



## CHARACTERISTICS OF LAMINAR FLOW IN PIPELINES OF HOMOGENOUS ALUM SLUDGE APPROXIMATED WITH USE OF THE VOČADLO MODEL FOR VISCOPLASTIC LIQUIDS

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**Abstract:** The study presents the manners of determination of the Darcy friction factor  $\lambda$  for a homogenous hydromixture of alum sludge of varied hydration and temperature for the laminar flow zone. The rheological evaluation of the hydromixture as a viscoplastic body has been conducted with use of measurements of viscosity. The curves of flow were approximated with use of the generalized Vočadlo model. The Darcy friction factor  $\lambda$  of the pipeline was determined with use of the non-dimensional criterion  $\lambda(Re_{gen})$  and  $\lambda(Re, He)$ .

### INTRODUCTION

The use of pipeline transport of hydromixtures as an optimum means of transport does not require a detailed justification. However, the determination of hydrotransport parameters influencing the costs of the planned venture may be questionable. The essential parameter of hydrotransport, which enables to determine the pressure loss of the transported mixture is the Darcy friction factor  $\lambda$ . The knowledge of this parameter enables eventually to conduct the analysis of the operation of the pump and pipeline system and to design it correctly. However, the regimen of flow for the planned transport has to be determined. The analysis of the diagrams of the correlation  $\lambda(Re, \epsilon)$  allows us to determine with certainty that for a given mixture, the critical value of the Reynolds number  $Re_{cr}$  (transition from laminar flow to the transition zone – turbulent flow) corresponds with the minimum value of the Darcy friction factor  $\lambda_{min}$  for a specific diameter of the pipe. Further slight decrease

in the value of the Darcy friction factor in the turbulent zone is however connected with the constant increase in the flow rate, which causes a significant pressure loss in the pipeline. To sum up, the final zone of laminar flow is characterised both by lower flow rates  $v$  and by lower values of the Darcy friction factor  $\lambda$ , which in turn leads to a decrease in the pressure loss in the designed pump and pipeline installation.

The economic aspect of pipeline hydrotransport should also be taken into account, as it requires transporting the hydrated material in the most efficient possible way. The fact that hydrotransport is preferred at high concentrations of the solid component is reasonable from the economic point of view.

The authors believe that the hydrotransport of homogenous mixtures should take place in the laminar flow zone, due to both hydraulic and economic reasons.

In order to provide the proper dimensions of the pump and pipeline installation for hydrotransport it is required to conduct the measurements of the viscosity of the mixture, necessary for the correct determination of the Darcy friction factor  $\lambda$  for the adopted flow parameters. Thus, the analysed problem is the determination of the rheological parameters of the mixture, and basing on that, of the pressure loss in the pipeline. This issue is important for designing the transport systems of concentrated sediments from water treatment and wastewater treatment [2, 3, 11].

## OBJECTIVE OF THE STUDY

The aim of the present analysis is to present the methodology of the determination of the rheological parameters and of the Darcy friction factor  $\lambda$  for a homogenous viscoplastic hydromixture, described by the tri-parametric Vočadlo model, basing on the analysis of the methods suggested by various authors [1, 6, 13, 14].

### *Physical and rheological properties of the analysed mixtures*

The study encompasses the sediments taken from alum sludge tanks of the waterworks utility collecting surface waters and using aluminium sulphate in the coagulation process. Apart from the rheological properties, the following characteristics were measured: the temperature, hydration and density of the sludge, chemical composition of the solid component and structure of the sediments. The rheological properties of the sludge were measured with use of a rotational rheometer Rheotest 2. The measurements determined the values of shear stress on the surface of the measurement cylinder for various assumed pseudo deformation rates. As a result of the measurements series of points were obtained, determining the pseudo-curves of flow. The measurements were conducted at the temperatures of 273.45 K and 293.15 K. Constant temperature of the sediment in the rheometer was maintained with use of an ultrathermostat equipped with contact thermometers. The hydration of the sediments was determined by the mass share of water and measured as the difference between the mass of humid sediment and the mass of the sediment dried to constant weight at the temperature of 378 K. The chemical composition of the solid fraction of the sludge was determined with use of chemical analysis, and its structure through the measurement of the isotherms of sorption of  $\text{CO}_2$  and  $\text{C}_6\text{H}_6$  in high-vacuum gravimetric apparatus [13].

