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PASSENGER'S ROUTES PLANNING IN STOCHASTIC COMMON-LINES' MULTI-MODAL TRANSPORTATION NETWORK THROUGH INTEGRATING GENETIC ALGORITHM AND MONTE CARLO SIMULATION

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Abstract:

In the urban transportation network, most passengers choose public transportation to travel. However, bad weather, accidents, traffic jams and other factors lead to uncertainty in transportation network. Besides, transport vehicles running on the same segments of routes and belonging to different modes or routes make the transportation network more complicated. In order to improve the efficiency of passenger's travel, this paper aim to introducing an approach for optimizing passenger travel routes. This approach takes the travel cost and the number of transfers as constraints to finding the shortest total travel duration of passenger in urban transportation network. The running duration and dwell duration of the vehicles in the network are uncertain, and the vehicles are running according to the timetables. As transportation modes, bus, rail transit and walk are considered. In terms of methodological contribution, this paper combines Genetic Algorithm (GA) and Monte Carlo simulation to deal with optimization problem under stochastic conditions. This paper uses Monte Carlo simulation to simulate the running duration and dwell time of vehicles in different scenarios to deal with the uncertainty of the network. The shortest path of passenger's travel is solved by GA. Two kinds of population management strategies including single population management strategy and multiple population management strategy are designed to guide the solution population evolving process. The two kinds of population management strategies of GA are tested in numerical example. The satisfactory convergence performance and efficiency of the model and algorithm is verified by the numerical example. The numerical example also demonstrated that the multiple population management strategy of GA can get better results in a shorter CPU time. At the same time, the influences of some significant variables on solution are performed. This paper is able to provide a scientific quantitative support to the path scheme selection under the influence of common-lines and timetables of different modes of transportation in stochastic urban multimodal transportation network.

Keywords: *stochastic networks, common-lines' multi-modal transportation, passengers' route choice, genetic algorithm, Monte Carlo simulation*

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1. Introduction

Nowadays, passenger travel within urban transportation network always need to consider planning of travel path. In the works of passenger travel path optimization, some literatures only considered one mode of transportation(Jin et al., 2017; Xiao & He, 2017; Zhu et al., 2019). However, multimodal transportation has become a necessary choice for passenger, it is difficult for passenger to find a suitable path in complicated routes (Abbaspour & Samadzadegan, 2010). Therefore, it is worth studying that passenger's routes planning under multiple modes of transportation.

Multiple lines share several segments of routes can be regarded as common-lines. This situation should be considered in the optimization of passenger travel path, because it increases the complexity of route selection. The common-lines problem was put forward by (Chriqui & Robillard, 1975) firstly. In his model, passenger selects a set of routes (called attractive lines) at origin node and board next vehicle to travel. (Nguyen et al., 1998) used a particular graph structure called hyperpaths, which uses different edges between nodes to represent different lines. Their model allows that passenger to choose attractive lines at every stop and also assume that passenger will take first arriving vehicle. In above studies, choice of route only is related to passenger waiting duration. In later work, (Nassir et al., 2019) took other factors into account when measuring the attraction of lines. Passenger waiting duration should not be the only factor when passenger choose routes. It may lead to more the number of transfers because the distances that lines carry passenger toward their destination is different. The number of common-lines' modes of transportation is a significant factor to be taken into account. (Artigues et al., 2013; Kang & Youm, 2017) searched an optimal travel path in multimodal networks where different lines of one mode of transportation are common-lines. Nevertheless, in multimodal urban networks, passenger face that lines from different modes of transportation are common-lines. (Liu et al., 2017) studied the multimodal shortest path problem under this condition.

In the all above cited literatures, the passenger travel path was studied under a deterministic environment. However, for passenger taking multi-modal transportation, one of significant sources of uncertainty is time, which cannot be estimated exactly (Ghavami, 2019). A few researchers pay attention to how to simulate total travel duration, especially waiting duration. (Pi et al., 2019) believed that it is reasonable to simplify the waiting duration of passengers in high-frequency traffic mode to a constant. (Omar Dib et al., 2018; Goerigk & Schmidt, 2017) also applied the waiting duration as a constant in their papers. However, (Cheng et al., 2019) adopted historical data to calculate the distribution of waiting duration. Moreover, to calculate the waiting duration more realistic, some researchers obtained waiting duration based on the arrival time of the passenger and the transfer line (Botea et al., 2019; López & Lozano, 2019; T. Zhang et al., 2018; Y. Zhang & Tang, 2018).

Suitable travel duration and travel cost are the factors that passenger pay attention to in their travels. In algorithm, the method of calculation of travel cost will influence on the choice of passenger travel path. One method for calculating travel cost is that multiplied the unit cost of different transportation modes by the travel distance (Dotoli et al., 2017; Faroqi & Mesgari, 2016; Narayan et al., 2020). And (Niksirat et al., 2012) calculated the travel cost of passenger by directly giving every cost of the edges. Sectional fare is commonly in rail traffic system, however, in above literatures, there is little information about sectional fare. Timetable provides public transport vehicles' visiting stops along a certain route at a specific time of a day. The importance of timetable has prompted some researchers to consider this factor in the optimization of passenger travel path (Dalkiliç et al., 2017; O. Dib et al., 2017).

As can be seen from our literature review, there are some more realistic aspects that have not been taken into account in previous studies and can be expressed as follows: (1) The distance that different transfer lines are able to carry passenger toward their destination have not been taken into account when passenger chose transfer line. (2) The factors that different transportation modes are common-lines and stochastic network have not been considered together in previous studies. (3) The sectional fare and timetable should be incorporated into solution procedure of passenger travel planning that different transportation modes are common-lines. Therefore, this paper study the travel path planning for passenger in stochastic urban transportation network where different modes of transportation on the same road have different timetables, and the selection strategy of vehicle transfer is given. GA is applied to solve the shortest transportation travel path with taking travel duration, dwell duration and transfer duration into account. Monte Carlo simulation is used to simulate different travel cases in step of calculating the fitness value of individual. The rest of this paper is organized as follows: Section 2 describes the passenger's routes planning problem in multimodal networks and a mathematic model is formulated. Section 3 introduces the process of GA and Monte Carlo simulation to solve the path optimization problem. Section 4 provides an example along with the computational results and numerical analysis. Finally, Section 5 presents the conclusions and future work.

2. Problem description

The problem is to optimize the planning of passenger travel in a stochastic multimodal transportation network (including the modes of Bus, Rail transit and Walk) with common-lines' different transport modes. Due to the factors such as weather, traffic conditions and the number of passengers boarding and alighting, the travel duration and dwell duration of vehicle are uncertain. Therefore, the challenge concerns dealing with the uncertainty of travel and dwell durations in order to enable the computation of the shortest path. This paper uses Monte Carlo simulation to simulate different scenarios to solve the uncertainty (the detailed description can be found in section 3.7).

This paper finds the shortest path from the perspective of minimizing passenger's total travel duration, and the constraints include travel cost and the number of transfers. The model formulation is developed under the following assumptions:

- Capacity of transit vehicles meets transfer needs of passenger.
- The number of intermediate nodes of different rail transit lines between the same transfer nodes is the same.
- Passenger chooses the first arrival vehicle which can transport them to a station closer to destination.
- The travel duration between transfer nodes and the dwell duration of transfer nodes obey a certain distribution.
- The earliest departure time from originating stop and the departure headways of every lines are known.
- The bus fare remains unchanged on the whole journey, and the charging rule of rail transit is the sectional fare which is defined as charging based on the number of stations passed by a passenger in travel.

2.1. Notation and definitions

Before this paper models this problem, notations which will be used to model the problem are as follows. It should be noted that stochastic parameters are represented by the $\sim$ symbol above them.

