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Novel Method of Physical Modes Generation for Reduced Order Flow Control-Oriented Models

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Abstract

Physical flow modes are of particular interest for Reduced Order Flow Control-Oriented Models. Computation of physical modes as the eigensolution of linearized Navier-Stokes equations is a cumbersome and difficult task, especially for large, 3D problems. Instead we propose the solution of Navier-Stokes equation in the frequency domain and investigation of the system response to local or global perturbation. The flow variables are perturbed around steady basic state and the system response is used to construct mode basis suitable for ROMs.

Keywords: Navier-Stokes equation, eigenmodes, Reduced Order Modelling, flow control.

1. Introduction

The Reduced Order Models (ROMs) of flow are often based on Galerkin Method [1]. This method strongly depends on flow mode basis. In flow modelling we can employ mathematical modes, empirical ones or eigenmodes of linearised system. The use of mathematical modes is rather a hypothetical solution as the mode basis can be hardly defined for general flow conditions. It has been proven that adequate use of both, empirical and physical modes assures high dynamical quality of the flow model [2].

There are many well established methods to generate the empirical modes basis. Traditionally, Proper Orthogonal Decomposition (POD) [3,4] and its modifications are used for this purpose. Recently Dynamic Mode Decomposition (DMD) [5,6] being the dynamical system identification method is widely used. In the same time there is a substantial progress in eigensolution of linearized Navier-Stokes equations [7,8,9] but eigensolution of generalized, non-hermitian, complex eigenvalue problem remains a cumbersome and difficult task. It is particularly pronounced for discretized 3D flow problems, described by systems of $(0)10^6$ Degrees of Freedom and requiring the eigensolution of such large eigenvalue problems.

We present here an alternative, novel method of physical modes generation. It is based on solution of linearized disturbance equation in frequency domain. Flow variables are perturbed around steady basic state. Flow responses to random or localized volume forces characterized by assumed frequencies closely resemble eigenmodes.

2. Governing equations

The incompressible fluid motion is described by the unsteady Navier-Stokes equation in the form:

$$\dot{V}_i + V_{i,j}V_j + P_{,i} - \tfrac{1}{Re}V_{i,jj} = 0 \tag{1}$$

The incompressibility condition is expressed by the continuity equation:

$$V_{i,i} = 0 \tag{2}$$

The Reynolds number is defined as:

$$Re = \tfrac{UL}{\mu} \tag{3}$$

where U is characteristic velocity, L characteristic length and μ kinematic viscosity of the fluid.

We assume that the unsteady solution of the Navier-Stokes equation (1) can be expressed as the sum of its steady solution and the disturbance:

$$\begin{aligned} V_i &= \bar{V}_i + \acute{V}_i \\ P &= \bar{P} + \acute{P} \end{aligned} \tag{4}$$

This assumption leads us to the disturbance equation, in the form:

$$\dot{\acute{V}}_i + \acute{V}_j\bar{V}_{i,j} + \bar{V}_j\acute{V}_{i,j} + \acute{V}_j\acute{V}_{i,j} + \acute{P}_{,i} - \tfrac{1}{Re}\acute{V}_{i,jj} = 0 \tag{5}$$

Further we assume small value of the disturbance and linearize equation (5). In the disturbance equation we separate the time and space dependence

$$\begin{aligned} \acute{V}_i(x,y,z,t) &= \tilde{V}_i(x,y,z)\, e^{\lambda t} \\ \acute{P}(x,y,z,t) &= \tilde{P}(x,y,z)\, e^{\lambda t} \end{aligned} \tag{6}$$

With introduction of equation (6) to the linearized disturbance equation (5) we obtain a differential eigenvalue problem having form:

$$\begin{aligned} \lambda\tilde{V}_i + \tilde{V}_j\bar{V}_{i,j} + \bar{V}_j\tilde{V}_{i,j} + \tilde{P}_{,i} - \frac{1}{Re}\tilde{V}_{i,jj} &= 0 \\ \tilde{V}_{i,i} &= 0 \end{aligned} \tag{7}$$

Discretization of (7) gives:

$$\lambda Bx + Ax = 0 \tag{8}$$

This equation represents the generalized complex, non-hermitian eigenvalue problem. The number of DOFs for (8) is usually very large, of order of $(0)10^5$ for two-dimensional problem and $(0)10^6$ for three-dimensional one.