The mathematical form of the model and the determination of values of rheological parameters were selected with use of methods of statistical analysis of flow curves. The

pseudo-curves of flow were transformed into curves of flow with use of the known Krieger – Elrod – Maron equation [8, 9, 10]. The values of  $m$  and  $dm/d\log\tau_R$  in this equation were determined from the polynomial approximation function of pseudo-curves of flow, determined in a logarithmic system of co-ordinates ( $\log G_p$ ,  $\log \tau_R$ ), with use of the classical method of least squares. Polynomial functions and the Bingham model are linear correlations with respect to their constant parameters. Due to this linearity, the application of the least squares method also leads to linear systems of normal equations for these parameters, so the solution thereof does not present any difficulties. On the other hand, the Vočadlo model is a non-linear correlation, which however can be reduced to a linear form. After the switching of the variables a linear form was obtained for new parameters, followed by the estimation of the value of one of the parameters and the application of the least squares method with respect to the linear forms of the analysed models. The limits of flow determined with use of the said models allow to test which of the measurement points fall within the zone of partial shearing of the sediment. Such points were rejected, and the described procedure was repeated until the condition of hydration of the whole sediment sample in the slit of the rheometer was fulfilled. Apart from the calculated values of the rheological parameters of the models, also the values of the correlation coefficient and standard deviation were calculated [13]. Values of the coefficient of correlation –  $R$  (and of the standard deviation –  $S$ ) are presented in Table 1 below.

Table 1. Statistical parameters of the flow curves

Temperature K	Hydration %	Models			
		Bingham		Vočadlo	
		R	S	R	S
293.15	93.60	0.9934	1.4039	0.9945	0.1546
	88.05	0.9531	5.4154	0.9985	0.4731
273.45	95.05	0.9904	2.1740	0.9989	0.1251
	88.00	0.9400	8.3210	0.9982	1.0634

These values enabled the evaluation of the compliance of the specified models with data obtained from the experiments. The tri-parametric model approximated the course of the flow curves of the sediments within the whole range of the shearing rate very well. Moreover, the applicability of the bi-parametric Bingham model for the description of the rheological properties of alum sludge was proven, in particular within the higher hydration range. The accurateness of this approximation decreased noticeably with the decrease in hydration.

The results of the analysis of the structure of sediments are presented in Table 2. The presented results allow to classify the analysed alum sludge as a sediment with a well-developed structure. This degree of development of the sediment structure depends on the content of aluminium hydroxide and has a vital influence on the rheological properties of the sludge.

Table 2. Physical properties of the sediment structure

properties of the structure					
ultramicropores	micropores	mesopores			total value
		(1.5÷3.0) μm	(3.0÷30) μm	(30÷100) μm	
pore capacity, cm <sup>3</sup> ·g <sup>-1</sup>					
0.007	0.012	0.011	0.111	0.040	0.181
pore surface area, m <sup>2</sup> ·g <sup>-1</sup>					
18.4	24.6	9.9	26.1	1.3	80.3

For the purpose of approximation of pseudo-curves of flow the triparametric, generalized Vočadlo model was used (1).

$$\dot{\gamma} = 0 \text{ for } \tau_R \leq \tau_0$$

$$\tau_R = \left( \frac{1}{\tau_0^n} + K_v \dot{\gamma} \right)^n \text{ for } \tau_R > \tau_0 \quad (1)$$

where:

$\tau_R$  – shear stress on the surface of cylinder [Pa]

$\tau_0$  – yield stress [Pa]

$\dot{\gamma}$  – shear rate [s<sup>-1</sup>]

$K_v$  – consistency index for Vočadlo model [Pa <sup>$\frac{1}{n}$</sup>  · s<sup>n</sup>]

n – flow behaviour index [-]

The physical and rheological properties of the analysed mixtures are presented in Table 3.