2.1.1. Index set

N_1 Set of transfer nodes where passenger can transfer

N_2 Set of intermediate nodes (between transfer nodes) where passenger not transfer at here

N Set of network nodes including transfer nodes and intermediate nodes, ($N = N_1 \cup N_2$)

E Set of edges

V Set of transportation modes: {B(bus), R(rail), W(walk)}

L Set of transport lines: {(rail line) L^R , (bus line) L^B }, and different lines running are restricted by timetables

2.1.2. Parameters

O the origin node of travel

D the destination node of travel

$v_{i,j}$ modes of transport from nodes i to j, $i, j \in N_1$

$\widetilde{t_{i,j}^a}$ travel duration (including the dwell duration of the intermediate nodes between transfer nodes) of lines a from node i to node j, $i, j \in N_1, a \in L$

$\widetilde{t_i^a}$ dwell duration of vehicle when passenger takes line a and does not transfer at node i, $i \in N_1, a \in L$

$t_i^{a,b}$ transfer duration (including three parts: ξ_i^1, ξ_i^2 and ξ_i^3) spent by passenger transferring from the lines a to b at node i , $i, \in N_1, a, b \in L$

ξ_i^1 alighting duration after passenger arrives at node i, $i, \in N_1$

ξ_i^2 possible walking duration of passengers transferring between different transport modes at node i, $i \in N_1$

ξ_i^3 waiting duration (ignores the boarding duration of transfer) when passenger transfers at node i, $i \in N_1$

T_i^a arrival time when a passenger takes the lines a to arrive nodes i, $i, \in N_1, a \in L$

T_O^a leave time when a passenger takes the lines a to leave node O

γ_i^h arrival time when transfer line h arrives at node i, $i, \in N_1, h \in L$

$\theta_{i,j}$ the number of stops passed by passenger from nodes i to j, $i, j \in N_1$

$C_a(\theta_{i,j})$ the cost of rail line from nodes i to j, $i, j \in N_1$, $a \in L^R$, it can easy gain from sectional charging rule of rail traffic

ω the number of different bus line taken by passenger in whole travel

C_b the fare for passenger to take one bus line

C_{max} maximum allowable travel cost acceptable by passenger

β_{max} maximum number of transfers acceptable by passenger

2.1.3. Decision variables

$$x_{i,j}^a = \begin{cases} 1, \text{if passenger selects line } a \text{ from nodes } i \text{ to } j (i, j \in N_1, a \in L) \\ 0, \quad else \end{cases}$$

$$y_i^{a,b} = \begin{cases} 1, \text{if } x_{q,i}^a = 1 \text{ and } x_{i,j}^b = 1 \ (q, i, j \in N_1 \ , i \ \neq S \ or \ D, a \ \neq b, a, \mathrm{b} \in L) \\ 0, \quad else \end{cases}$$

2.2. Formulation

The objective function in this study is minimizing the passenger's total travel duration with the number of transfers and the total travel cost acceptable by passenger. It can be formed as follows:

$$minz = \sum_{i,j \in N_1} \sum_{a,b \in L} ((\widetilde{t_{i,j}^a} + t_i^{a,b} * y_i^{a,b} + \widetilde{t_i^a} * (1 - y_i^{a,b})) * x_{i,j}^a) \tag{1}$$

The constraints of this problem are given as follows:

$$C_b * w + \sum_{i,j \in N_1} C_a(\theta_{i,j}) \le C_{max} \tag{2}$$

$$\sum_{i \in N_1} \sum_{a,b \in \mathrm{L}} y_i^{a,b} \le \beta_{max} \tag{3}$$

$$\sum_{j \in N_1} \sum_{a \in \mathrm{L}} x_{O,j}^a = 1 \, , \mathrm{j} \neq \mathrm{O} \tag{4}$$

$$\sum_{i \in N_1} \sum_{a \in \mathrm{L}} x_{i,z}^a - \sum_{j \in N_1} \sum_{a \in \mathrm{L}} x_{z,j}^a = 0, \mathrm{z} \in N_1 / \{O, D\}, i \neq j \tag{5}$$

$$\sum_{i \in N_1} \sum_{a \in \mathrm{L}} x_{i,D}^a = 1 \tag{6}$$

$$\sum_{a \in \mathrm{L}} x_{i,j}^a \le 1, \ \ i, j \in N_1 \tag{7}$$

$$\sum_{a,b \in \mathrm{L}} y_i^{a,b} \le 1, i \in N_1 \tag{8}$$

$$(T_i^a + \ \xi_i^1 + \xi_i^2) y_i^{a,h} \le \gamma_i^h y_i^{a,h}, i \in N_1 and \ i \neq D, a, h \in \mathrm{L} \tag{9}$$

$$T_j^a x_{i,j}^a (1 - y_i^{a,b}) = (T_i^a + \widetilde{t_i^a} + \widetilde{t_{i,j}^a}) x_{i,j}^a (1 - y_i^{a,b}), i, j \in N_1, i \neq j \ and \ i \neq D, a \in \mathrm{L} \tag{10}$$

$$T_j^b x_{i,j}^b y_i^{a,b} = (T_i^a + t_i^{a,b} + \widetilde{t_{i,j}^b}) x_{i,j}^a y_i^{a,b}, i, j \in N_1, i \neq j \ and \ i \neq D, a, b \in \mathrm{L} \tag{11}$$

$$\xi_i^3 y_i^{a,h} = (\gamma_i^h - \xi_i^1 - \xi_i^2 - T_i^a) y_i^{a,h}, i \in N_1 \ and \ i \neq D, a, h \in \mathrm{L} \tag{12}$$

$$x_{i,j}^a \in \{0,1\}, a \in \mathrm{L}, i, j \in N_1 \ and \ i \neq j \tag{13}$$

$$y_i^{a,b} \in \{0,1\}, a, b \in \mathrm{L}, i \in N_1 \ and \ i \neq D \tag{14}$$

Constraints (2) and (3) ensure that passenger's total travel cost and transfer duration are within the acceptable range of passengers. Constraints (4), (5) and (6) ensure that passenger arrives at their destination from starting point. Constraint (7) ensures that only one mode of transport is selected between two transfer nodes. Constraint (8) prevents multiple transfers at transfer nodes. Constraint (9) ensures that passenger can only board the transfer vehicle after passing the necessary transfer steps. Equations (10) and (11) represent the relationship of arrival time between two transfer nodes Transfer nodes that passenger needs to pass. Equation (12) is the calculation formula for waiting duration. Equations (13) and (14) mean that all the decision variables are binary.

Take Figure.1 as an example and the network conditions are shown in Table 1 and Table 2. Suppose that a passenger takes line 201 from node 1 to node 2, transfer 202 to node 5, take line 103 to node 8 and then take 201 to node 9. The number of stops (includes intermediate nodes) that passenger travelled by rail transit arriving node 5 is 6, therefore, the cost of the two rail transit lines is $2. And the cost for the passenger from node 5 to node 8 and from node 8 to node 9 is $2 respectively. The total cost of the passenger from node 1 to node 9 is $6.

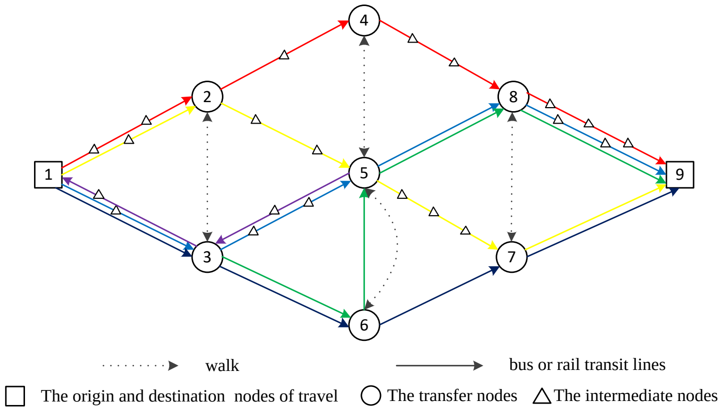

Fig. 1. An example of transit network

Table 1. Information of running lines

Line colour	Line ID	The earliest departure time from originating stop (AM)	Departure headways(min)	The transport nodes that pass through sequentially
Red	101	6:00	10	2-4-8
	106	6:10	10	4-8-9
	201	6:00	10	1-2-4-8-9
Yellow	102	6:10	8	1-2-5-7-9
	202	6:00	6	1-2-5-7-9
Blue	103	6:00	10	1-3-5-8-9
	203	6:00	6	3-5-8
Green	104	6:00	10	3-6-5-8-9
Cyan	105	6:00	10	1-3-6-7-9
Purple	204	6:00	6	5-3-1

Note: The bus Line ID from 101 to 106, the rail traffic line ID from 201 to 204 and the line 3 represents walk.

