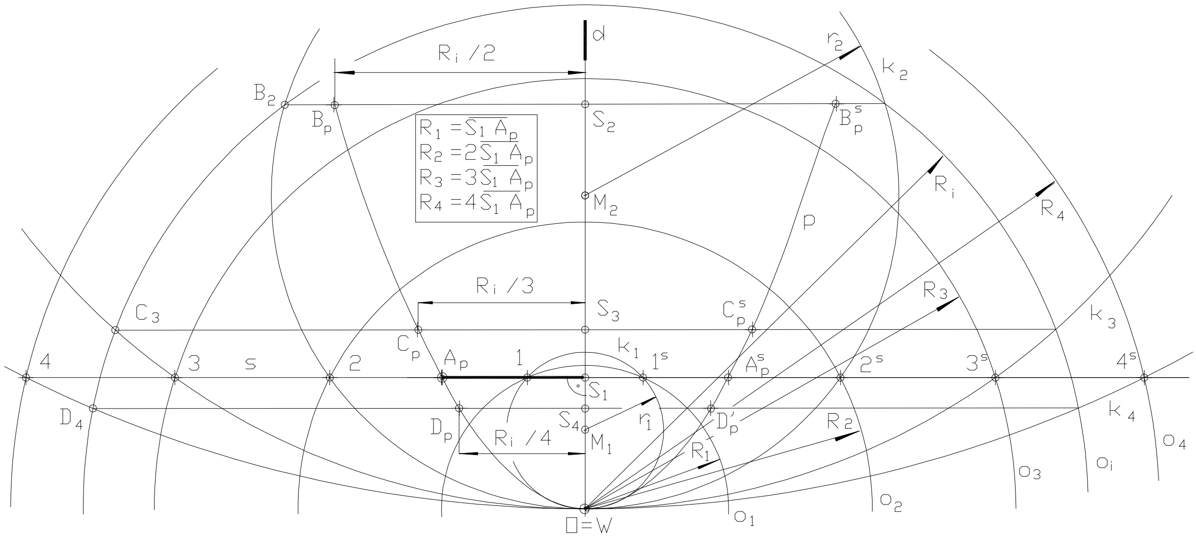

Fig. 3

Construction of the points of parabola presented above does not depend on the selection of auxiliary circles $o_n(O, R_n= nSA_p)$. The result of the definition of this transformation is that a random segment SA_p, perpendicular to the axis d (coordinate of parabola), can be assumed as n-part of the radii R_i or of their multiple (n=1,2,3…). Thus as long as the circles $o_n(O, R_n= nSA_p)$ intersect the straight line s in the real points (different or united) they generate the pencil of the tangent circles k_n (M_n, r_n) with the vertex W=O (Fig. 3). The image of these circles in this transformation is the same, univocally determined parabola $p(W,d, A_p)$.

References:

[1] Ochoński S.: *Equilateral Triangles whose Vertices Belong to Three Given Straight Lines*. The Journal BIULETYN of Polish Society for Geometry and Engineering Graphics, Vol. 19 (2009), 15-26.

KONSTRUKCJA PARABOLI

W artykule podano interesującą i stosunkowo prostą konstrukcję bieżących punktów paraboli wykorzystując własności przekształcenia okręgu za pomocą pęku współśrodkowych okręgów.
W pierwszej części pracy zdefiniowano przekształcenie oraz wykazano, że obrazem okręgu w tak określonym przekształceniu jest pęk parabol o wspólnym wierzchołku i osi. Ponadto stwierdzono iż w przypadku gdy środek koncentrycznych okręgów należy do przekształcanego okręgu to wierzchołek tych parabol jednoczy się z nim.
W drugiej części artykułu podano algorytm konstrukcji kolejnych punktów paraboli zadanej poprzez podanie wierzchołka, osi oraz jej dowolnego punktu.