Table 3. Physical and rheological properties of the analysed mixtures

Hydration [%]	93.60	88.05	95.05	88.00
Density [kg/m <sup>3</sup> ]	1022	1041	1016	1041
Temperature [K]	293.15		273.45	
$K_v \left[ Pa^{\frac{1}{n}} \cdot s^n \right]$	0.0060	4.6118	0.0169	28.546
$\tau_0$ [Pa]	1.035	6.446	0.787	12.725
n [-]	0.820	0.368	0.680	0.348

***Determination of the parameters of flow of the mixture***

For laminar flow of Newtonian liquids the Darcy friction factor  $\lambda$  is calculated with use of formula (2)

$$\lambda = \frac{64}{\text{Re}} \quad (2)$$

where the Reynolds number  $\text{Re}$  is calculated with use of equation (3)

$$\text{Re} = \frac{vD\rho}{\eta} \quad (3)$$

Analogically to the Newtonian liquid, Parzonka [12], Kempinski [6], Sozański [13] adopt the determination of parameter  $\lambda$  for non-Newtonian liquids with use of formula (4)

$$\lambda = \frac{64}{\text{Re}_{gen}} \quad (4)$$

where  $\text{Re}_{gen}$  is the generalized Reynolds number for the adopted rheological model. In this case, the determination of the dimensions of the installation is based on the unconditional criterion  $\lambda(\text{Re}_{gen})$ .

Vočadlo and Charles [14] and Czaban [1] suggest that the determination of dimensions within the laminar zone of the flow should be based on the criterion  $\lambda(\text{Re}, \text{He})$ , thus creating a family of curves shifted parallelly depending on the value of the Hedström number  $\text{He}$ . A similar approach was suggested by Govier and Aziz [4].

***Non-dimensional criterion  $\lambda(\text{Re}_{gen})$*** 

The application of this criterion requires the knowledge of the generalized Reynolds number  $\text{Re}_{gen}$  for the adopted rheological model describing the viscoplasticity of homogenous hydromixture. Such hydromixtures are characterised by the occurrence of the so-called yield stress  $\tau_0$ . The omission of this value causes errors in the determination of the Reynolds number [7]. The lack of knowledge of  $\text{Re}_{gen}$  led to the abandonment of this method of determination of dimensions. Kempinski [6] presented a full generalized Reynolds number for the Vočadlo model along with a precise approach for the application thereof. The formulas quoted below enable to determine the required stress on the pipeline wall  $\tau_w$  (6), the generalized Reynolds number (5) and the Darcy friction factor  $\lambda$  (4).

Generalized Reynolds number for the Vočadlo model according to Kempinski [6]

$$\frac{\overline{\Delta p}}{L} = 4 \left( 1 + \frac{1}{3n} \right)^n \frac{\tau_0}{D} \left( 1 + \frac{6vK}{D\tau_0^n} \right)^n \quad (5)$$

Shear stress on the surface of the pipeline according to Kempieński [6]

$$\tau_w = \frac{(2\nu K)^n}{D^n \left[ \frac{n}{3n+1} - \frac{1}{3} \left( \frac{\tau_0}{\tau_w} \right)^{\frac{1}{n}} + \frac{1}{3(3n+1)} \left( \frac{\tau_0}{\tau_w} \right)^{3+\frac{1}{n}} \right]^n} \quad (6)$$

Sozański [13] presented the generalized Reynolds number  $Re_{gen}$  (7) for viscoplastic mixtures as a modification of the known Reynolds number for pseudoplastic bodies  $Re$  (8), by means of introduction of the plasticity number  $Lp$  (9), and relative error  $\delta_{\Delta p}$  (10).