Table 2. Sectional charging rule of rail traffic

Number of passing nodes	[1,6]	(6,11]	(11,17]	(17,24]	(24,32]	(32,+∞)
fares ($)	2	3	4	5	6	7

3. Solution methodology

GA has been widely applied to optimization problem based on its strong random search ability(Bagheri et al., 2020). However, the optimization efficiency decline of single heuristic algorithms results from the complexity and uncertainty of real-world problems. This problem is better solved by simulation optimization algorithm (A. A. Juan et al., 2015). Combined by heuristic algorithm and simulation optimization algorithm, hybrid simulation optimization algorithm significantly improves the efficiency of heuristic algorithm (Guimarans et al., 2018). The development of pseudo-random number generators has created conditions for solving optimization problems by using probabilistic methods such as Monte Carlo simulation (Angel A. Juan et al., 2010). Monte Carlo simulation is a method to solve random problems with random simulations and statistical experiments. Integrated with heuristic algorithms, Monte Carlo simulations are more efficient to solve stochastic optimization problems (Yeh et al., 2010). The randomness of fitness value evaluation is the feature of stochastic optimization problems. This difficulty is solved by the simulation technique which regards a given number of the samples' average value as the true fitness value of the solution (S. Zhang et al., 2017).

Many probabilistic uncertainty problems can be solved by Monte Carlo simulation (Janssen, 2013). Stochastic of urban transport network can be simulated by Monte Carlo simulation (see, for instance (Chen et al., 2016; Luan et al., 2019)). When road section's travel duration is independently distributed, there are two methods to solve the shortest path problem in a stochastic network. One method obtains the cumulative distribution function of the path travel duration through traditional calculus method. However, this method cannot solve all the shortest path problems in a stochastic network (Ji et al., 2011). For example, when road section's travel duration is correlated, it is a difficult task to obtain the cumulative distribution function of path travel duration. Moreover, because of the factors that vehicles are restricted by the schedules and transportation modes with different travel time distribution function are common-lines, it is more difficult to solve the cumulative distribution function. Another approach to solve stochastic shortest path problems is through simulation (Zockaie et al., 2013). The simulation-based method has the advantages of simple implementation, flexible structure, and can adapt to a variety of problems. In addition, the approach seems to provide a good alternative plan in cases where other algorithms encounter difficulties (Zockaie et al., 2014). In realistic transportation network, the distribution function of travel duration and dwell duration of different transportation modes on road section can be obtained. Therefore, time parameters of various travel situations can be simulated by Monte Carlo simulation, so as to transform the stochastic shortest path problem into the shortest path problem under the determined network.

To the best of our knowledge, there is less previous work concerning such stochastic methods in travel path optimization literature. In this regard, this paper proposes a solving method combining GA and Monte Carlo simulation. The rest of this section is devoted to specific simulation methods and all the examples mentioned in this section are based on Figure. 1.

3.1. Encoding and decoding scheme

This study uses two-part integer encoding to encode passenger travel plan. The encoding of first paragraph represents the sequence of the transfer nodes in travel plan. The encoding of second paragraph represents the lines between nodes. When nodes are disconnected, the corresponding encoding of second paragraph is represented by 0. It needs to judge whether the routes' codes of the encoding of second paragraph before and after the transfer node are same. If the two codes represented different lines, transfer behaviour happens at the node, otherwise, the transfer behaviour not happen. The schematic diagram of encoding and decoding is shown in Figure. 2.

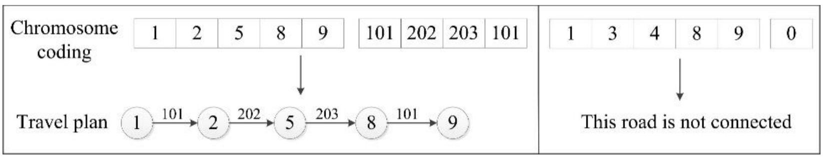

Fig. 2. Schematic diagram of encoding and decoding

3.2. The generation of the initial population

Section 3.1 introduces the encoding and decoding scheme of a single chromosome. Many such chromosomes constitute a population. In this paper, population size (ps) is used to represent the number of chromosomes in a population. The following steps explain how to generate an initial population:

Step1: The path with the shortest expected time between O and D is found by Floyd algorithm, which is regarded as the individual's encoding of first paragraph.

Step2: The transportation modes between transfer nodes in the encoding of first paragraph are selected randomly from existing transportation modes between transfer nodes.

Step3: Select lines according to the selected transportation modes and route information between transfer nodes, then, the individual's encoding of second paragraph is generated.

The detailed method of line determination is shown in Table 3.

Step4: The whole algorithm terminates, when current population number reaches required number. Otherwise, the expected time between transfer nodes of the shortest path found previously is expanded 1.2 times by the algorithm (to ensure the diversity of the population), and performs Step1 to seek the shortest path from O to D.

3.3. Selection and elite retention strategy

The reciprocal of individual's fitness value (passenger's travel duration) is taken as the evaluation function, and roulette selection is applied to select individuals. Elite preservation strategy is adopted in the genetic process.

3.4. Crossover

There are two kinds of crossover operation. The first kind of operation is shown in Figure.3. If the same transfer nodes exist in two individuals' the encoding of first paragraph (except for O and D), an identical

Table 3. The method of line determination

Line determinate Strategy
Input: i, p, q : the transfer nodes of shortest path searched by Floyd algorithm; p : the previous node of i, if $i \neq O$; q : the latter node of i, if $i \neq D$; $v_{p,i}$, $v_{i,q}$; Route information sheet for mode of transportation; **Output:** The encoding of second paragraph; 1: **For** i from O to D 2: **If** $i = O$ **then** 3: According to $v_{i,q}$ random select line to extend to node q; 4: **else** 5: **If** $i \neq O$ or D **then** 6: **If** $v_{p,i} = v_{i,q}$ and the selected line $a(a \in L)$ between nodes p and i can extend to node q **then** 7: Select the same line a between nodes i and q; 8: **else** 9: Select randomly the line on the basis of $v_{i,q}$ to extend to node q; 10: **else** 11: pass; 12: **End if** 13: **End if** 14: **End if** 15: **Return** The encoding of second paragraph

node is selected as the crossover position to exchange gene fragment. Then the corresponding crossover operation is performed in the two individuals' the encoding of second paragraph; The second kind of operation is shown in Figure.4. If two individuals do not have the same transfer nodes in the first paragraph of the code, the crossover position is selected randomly at the same transfer nodes (except for O and D) of the two individuals. It is worth noticing that the new connections that do not connect directly may be generated between two transfer nodes. In this condition, the intermediate transfer node of the two transfer nodes will be repaired by Floyd algorithm. The missing part of the encoding of second paragraph caused by the crossover operation will be replenished by Step2 and Step3 of the generation of the initial population. During the model solving process, we use the crossover probability (Pc) to control whether the chromosome performs the above crossover operation. The higher the Pc, the higher the probability that the chromosome will perform the crossover operation. On the contrary, the probability is smaller.