Particularly three-dimensional eigensystem is a challenging numerical problem to be solved. Eigenvalues are often complex conjugate pairs what causes additional problems for solution algorithms.

Instead of direct solution of (9) we investigate frequency response function having form:

$$Ax_{Re} + \lambda_{Re} B x_{Re} + \lambda_{Im} B x_{Im} = F_{Re}$$
$$Ax_{Im} + \lambda_{Re} B x_{Im} - \lambda_{Im} B x_{Re} = F_{Im} \qquad (9)$$

to localized or random forcing. In equation (9) we split real and imaginary part of the equation to use real value computer algebra.

3. Flow solver for 2D and 3D computations

Solution of disturbance equation in frequency domain required development of adequate numerical solver. It is based on our earlier UNS3 system (MF3 for structural problems) widely used in flow stability, control and Reduced Order Modelling [1]. UNS3 is based on unstructured FEM in penalty formulation and employs second and third order triangular (2D) or tetrahedral elements (3D).

The three-dimensional version of the program is parallelized and based on METIS [10] domain partitioning. UNS3 scales linearly up to hundreds of CPUs enabling computation on grids having several millions of grid points. Two-dimensional solver uses scalar, single processor code.

For purpose of frequency domain computation the Finite Element has doubled (for real and imaginary part) number of DOFs in comparison to our regular Navier-Stokes computations. In Figure 1 example grid partitioning to 16 domains, used later for the computations of a flow around sphere is shown.

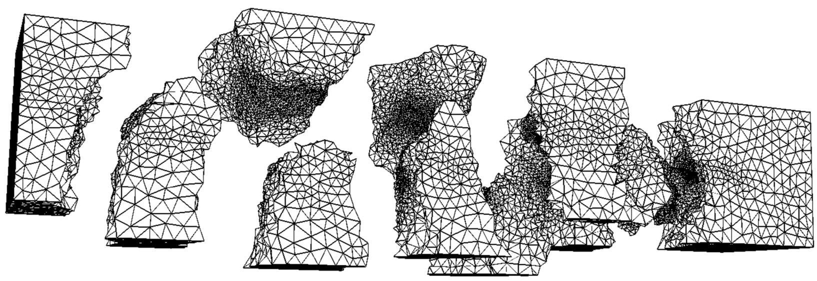

Figure 1. Finite Element (FEM) grid, domain partitioning and grid refinement for flow around a sphere

4. Numerical results

Solution of disturbance equation requires firstly steady basic solution of the Navier-Stokes equations. It has been obtained with classical version of UNS3 code. In Figure 3 the example steady solution for flow around the sphere at Re=250 is shown.

In Figure 2 we present real and imaginary parts of the modes. It is relatively easy to obtain von Karman mode as depicted in the first row. It is the response of the flow for random forcing and the complex value of (0.1 + i 0.87) yields neutral stability corroborating the results of flow stability analysis based on eigensolutions of the equation (8). In the second row the response to point volume force at different frequencies and locations is shown.

With the 2D results, closely following earlier stability analysis we apply the method to three-dimensional flows. For 3D flow the stability investigations are much more rare [8].

The results for the flow around a sphere at Re=250 are depicted in Figure 3. The solution is obtained for random perturbation of the flow and develops around steady solution shown in the left part of the figure. Dominating mode is shown with the use of Lambda2 criterion. It shows characteristic periodicity of the flow and spreading angle of the shedding vortices.

It should be noted that dominating mode can be relatively easy retrieved also with DMD or POD method. In the case of POD physical modes can be found by analysis of time history of slightly perturbed steady solution.