The generalized Reynolds number according to Sozański [13] is determined with use of the formula (7)

$$Re_{gen} = Re \left( 1 + \frac{1}{6} Lp \right)^{-n} \left( 1 - \delta_{\Delta p} \right)^{-1} \quad (7)$$

where

$$Re = \frac{\nu^{2-n} D^n \rho}{\frac{K^n}{8} \left( 6 + \frac{2}{n} \right)^n} \quad (8)$$

The plasticity number (9)

$$Lp = \frac{D \tau_0^{\frac{1}{n}}}{\nu K} \quad (9)$$

Relative error (10)

$$\delta_{\Delta p} = \frac{\overline{\sigma_{\Delta p}}}{\frac{\Delta p}{L}} \quad (10)$$

Absolute error is determined with use of formula (11)

$$\overline{\sigma_{\Delta p}} = \frac{\overline{\Delta p}}{L} - \frac{\Delta p}{L} \quad (11)$$

where

$$\frac{\overline{\Delta p}}{L} = 4 \left( 1 + \frac{1}{3n} \right) \frac{\tau_0}{D} \left( 1 + \frac{6\nu K}{D \tau_0^{\frac{1}{n}}} \right)^n \quad (12)$$

this is an approximate value of pressure loss, while  $\frac{\Delta p}{L}$  is an exact value of pressure loss which, according to the methodology adopted by Sozański [13] can be determined numerically, e.g. by means of halving the range with use of equation (13).

$$\left(\frac{\Delta p}{L}\right)_1 = \frac{\overline{\Delta p}}{L} \left( 1 - \frac{1}{3 \left(1 + \frac{1}{3n}\right)^{1+3n} \left(1 + \frac{6\nu K}{D \tau_0^n}\right)^{1+3n}} \right) \tag{13}$$

as the first approximation of the searched value.

**Non-dimensional criterion  $\lambda(\text{Re}, \text{He})$**

Vočadlo and Charles [14] determined the friction factor  $f(\text{Re}, X)$  in form of the formula (14)

$$f = \frac{16}{\text{Re} \left\{ 1 - \left(\frac{X}{f}\right)^{\frac{1}{n}} \text{Re}^{\frac{2}{n(n-2)}} \left[ 1 + \frac{1}{3n} \left( 1 - \left(\frac{X}{f}\right)^3 \text{Re}^{\frac{6}{n-2}} \right) \right] \right\}^n} \tag{14}$$

where

$$f = \frac{\lambda}{4} \tag{15}$$

and  $\text{Re}$  is the Reynolds number as for the pseudoplastic model, determined by formula (8).

The parameter  $X$  is determined by the correlation (16)

$$X = \tau_0 2^{\frac{8-n}{2-n}} \left[ \frac{D \sqrt{\rho}}{2K_v \left(3 + \frac{1}{n}\right)} \right]^{\frac{2n}{2-n}} \tag{16}$$

Czaban [1] suggested a solution based on the correlation  $\lambda(\text{Re}, \text{He}, \beta)$ :

$$\lambda = \frac{64}{\text{Re} \left[ 1 - \frac{1+3n}{3n} \left(\frac{\beta}{\lambda}\right)^{\frac{1}{n}} + \frac{1}{3n} \left(\frac{\beta}{\lambda}\right)^{\frac{1}{n}+3} \right]^n} \tag{17}$$

$$He = \frac{\tau_0}{\rho v^2} Re^2 \quad (18)$$

where  $Re$  is, as in the solution of Vočadlo and Charles, the Reynolds number determined as in the pseudoplastic model (8), and  $He$  is the Hedström number (18),

$$\beta = \frac{8He}{Re^2} \quad (19)$$

while  $\beta$  can be calculated with use of formula (19).