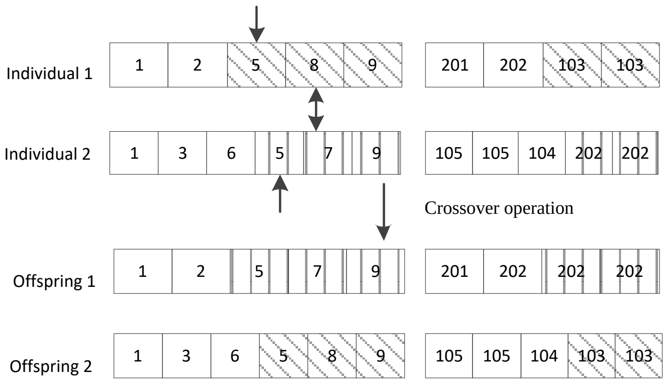

Fig. 3. Crossover operation with the same node

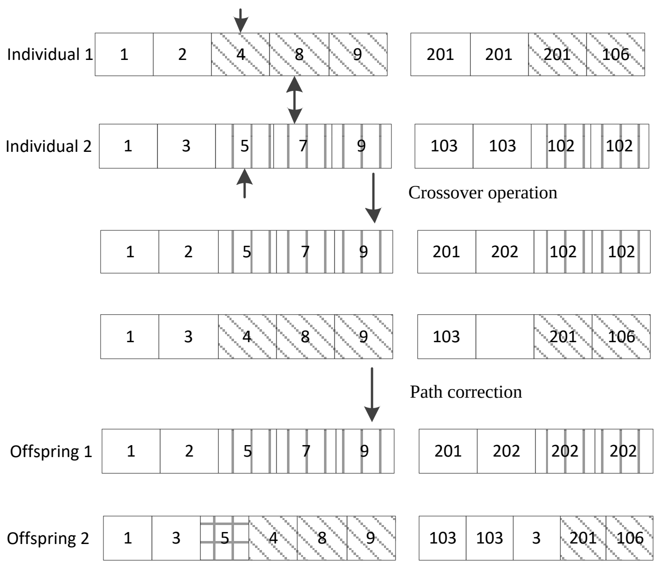

Fig. 4. Crossover operation with no the same node

3.5. Mutation

The mutation operator expands the search range of GA. For the solution structure of the problem, a mutation operator based on replacing a node is designed for GA, which is described in Figure.5. A node selected randomly (except for O and D) in individual's the encoding of first paragraph is deleted by mutation operation. Then an alternative path between the one previous and latter transfer nodes of the transfer node is searched by Floyd algorithm. If there is no valid path, the latter transfer nodes of the selected transfer node will move backward one transfer node and search again until a valid path is found. The missing part of code in the encoding of second paragraph can be completed by Step2 and Step3 of the generation of the initial population. In the process of solving the model, the mutation probability (Pm) is used to control the chromosome to perform the above mutation operation. The higher the Pm, the higher the probability of chromosome mutation. On the contrary, the probability is smaller.

It is worth noticing that the same transfer nodes may exist on the encoding of first paragraph after the crossover and mutation operation, which leads to loops. The method of avoiding loops is as follows: delete the fragment between two identical transfer nodes and keep one of the same transfer nodes. Then the encoding of second paragraph deletes the relevant gene fragment according to the first paragraph. Figure.6 depicts the steps of avoiding loops' method.

3.6. Stopping conditions and population management strategy

When the number times of current iterations exceed the maximum number of iterations (MAXGEN), the entire algorithm terminates.

In this paper, we adopt two kinds of population management strategies including single population and multiple population, which both meet a given population size. The single population management strategy performs the operations of select, crossover and mutation for the whole population. However, the multiple population management strategy performs iterative evolution in multiple independent and parallel sub-populations, and carry out gene exchange among the populations after fixed intergenerational

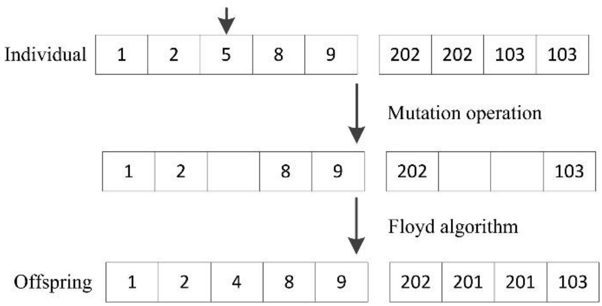

Fig. 5. Mutation operation

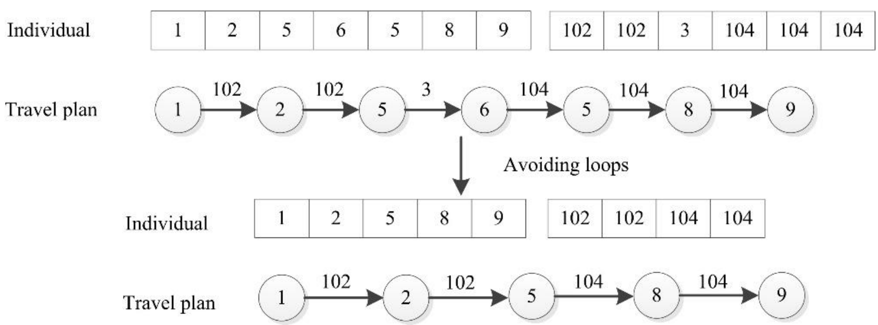

Fig. 6. Handling methods for loops

evolution, so as to realize different process of optimal search. Afterwards, the multiple population management strategy performs crossover operation by selecting the best individual in each generation to generate new individuals and evaluate the fitness values. Moreover, it selects the optimal solution from the local optimal solutions, the fitness value of new individuals and the current global optimal solutions. Then updates the global optimal solution immediately and reserves the optimal individual to the next generation of population. Considering the superiority of the Monte Carlo simulation in dealing with uncertain problems described above, the paper combines the single population management strategy and the multiple population management strategy with the Monte Carlo simulation. The former is that the single population management strategy based on the Monte Carlo simulation (*MCGA-I*) and the latter is that the multiple population management strategy based on Monte Carlo simulation (*MCGA-II*).

3.7. Fitness value calculation based on Monte Carlo simulation

This paper uses Monte Carlo simulation to simulate vehicle running duration and dwell. We use m to represent the m-th scenario of Monte Carlo simulation, and use M to represent the total number of scenarios to be simulated. In every scenario, calculate the shortest passenger travel duration based on the randomly obtained vehicle running duration and dwell duration. Then, take the average of the shortest passenger travel duration of M scenarios as the individual objective function. The detailed calculation process is shown in Table 4.

Table 4. The process of calculating individual fitness value

Individual fitness value calculation

Input: The earliest departure time from originating stop, Departure headways;
Transfer nodes that passenger needs to be gone through in travel plan;
Time distribution function of different transportation modes;
i : the transfer nodes of shortest path searched by Floyd algorithm;
Output: Individual fitness value, $Av\ Fitness$;
$m \leftarrow 1$, m represents the m-th scenario;
$H \leftarrow 0$, H is used to save the sum of the duration spent by passengers in m scenarios;
$H_m \leftarrow 0$, H_m is used to record the travel duration of passenger in the m-th scenario;
If $i \neq D$, $i+1$ represents the latter node of i;
Assume that passenger goes to node O by walk (i.e. the line 3). In other words, no matter which bus or rail transit line the passenger chooses to take at node O, the passenger needs to transfer at node O;
While $m \leq M$ **do**
  For i from O to D
    Generate $\widetilde{t_i^a}$ and $\widetilde{t_{i,i+1}^a}$ according to the time distribution function;
    If $a = b$ **then**
      $T_{i+1}^a \leftarrow$ Equation (10);
    else
      Calculate $t_i^{a,b}$;
      $T_{i+1}^b \leftarrow$ Equation (11);
    End If
    $H_m \leftarrow \sum_{a \in L} T_D^a - \sum_{a \in L} T_O^a$;
    $H \leftarrow H_m + H$;
    $m \leftarrow m + 1$;
End While
$Av\ Fitness \leftarrow H\ /\ M$;
Return $Av\ Fitness$

3.8. Calculation of passenger waiting duration

Passenger waiting duration is determined by the first arriving vehicle of the transfer line. In urban multi-modal transportation network, there are multiple transfer lines for passenger to choose. At transfer nodes, passenger generally tend to choose the first arriving vehicle which can transport them to a station closer to destination. Two transfer lines in Figure.7 are taken as an example. Suppose that a passenger will transfer at node 5 and the time after the behaviours of alighting vehicle and possible walking are fixed. We assume that the transfer vehicle arrives just at the moment that passenger finish the two above behaviours. The passenger waiting duration can be calculated by the difference between the actual departure time of originating stop and the originating stop's hypothetical departure time that comes from the backward derivation of dwell duration and running duration. In Figure.7, assume that the dwell duration of both lines the 104 and 103 are 1min (it is obtained randomly by the distribution function in the program) and ξ^1 is approximately 10s. The transportation mode between node 5 and node 9 is assumed as bus and both the lines 103 and 104 can take passenger from node 5 to node 9. The timetables of the lines 103 and 104 is shown in Figure.1. The time when passenger arrives at node 5 is 7:00, so the hypothetical departure time of line 104 is calculated by backward derivation as 6:51:10. Comparing it with the actual departure time 7:00, the waiting duration of passenger at node 5 is 8min50s. In the same way, the hypothetical departure time of line 103 is 6:48:10 and the actual departure time is 6:50, so the waiting duration of passenger at node 5 is 1min50s.