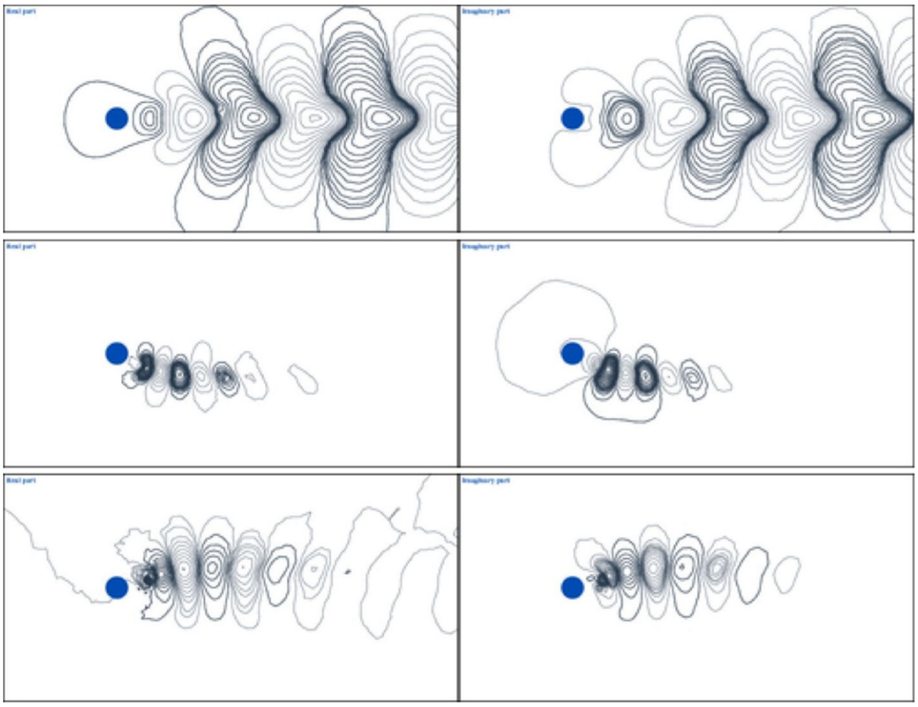

Figure 2. Real (left) and imaginary (right) part of the mode for different forcing of the flow around two-dimensional circular cylinder at Re = 100

While for finding the dominating mode any of the mentioned method can be used, physical modes characterized by higher frequencies are difficult to determine. DMD method shows Fourier modes, doubling the mode frequency and POD determines energetically optimal modes for the limit cycle being, however, not physical ones.

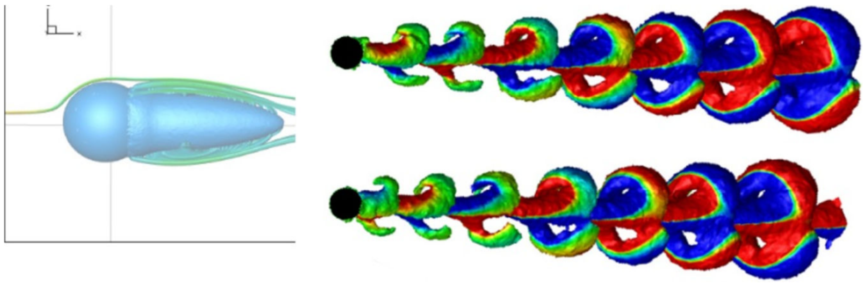

Figure 3. Steady flow around a sphere at Re=250 (left). Streamlines and wake contour is shown (isosurface of Vx = 0). Right: real (top) and imaginary (bottom) part of the dominating shedding mode visualized with the Lambda2 criterion

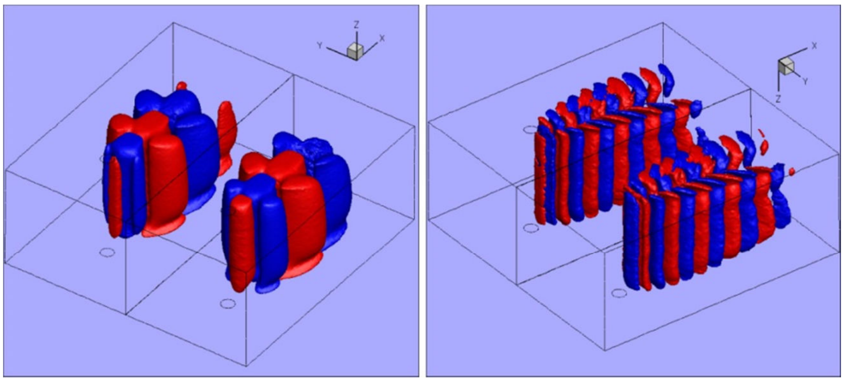

Figure 4. Real and imaginary part of modes for the flow around the wall-mounted cylinder. Left: von Karman mode, right: higher, shear-layer mode