### ANALYSIS OF THE RESULTS OF THE CALCULATIONS

The initial viscometric measurements enabled us to select the correct rheological model along with the determination of its parameters. This in turn allowed to conduct the calculation of the parameters of hydrotransport, in particular the Darcy friction factor  $\lambda$ , according to the methodologies discussed here above, suggested by various authors. The parameters of flow of alum sludge were calculated for an adopted pipeline diameter  $D = 200$  mm, within the range of flow rates from 0.1 to 2.4 m/s, depending on the hydration and temperature of the mixture, basing on the adopted critical Reynolds number  $Re_{cr} = 2300$ .

The values of generalized Reynolds numbers  $Re_{gen}$  and of the Darcy friction factors of the pipeline  $\lambda$ , calculated basing on the correlations presented by various authors, are presented in Tables 4, 5, 6, 7.

The results of calculation presented in form of tables confirm the similarity of methodological solutions proposed by various authors and enable us to calculate the Darcy friction factor  $\lambda$  in an unequivocal manner. The methodology developed by Kempniński [6] and Sozański [13] enables however, apart from the determination of the  $\lambda$  factor, to evaluate the nature of flow of a homogenous hydromixture (laminar or turbulent), basing on the generalized Reynolds number  $Re_{gen}$ . However, the evaluation of the regimen of flow of viscoplastic mixtures basing on the Reynolds number  $Re$  calculated as for pseudoplastic bodies may lead to significant errors [7]. Significant differences between

Table 4. Hydraulic parameters of sludge mixture of the hydration of 93.6%, at temperature  $T = 293.15$  K

v [m/s]	Kempniński		Sozański		Vočadlo		Czaban	
	$Re_{gen}$	$\lambda$	$Re_{gen}$	$\lambda$	Re	$\lambda$	Re	$\lambda$
0.1	71.5	0.8953	74.9	0.8543	1666	0.8947	1666	0.8947
0.2	274	0.2338	283	0.2261	3775	0.2319	3775	0.2319
0.4	1029	0.0622	1050	0.0609	8554	0.0618	8554	0.0618
0.6	2206	0.0290	2236	0.0286	13803	0.0290	13803	0.0290
0.8	3762	0.0170	3799	0.0168	19381	0.0170	19381	0.0170



Table 5. Hydraulic parameters of sludge mixture of the hydration of 88.05%, at temperature  $T = 293.15$  K

v [m/s]	Kempiński		Sozański		Vočadlo		Czaban	
	$Re_{gen}$	$\lambda$	$Re_{gen}$	$\lambda$	Re	$\lambda$	Re	$\lambda$
0.2	42.0	1.5222	42.4	1.5084	77.4	1.5215	77.4	1.5215
0.4	155	0.4133	155	0.4117	240	0.4132	240	0.4132
0.6	328	0.1953	328	0.1949	465	0.1953	465	0.1953
0.8	554	0.1155	555	0.1154	743	0.1155	743	0.1155
1.0	830	0.0772	830	0.0771	1070	0.0771	1070	0.0771
1.2	1150	0.0556	1151	0.0556	1441	0.0556	1441	0.0556
1.4	1514	0.0423	1514	0.0423	1854	0.0423	1854	0.0423
1.6	1918	0.0334	1918	0.0334	2305	0.0334	2305	0.0334
1.8	2360	0.0271	2361	0.0271	2794	0.0271	2794	0.0271

Table 6. Hydraulic parameters of sludge mixture of the hydration of 95.05%, at temperature  $T = 273.45$  K

v [m/s]	Kempiński		Sozański		Vočadlo		Czaban	
	$Re_{gen}$	$\lambda$	$Re_{gen}$	$\lambda$	Re	$\lambda$	Re	$\lambda$
0.2	315	0.2030	318	0.2001	1175	0.2024	1175	0.2024
0.4	1122	0.0571	1126	0.0568	2934	0.0568	2934	0.0568
0.6	2306	0.0278	2310	0.0278	5011	0.0277	5011	0.0277
0.8	3800	0.0168	3803	0.0169	7326	0.0169	7326	0.0169