3.9. The discriminant method of common-lines

A line from many common-lines can be randomly selected by passenger to travel. For example, the travel plan is that passenger will go through the nodes 1-2-5-8-9 in sequence and the lines through these nodes are shown in Figure.8. Suppose that the modes of transportation are B-R-B-B in the passenger travel plan. It is inevitable that passenger will transfer at the nodes 2 and 5 due to the different transport modes before and after the nodes 2 and 5. Lines 101 and 102 meet the passenger travel plan from node 1 to node 2, in other words, the common-lines from node 1 to node 2 are lines 101 and 102. Although both lines 102 and 202 can take passenger from node 2 to node 5, the transportation mode is rail transit from node 2 to node 5 in the passenger travel plan. Therefore, passenger is only able to take line 202 to node 5, and there is no common-lines at node 2 in this passenger travel plan. Passenger transfers at node 5 and the mode of transportation from node 5 to node 8 and node 8 to node 9 are bus. Therefore, the lines 103 and 104 are common-lines, because them conform to the mode of transportation in passenger travel plan from node 5 to node 9.

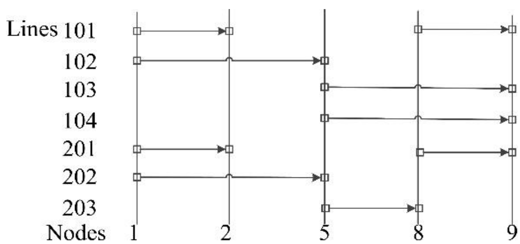

Fig. 8. Running lines passed through the nodes of travel plan

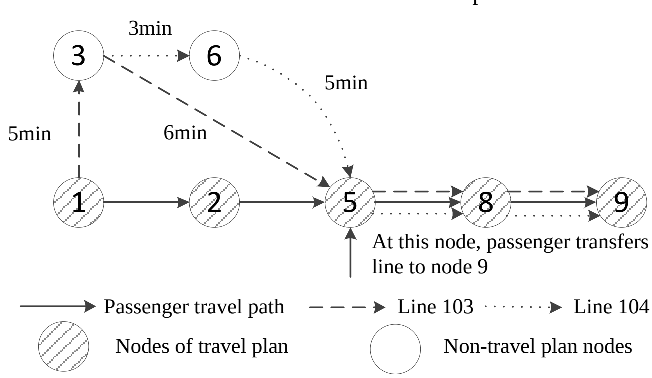

Fig. 7. Passenger travel diagram

After introducing the algorithm approach proposed by us, it seems useful to employ Figure. 9 to summarize the method. The distinguishing features of the proposed algorithm are highlighted with a different colour.

4. Numerical examples

4.1. Problem example

To examine the effectiveness of the algorithm, this paper simulates a multimodal transport network with 30 nodes (nodes 1 and 30 represent the origin node and destination node of passenger travel, respectively, other 28 nodes are transfer nodes). Because of charging rule, the number of intermediate nodes of rail transit line is shown in Table 5 and the intermediate nodes of bus line are not given. The fare system of rail transit is shown in Table 2 and the flat fare of bus is $2. The dwell duration of rail transit and bus at the intermediate nodes are included in the vehicle's running duration. The distribution function of vehicle's running duration between nodes are shown in Figure.10. The dwell duration of the vehicle at transfer nodes is subject to the uniform distribution function of U (1.5,2). The alighting duration and possible transfer walking duration of passenger are 10s and 2min, respectively. The conditions that the number of transfers are not more than 3 times and the allowable travel cost should be less than $8. Table 6 shows timetables of lines and the transfer nodes that lines pass through.

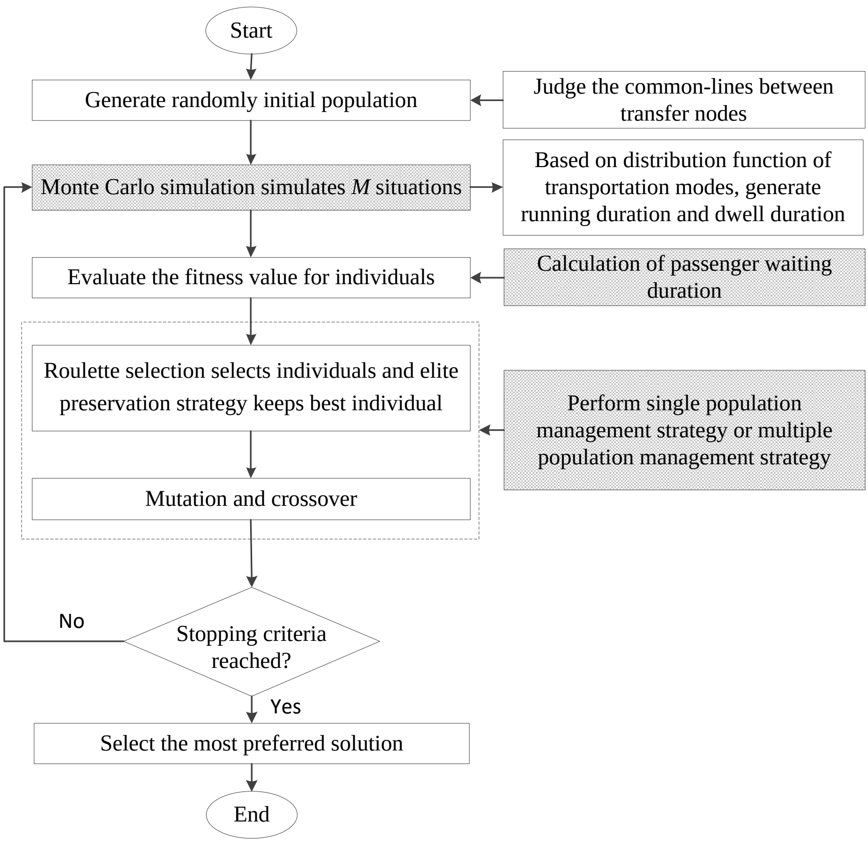

Fig. 9. Flowchart of the proposed algorithm

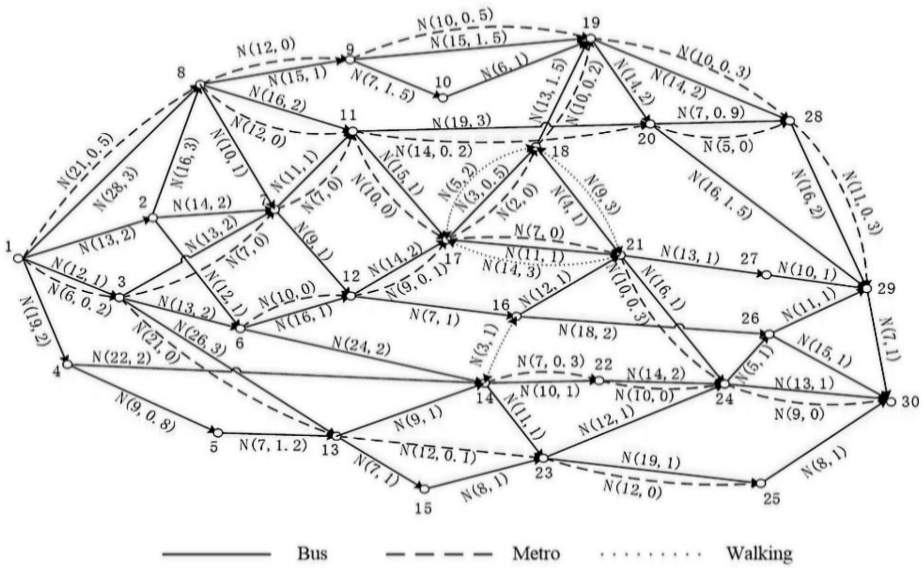

Fig. 10. Schematic diagram of transportation network

Table 5. Number of intermediate nodes between transfer nodes of lines of rail traffic

O	D	Number of intermediate nodes	O	D	Number of intermediate nodes
1	3	1	12	17	2
1	8	6	13	23	4
3	7	1	14	22	1
3	13	8	17	21	2
6	12	3	18	19	3
7	11	1	20	28	1
8	9	3	21	24	3
8	11	1	22	24	2
9	19	2	23	25	4
11	17	3	24	30	2
11	20	2	28	29	3

Note: There are no routes or intermediate nodes between the nodes not mentioned in above table.