In the next Figure 4 we show both, dominating Karman mode and higher one, determined with the presented here method. Flow develops around a wall-mounted cylinder.

In Figure 5 the Lambda2 visualization (top view) of unsteady flow is also shown as the reference.

5. Conclusions

We presented a novel method of finding physical modes for complex, two- and three-dimensional flows with the use of frequency domain solutions of linearized Navier-Stokes equations. It has been shown that the method enables computation of dominant modes as well as higher frequency ones. The modes determined with this method will serve as the basis for Reduced Order Models of the flow. The method can be also applied for investigation of effect of flow actuators. Both placement and actuation character can be modelled with the presented method. In this way flow control can be more effectively planned.

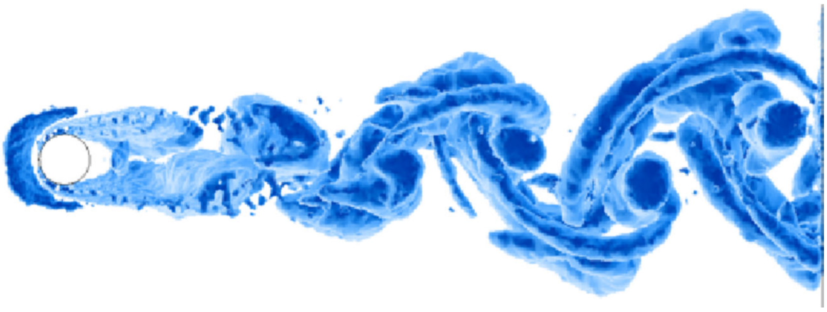

Figure 5. Unsteady flow around wall-mounted cylinder (top view) visualized with the Lambda2 criterion

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References

1. B.R. Noack, M. Morzyński, G. Tadmor. *Reduced-Order Modelling for Flow Control.* Number 528. Springer, 2011.
2. B.R. Noack, K. Afanasiev, M. Morzyński, G. Tadmor, F. Thiele, *A hierarchy of low-dimensional models for the transient and post-transient cylinder wake,* J. Fluid Mech., **497** (2003), 335-36
3. P. Holmes, J. Lumley, G. Berkooz, *Turbulence, Coherent Structures, Dynamical Systems and Symmetry,* Cambridge University Press, Cambridge, 1998.
4. L. Sirovich, *Turbulence and the dynamics of coherent structures.* Quart. Appl. Math., **45** (1987), 561–590.
5. P.J. Schmid, J. Sesterhenn*, Dynamic Mode Decomposition of Numerical and Experimental Data,* J. Fluid Mech., **656** (1) (2010), 5-28.
6. Ch.J. Mack, P.J Schmid. *Global stability of swept ow around a parabolic body: features of the global spectrum*. J. Fluid Mech., **669,** (2011) 375-396.
7. M. Morzyński, K. Afanasiev, F. Thiele, *Solution of the eigenvalue problems resulting from global non-parallel flow stability analysis,* Comput. Meth. Appl. Mech, Engrng., **169** (1999), 161-176.
8. V. Theofilis. *Global linear instability* Annual Review of Fluid Mechanics, **43**, (2011), 319-352.
9. W. Stankiewicz, M. Morzyński, B.R. Noack, F. Thiele, *Stability Properties Of 2D Flow Around Ahmed Body,* Math. Model. Analysis (2005), 129-134.
10. G. Karypis, V. Kumar, *A Fast and Highly Quality Multilevel Scheme for Partitioning Irregular Graphs,* SIAM Journal on Scientific Computing, Vol**. 20**, (1), (1999), 359-392.