Table 7. Hydraulic parameters of sludge mixture of the hydration of 88.00%, at temperature  $T = 273.45$  K

v [m/s]	Kempiński		Sozański		Vočadlo		Czaban	
	$Re_{gen}$	$\lambda$	$Re_{gen}$	$\lambda$	Re	$\lambda$	Re	$\lambda$
0.2	22.3	2.875	22.6	2.8366	44.0	2.8745	44.0	2.8745
0.4	83.4	0.7674	84.0	0.7623	138	0.7672	138	0.7672
0.6	179	0.3582	179	0.3568	270	0.3581	270	0.3581
0.8	305	0.2098	306	0.2092	435	0.2097	435	0.2097
1.0	460	0.1390	461	0.1388	629	0.1390	629	0.1390
1.2	643	0.0995	644	0.0994	850	0.0996	850	0.0996
1.4	851	0.0752	852	0.0751	1096	0.0752	1096	0.0752
1.6	1084	0.0590	1085	0.0590	1367	0.0591	1367	0.0591
1.8	1340	0.0477	1341	0.0477	1660	0.0478	1660	0.0478
2.0	1619	0.0395	1620	0.0395	1976	0.0395	1976	0.0395
2.2	1920	0.0333	1921	0.0333	2313	0.0333	2313	0.0333
2.4	2242	0.0285	2243	0.0285	2670	0.0285	2670	0.0285
2.6	2585	0.0248	2585	0.0248	3048	0.0248	3048	0.0248

$Re_{gen}$  and  $Re$  are noticeable. These differences grow with the increase of the rheological parameter  $n$ . Figures 1, 2, 3 and 4 present the diagrams of correlations  $Re_{gen}(v)$  and  $Re(v)$  that enable us to calculate the maximum acceptable values of flow rate  $v_{max}$ , that meet the conditions of laminar flow. It is clearly visible that the zone of laminar flow determined basing on the diagram of  $Re_{gen}(v)$  increases along with the decrease in hydration, thus for the hydrations 95.05% and 93.60% the acceptable actual rate  $v_{max}$  is, respectively, 0.60 m/s and 0.61 m/s, while for lower hydration values 88.05% and 88.00% this rate is, respectively, 1.78 m/s and 2.4 m/s. The acceptable range of flow rates increased 3–4 times in this case.

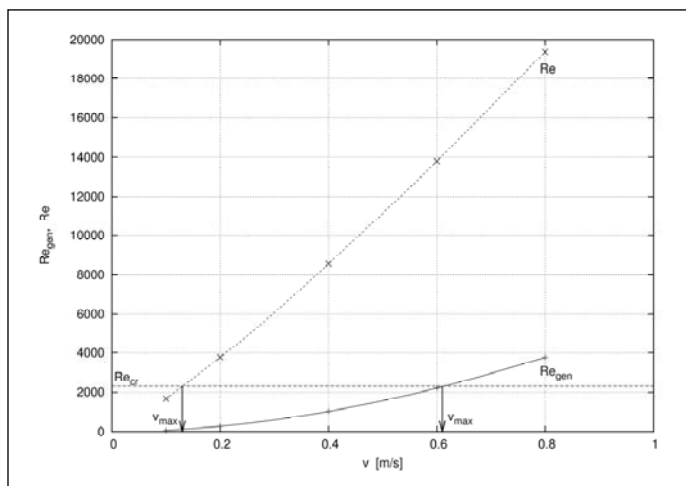


Fig. 1. Correlations  $Re_{gen}(v)$  and  $Re(v)$  for sludge of the hydration of 93.60%, at temperature  $T = 293.15$  K