The passenger itinerary planning is optimized according to the network conditions in Table 2, Table 5 and Table 6. Some parameters are set as follows: Pc =0.9; Pm =0.5; MAXGEN =50; M=20; ps =50; the size of two sub-populations of *MCGA-II* are 20 and 30, respectively. The convergence process at high Pm and Pc is shown in Figure.11. We can see that both the curve of *MCGA-I* and *MCGA-II* can keep stable within 50 generation, which means the algorithm is effective.

Table 6. Data of running lines

Line ID	The transport nodes that pass through sequentially	The earliest departure time from originating stop (AM)	Departure headways (min)
101	1-2-8-11-17-18-21-24	6:00	10
102	1-3-6-12-16-21-27	6:10	8
103	1-3-13-14-22-24-26-29	6:00	10
104	4-7-11-20-29-30	6:00	10
105	1-4-5-13-15-23-25	6:00	10
106	4-14-23-24-30	6:10	10
107	8-7-12-16-26-30	6:10	8
108	1-8-9-19-28-29	6:00	10
109	2-7-12-17-18-19-20-28	6:00	8
110	2-8-9-10-19-20-29-30	6:10	8
111	2-6-14-23-24-30	6:00	10
112	5-13-15-23-25-30	6:10	10
113	6-12-17-21-27-29-30	6:00	10
114	2-7-12-16-26-30	6:10	8
201	1-8-9-19-28-29	6:00	6
202	1-3-7-11-20-28	6:00	6
203	3-13-23-25	6:00	6
204	14-22-24-30	6:00	6
205	8-11-17-21-24-30	6:00	6
206	6-12-17-18-19	6:00	6

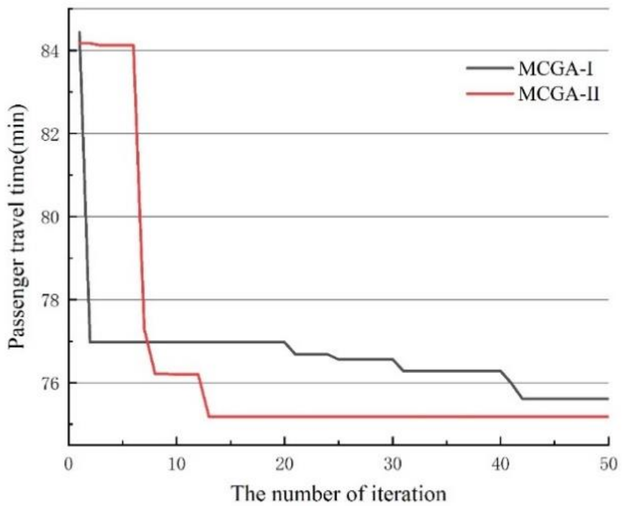

Fig. 11. Algorithm convergence process under high Pc and Pm

The crossover probability and mutation probability are significant parameters affecting the entire evolution process. For determining the two parameters, this paper changes Pc from 0.6 to 0.9 and change Pm from 0.1 to 0.5, respectively. The average value of fitness values is calculated under different Pm and Pc probabilities and the results of the five times repeated experiments are shown in Figure.12 and 13. In these figures, the change of fitness value from high to low corresponds to the change of colour from dark red to purple on the surface. The figure on the surface where the Pm and Pc axis intersect is a projection of a three-dimensional surface, and the spherical point represents the fitness value corresponding to the scale on the axis of Pm and Pc. From these figures, we can see that the fitness values of both *MCGA-I* and *MCGA-II* are a range of 69 to 84, which means that Pm and Pc have an influence on the solution result of the both algorithms. Purple and blue represent low fitness values. We need to find the Pm and Pc with lower fitness values for both algorithms to compare those algorithms. A higher Pm and Pc may make the optimal solution more likely to be found, but it will cost more time. Therefore, in order to reduce both algorithms' solving time under the condition of similar fitness values, we set the values of Pc, Pm as 0.7 and 0.2 respectively.

Take Pc=0.7; Pm=0.2; ps=50 or 80 as an example. The size of two sub-populations of *MCGA-II* are (20 and 30) or (30 and 50), respectively. Table 7 summarizes the results of 10 times repeated experiments under different population size. As can be understood from Table 7, no matter how the two population sizes of *MCGA-I* and *MCGA-II* changes, *Best*, *Max* and *A-fit* of two algorithms are almost same. Therefore, both of them can find the optimal solution. However, *Nt* and *Time* of *MCGA-II* are smaller, which means that it can find the optimal solution faster. Figure.14 depicts the disturbance of *Nt*, it shows that the degree of change of *Nt* of *MCGA-II* is steadier than *MCGA-I's* whether ps is 50 or 80. Moreover, the average value of *Nt* is the lowest when ps is 50. As stated above, the optimization effect of *MCGA-II* is relatively better. This paper sets ps=50 to get the result of passenger itinerary planning.

The convergence process of the experiment (ps=50) is shown in Figure.15. As can be found from the figure, the curve keeps stable after 25 generations. The transfer nodes that passenger passed in final optimized solution is 1-3-7-11-20-29-30. Passenger first takes line 202 to node 20 and then transfer line 110 to node 30, which will cost passenger 69.63 mins and $6.

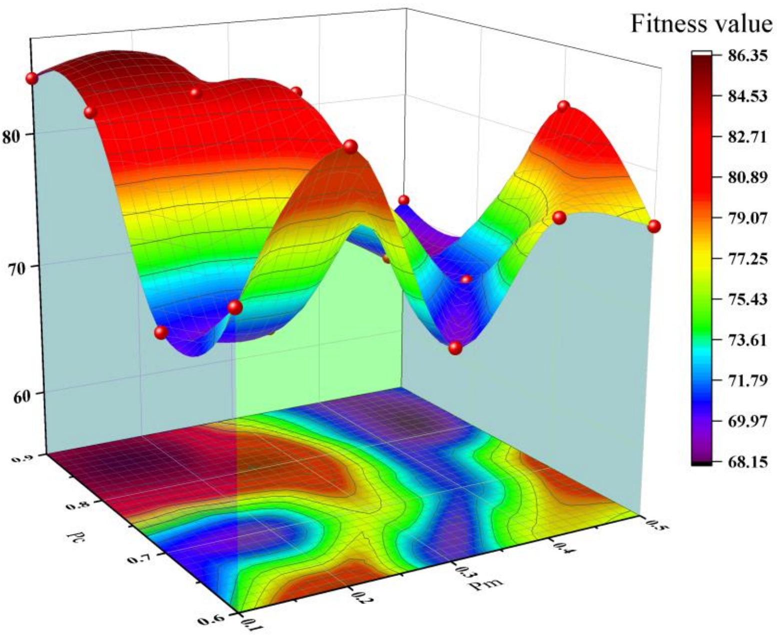

Fig. 12. Fitness values of *MCGA-I* under different Pc and Pm

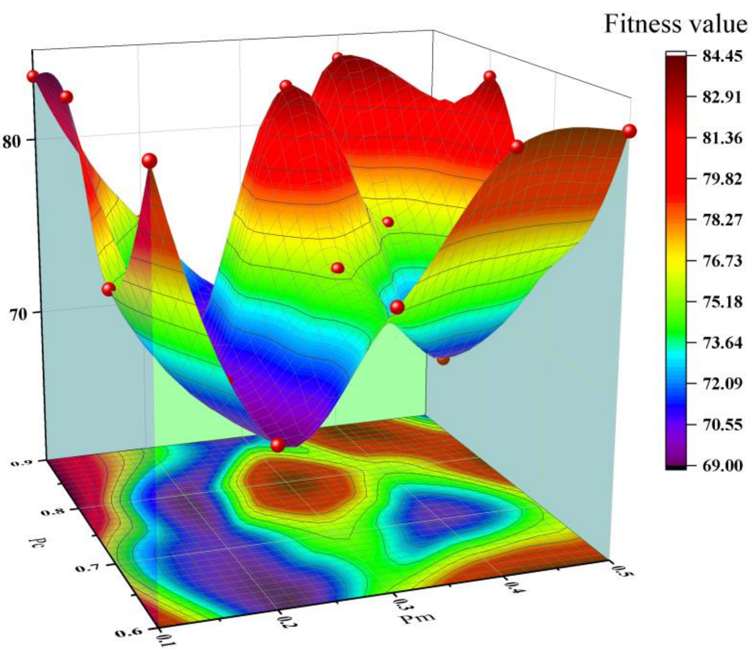

Fig. 13. Fitness values of *MCGA-II* under different Pc and Pm

Table 7. The results of the two algorithms are compared

parameter	MCGA-I					MCGA-II				
	Best(min)	*Max*(min)	*A-fit*(min)	*Time*(s)	*Nt*	*Best*	*Max*	*A-fit*	*Time*	*Nt*
ps=50 or (20+30)	69.67	83.81	76.81	387.4	28	70.09	83.62	75.19	352.1	14
ps=80 or (30+50)	70.89	87.26	78.35	605.2	29.6	69.94	83.42	75.06	540.5	16.6

Note: *Best* is the smallest fitness value in 10 repeated experiments; *Max* is the largest fitness value in 10 repeated experiments; *A-fit* is the average of fitness value in 10 repeated experiments; *Time* is the average of the CPU time consumed to get the best fitness value in 10 repeated experiments; *Nt* is the average of the number of iterations to first reach the optimal fitness in 10 repeated experiments.

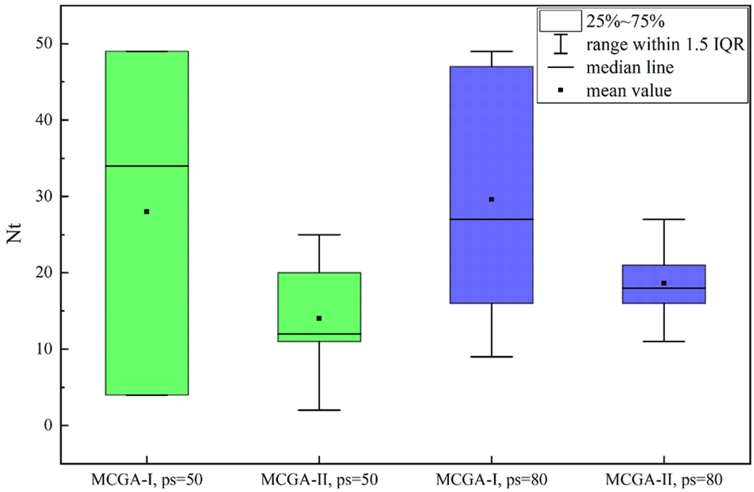

Fig. 14. The distribution of *Nt*

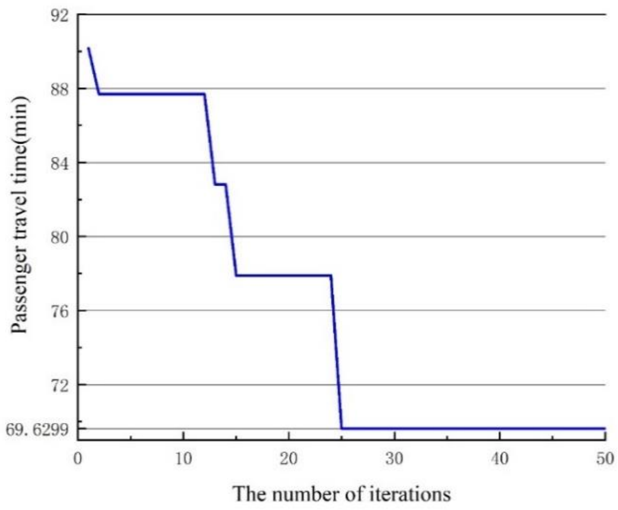

Fig. 15. Convergence process of travel plan

4.2. Numerical analysis

We perform some analyses that the relationships between total travel duration and travel cost or the number of transfers, respectively. The number of transfers and travel cost not only both affect the total travel duration, but also affect each other. In order to avoid the two parameters to influence each other, this paper repeats experiments regardless of constraints. Two best passenger travel plans are presented in Table 8. According to the Table 8, passenger only needs to transfer one time and spend $6.

Table 8. Data of travel plan under high number of transfers and travel cost

The serial number	Passenger takes lines	Travel duration (min)	Actual travel cost ($)
1	202-202-202-202-110-110	69.464	6
2	202-202-202-202-104-104	69.527	6

This paper sets the number of transfers is three times and changes the allowable travel cost from $2 to $8. Table 9 provides the optimal passenger travel plans under the different allowable travel cost. The influence of the allowable travel cost on the total travel duration can be divided into three parts. The allowable travel cost of the first part ranges from 2 to 3, the second part ranges from $4 to $5 and the third part equals to or more than $6. In the first part, algorithm cannot supply optimal path to passenger because passenger cannot take only one line from origin node to destination node in the experiment. In the second part, passenger can only choose bus lines or rail transit lines with shorter travel distances, which may lead passenger spend more time during travel. In the third part, appropriate increment of the allowable travel cost leads passenger to choose the lines with shorter travel duration but higher cost such as the allowed travel cost reaches $6 yuan from $4. However, when the allowed travel cost exceeds $6, it cannot cause the decline of travel duration, which is apparent from Table 8. As for number of transfers, algorithm keeps it at one time. The reason is that a greater number of transfers means that passenger will spend more time for transferring vehicles, which increases the travel duration of passenger.

Table 9. Data of travel plan under different travel cost

Allowable travel cost ($)	Passenger take lines	Travel duration (min)	Actual travel cost ($)
4	202-202-114-114-114-114	75.085	4
5	202-202-114-114-114-114	76.014	4
6	202-202-202-202-110-110	69.464	6

5. Conclusions

The main result of this paper is to establish a combined optimization model that considers multiple modes of transportation, timetables of public transportation, passenger's travel cost, and the number of transfers under the condition of uncertain vehicle running duration and dwell. From a practical point of view, the research results of this paper are mainly to provide travel plan that meet the needs of passenger, improve passenger's travel efficiency, and encourage more passengers to choose public transportation to travel. Beyond the reported results, this study also emphasizes the importance of considering the uncertainty of the urban transportation network and the common-lines of multiple transportation modes in order to obtain a more realistic passenger travel route with the shortest travel time. The key conclusions are summarized as follows:

(1) The proposed combination algorithm performed well in the numerical example. The different values of Pc and Pm have effect on the optimal result. And the population size has little effect on the results of *MCGA-I* and *MCGA-II* to obtain the optimal fitness value.

(2) Both *MCGA-I* and *MCGA-II* can solve the problem, however, *MCGA-II* can get better results in a shorter CPU time.

(3) Through the numerical analysis, the allowable travel cost has a positive correlation with travel duration within an appropriate range. However, the impact of changes in the numbers of transfers on travel duration is not obvious.

(4) In the optimization of passenger travel routes, different transportation modes include different transportation lines. The line's timetable and the dwell duration of vehicle at the node will have impact on the number of transfers and travel duration of passengers.

Future research could be performed from the following aspects:

(1) The algorithm used in this paper is effective in the proposed passenger travel scenarios, but whether it is still applicable to more complex passenger travel scenarios remains to be further studied.

(2) There are many algorithms to solve the travel route optimization problem, but how to find the optimal route faster and more efficiently still needs further research.

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References

[1] Abbaspour, R. A., & Samadzadegan, F. (2010). An evolutionary solution for multimodal shortest path problem in metropolises. *Computer Science and Information Systems*. https://doi.org/10.2298/CSIS090710024A

[2] Artigues, C., Huguet, M. J., Gueye, F., Schettini, F., & Dezou, L. (2013). State-based accelerations and bidirectional search for bi-objective multi-modal shortest paths. *Transportation Research Part C: Emerging Technologies*. https://doi.org/10.1016/j.trc.2012.08.003

[3] Bagheri, M., Ghafourian, H., Kashefiolasl, M., Pour, M. T. S., & Rabbani, M. (2020). Travel management optimization based on air pollution condition using markov decision process and genetic algorithm (case study: Shiraz city). *Archives of Transport*, *53*(1), 89–102. https://doi.org/10.5604/01.3001.0014.1746

[4] Botea, A., Kishimoto, A., Nikolova, E., Braghin, S., Berlingerio, M., & Daly, E. (2019). Computing Multi-Modal Journey Plans under Uncertainty. *Journal of Artificial Intelligence Research*. https://doi.org/10.1613/jair.1.11422

[5] Chen, L., Gendreau, M., Hà, M. H., & Langevin, A. (2016). A robust optimization approach for the road network daily maintenance routing problem with uncertain service time. *Transportation Research Part E: Logistics and Transportation Review*. https://doi.org/10.1016/j.tre.2015.11.006