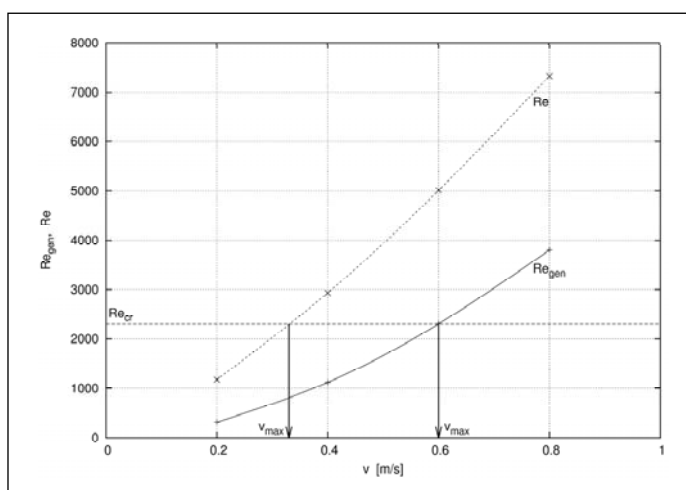


Fig. 2. Correlations  $Re_{gen}(v)$  and  $Re(v)$  for sludge of the hydration of 95.05%, at temperature  $T = 273.45$  K

The rates of laminar flow, determined basing on the correlation  $Re(v)$  narrow the zone of flow significantly, in particular for high hydration values, limiting the flow to the rate  $v_{max}$  ranging from 0.13 m/s to 0.33 m/s (Figure 1, 2). For lower hydration values the rate  $v_{max}$  ranges from 1.6 m/s to 2.2 m/s (Figures 3, 4).

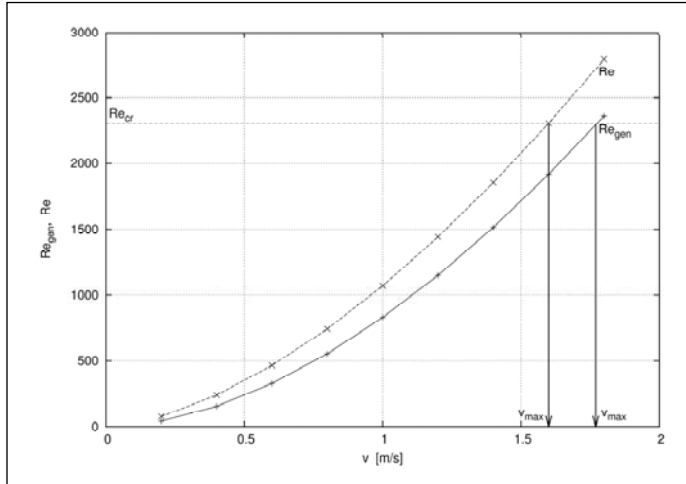


Fig. 3. Correlations  $Re_{gen}(v)$  and  $Re(v)$  for sludge of the hydration of 88.05%, at temperature  $T = 293.15$  K

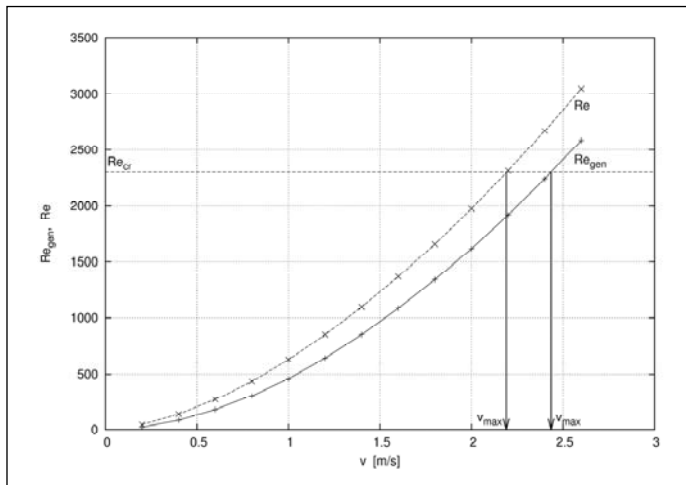


Fig. 4. Correlations  $Re_{gen}(v)$  and  $Re(v)$  for sludge of the hydration of 88.00%, at temperature  $T = 273.45$  K

The approach developed by Kempniński [6], based on indirect determination of shear stress on the surface of the pipe  $\tau_{w^2}$ , enables also to determine, with use of this parameter

and the value of the yield stress  $\tau_0$ , the radius of the core of the cross-section  $r_0$  for the laminar flow of homogenous viscoplastic hydromixture. This enables us to obtain deeper knowledge about the nature of flow of the analysed hydromixture.

## SUMMARY AND CONCLUSIONS

The description of rheological properties of homogenous viscoplastic hydromixtures should be based on tri-parametric, generalized rheological models, such as the Vočadlo model, which is simplified to simpler, bi- and uni-parametric models. The selection of the rheological model should be based on the statistical analysis of the results obtained from experimental viscometric tests.

The authors suggest to apply, for the purpose of determination of dimensions of pump and pipe installations, the generalized, non-dimensional criterion  $\lambda(\text{Re}_{\text{gen}})$ . It allows not only to determine the Darcy friction factor  $\lambda$ , but also, basing on the knowledge of the generalized Reynolds number  $\text{Re}_{\text{gen}}$ , to determine the range of acceptable flow rates ensuring the laminar flow of the analysed homogenous hydromixture.

The criterion based on the correlation  $\lambda(\text{Re}, \text{He})$ , which also enables the precise determination of the Darcy friction factor  $\lambda$ , does not allow for the evaluation of the regimen of flow. The evaluation of the flow rate range in laminar flow is another fundamental task of the designer. The lack of possibility to evaluate the critical Reynolds number  $\text{Re}_{\text{cr}}$  directly for the Vočadlo model allows the authors to adopt a generally applied value of  $\text{Re}_{\text{cr}}$  on the level 2300.

For the analysed mixtures of alum sludge the following conclusions, resulting from theoretical considerations on the parameters on their flow, may be drawn:

- the Vočadlo model describes the rheological properties of mixtures correctly;
- for the adopted pipeline of a diameter  $D = 200$  mm, the maximum rates  $v_{\text{max}}$  for laminar flow fell within the range 0.6–2.4 m/s, depending on the temperature and concentration of the mixtures;
- the Darcy friction factor varied within the range from 0.0168 to 2.875, depending on the flow rate and the temperature and hydration of the mixtures;
- the analysed theoretical methods of determination of the Darcy friction factor  $\lambda$  lead to similar obtained results of calculations and thus can be considered fully applicable in engineering calculations.

The models of applied rheology were used to describe the flow characteristics of alum sludge hydromixtures. Importance of the results described here will continuously grow with the development of water treatment plants, increase of water pollution or increasing length of pipelines for the waste conveying.

The issues of waste transport and utilization are closely related to environmental engineering in general [3] and, particularly, to sediment management at water treatment plants and sewage treatment plants [5, 15]. The presented work expands the area of applicable knowledge of hydrotransport methods in design of sludge management equipment.

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#### CHARAKTERYSTYKA LAMINARNEGO PŁYNIĘCIA W RUROCIĄGACH JEDNORODNYCH OSADÓW POKOAGULACYJNYCH APROKSYMOWANYCH MODELEM CIECZY PLASTYCZNO-LEPKICH VOČADLI

Praca przedstawia sposoby określenia współczynnika oporów rurociągu  $\lambda$ , dla jednorodnej hydromieszaniny osadów pokoagulacyjnych o różnym uwodnieniu i różnej temperaturze, dla laminarnej strefy przepływu. Ocenę reologiczną hydromieszaniny, jako ciała plastyczno-lepkiego, wykonano na podstawie pomiarów wiskozymetrycznych, przy zastosowaniu do aproksymacji krzywych płynięcia uogólnionego modelu Vočadli. Określenie współczynnika oporu rurociągu  $\lambda$  przeprowadzono z wykorzystaniem bezwymiarowego kryterium  $\lambda(\text{Re}_{\text{gen}})$  oraz  $\lambda(\text{Re}, \text{He})$ .