[6] Cheng, P., Xu, C., Lebreton, P., Yang, Z., & Chen, J. (2019). TERP: Time-event-dependent route planning in stochastic multimodal transportation networks with bike sharing system. *IEEE Internet of Things Journal*. https://doi.org/10.1109/JIOT.2019.2894511

[7] Chriqui, C., & Robillard, P. (1975). COMMON BUS LINES. *Transportation Science*. https://doi.org/10.1287/trsc.9.2.115

[8] Dalkiliç, F., Doğan, Y., Birant, D., Kut, R. A., & Yilmaz, R. (2017). A Gradual Approach for Multimodel Journey Planning: A Case Study in Izmir, Turkey. *Journal of Advanced Transportation*. https://doi.org/10.1155/2017/5656323

[9] Dib, O., Moalic, L., Manier, M. A., & Caminada, A. (2017). An advanced GA–VNS combination for multicriteria route planning in public transit networks. *Expert Systems with Applications*. https://doi.org/10.1016/j.eswa.2016.12.009

[10] Dib, Omar, Caminada, A., Manier, M. A., & Moalic, L. (2018). Computing multicriteria shortest paths in stochastic multimodal networks using a memetic algorithm. *Proceedings - International Conference on Tools with Artificial Intelligence, ICTAI*. https://doi.org/10.1109/ICTAI.2017.00177

[11] Dotoli, M., Zgaya, H., Russo, C., & Hammadi, S. (2017). A Multi-Agent Advanced Traveler Information System for Optimal Trip Planning in a Co-Modal Framework. *IEEE Transactions on Intelligent Transportation Systems*. https://doi.org/10.1109/TITS.2016.2645278

[12] Faroqi, H., & Mesgari, M. S. (2016). Performance Comparison between the Multi-Colony and Multi-Pheromone ACO Algorithms for Solving the Multi-objective Routing Problem in a Public Transportation Network. *Journal of Navigation*. https://doi.org/10.1017/S0373463315000594

[13] Ghavami, S. M. (2019). A web service based advanced traveller information system for itinerary planning in an uncertain multimodal network. *Geocarto International*.

https://doi.org/10.1080/10106049.2019.1583773

[14] Goerigk, M., & Schmidt, M. (2017). Line planning with user-optimal route choice. *European Journal of Operational Research*. https://doi.org/10.1016/j.ejor.2016.10.034

[15] Guimarans, D., Dominguez, O., Panadero, J., & Juan, A. A. (2018). A simheuristic approach for the two-dimensional vehicle routing problem with stochastic travel times. *Simulation Modelling Practice and Theory*. https://doi.org/10.1016/j.simpat.2018.09.004

[16] Janssen, H. (2013). Monte-Carlo based uncertainty analysis: Sampling efficiency and sampling convergence. *Reliability Engineering and System Safety*. https://doi.org/10.1016/j.ress.2012.08.003

[17] Ji, Z., Kim, Y. S., & Chen, A. (2011). Multi-objective α-reliable path finding in stochastic networks with correlated link costs: A simulation-based multi-objective genetic algorithm approach (SMOGA). *Expert Systems with Applications*. https://doi.org/10.1016/j.eswa.2010.07.064

[18] Jin, F., Yao, E., Zhang, Y., & Liu, S. (2017). Metro passengers' route choice model and its application considering perceived transfer threshold. *PLoS ONE*, *12*(9), 1–17. https://doi.org/10.1371/journal.pone.0185349

[19] Juan, A. A., Rabe, M., Faulin, J., & Grasman, S. E. (2015). Guest editorial. *Journal of Simulation*, *9*(4), 261–262. https://doi.org/10.1057/jos.2015.18

[20] Juan, Angel A., Faulin, J., Ruiz, R., Barrios, B., & Caballé, S. (2010). The SR-GCWS hybrid algorithm for solving the capacitated vehicle routing problem. *Applied Soft Computing Journal*. https://doi.org/10.1016/j.asoc.2009.07.003

[21] Kang, Y., & Youm, S. (2017). Multimedia application to an extended public transportation network in South Korea: optimal path search in a multimodal transit network. *Multimedia Tools and Applications*. https://doi.org/10.1007/s11042-016-4015-9

[22] Liu, L., Mu, H., & Yang, J. (2017). Toward algorithms for multi-modal shortest path problem and their extension in urban transit network. *Journal of Intelligent Manufacturing*. https://doi.org/10.1007/s10845-014-1018-0

[23] López, D., & Lozano, A. (2019). Shortest hyperpaths in a multimodal hypergraph with real-time information on some transit lines. *Transportation Research Part A: Policy and Practice*. https://doi.org/10.1016/j.tra.2019.09.020

[24] Luan, S., Chen, X., Su, Y., Dong, Z., & Ma, X. (2019). Modeling travel time volatility using copula-based Monte Carlo simulation method for probabilistic traffic prediction. *Transportmetrica A: Transport Science*. https://doi.org/10.1080/23249935.2019.1692959

[25] Narayan, J., Cats, O., van Oort, N., & Hoogendoorn, S. (2020). Integrated route choice and assignment model for fixed and flexible public transport systems. *Transportation Research Part C: Emerging Technologies*. https://doi.org/10.1016/j.trc.2020.102631

[26] Nassir, N., Hickman, M., & Ma, Z. L. (2019). A strategy-based recursive path choice model for public transit smart card data. *Transportation Research Part B: Methodological*. https://doi.org/10.1016/j.trb.2018.01.002

[27] Nguyen, S., Pallottino, S., & Gendreau, M. (1998). Implicit enumeration of hyperpaths in a logit model for transit networks. *Transportation Science*. https://doi.org/10.1287/trsc.32.1.54

[28] Niksirat, M., Ghatee, M., & Mehdi Hashemi, S. (2012). Multimodal K-shortest viable path problem in Tehran public transportation network and its solution applying ant colony and simulated annealing algorithms. *Applied Mathematical Modelling*. https://doi.org/10.1016/j.apm.2012.01.007

[29] Pi, X., Ma, W., & Qian, Z. (Sean). (2019). A general formulation for multi-modal dynamic traffic assignment considering multi-class vehicles, public transit and parking. *Transportation Research Part C: Emerging Technologies*. https://doi.org/10.1016/j.trc.2019.05.011

[30] Xiao, Q., & He, R. C. (2017). Carpooling scheme selection for taxi carpooling passengers: A multi-objective model and optimisation algorithm. *Archives of Transport*, *42*(2), 85–92. https://doi.org/10.5604/01.3001.0010.0530

[31] Yeh, W. C., Lin, Y. C., Chung, Y. Y., & Chih, M. (2010). A particle swarm optimization approach based on monte carlo simulation for solving the complex network reliability problem. *IEEE Transactions on Reliability*. https://doi.org/10.1109/TR.2009.2035796

[32] Zhang, S., Xu, J., Lee, L. H., Chew, E. P., Wong, W. P., & Chen, C. H. (2017). Optimal Computing Budget Allocation for Particle Swarm Optimization in Stochastic Optimization. *IEEE Transactions on Evolutionary Computation*. https://doi.org/10.1109/TEVC.2016.2592185

[33] Zhang, T., Dong, S., Zeng, Z., & Li, J. (2018). Quantifying multi-modal public transit accessibility for large metropolitan areas: a time-dependent reliability modeling approach. *International Journal of Geographical Information Science*. https://doi.org/10.1080/13658816.2018.1459113

[34] Zhang, Y., & Tang, J. (2018). Itinerary planning with time budget for risk-averse travelers. *European Journal of Operational Research*. https://doi.org/10.1016/j.ejor.2017.11.023

[35] Zhu, W., Fan, W. li, Wahaballa, A. M., & Wei, J. (2019). Calibrating travel time thresholds with cluster analysis and AFC data for passenger reasonable route generation on an urban rail transit network. *Transportation*, *0123456789*. https://doi.org/10.1007/s11116-019-10040-8

[36] Zockaie, A., Nie, Y. M., & Mahmassani, H. S. (2014). Simulation-based method for finding minimum travel time budget paths in stochastic networks with correlated link times. In *Transportation Research Record*. https://doi.org/10.3141/2467-15

[37] Zockaie, A., Nie, Y., Wu, X., & Mahmassani, H. (2013). Impacts of correlations on reliable shortest path finding. *Transportation Research Record*. https://doi.org/10.3141/2334-01