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Łukasz Kutrowski, Mateusz Miara
Czestochowa University of Technology
Institute of Mechanics and Machine Design Fundamentals
ul. Dąbrowskiego 73, 42-201 Częstochowa
e-mail: kutrowski@imipkm.pcz.pl, miara@imipkm.pcz.pl

STUDIES OF THE SYSTEM: BEAM - VARIABLE MASS ELEMENT MODELING OVERHEAD CRANE

Abstract. This paper presents the free vibration problem of beam with additionally attached mass element. The considered system can be treated as simplified to two-dimensional model of the bridge crane. In this model the displacement of the hoist along the girder is analyzed and the longitudinal motion of the bridge crane does not occur. Based on the Hamilton's principle, the boundary problem for this system has been formulated. On the basis of numerical simulations the influence of the position of the mass element (representing the hoist and mass of the load) on the free vibration frequencies of the system has been presented. The accepted mathematical model has been verified by the experimental studies.
Keywords: bridge crane, free vibrations, beam.

BADANIA UKŁADU: BELKA – ZMIENNA MASA SKUPIONA MODELUJĄCEGO SUWNICĘ

Streszczenie. W niniejszej pracy rozważano problem drgań swobodnych belki z dodatkowo zamocowanym elementem masowym. Układ ten może modelować konstrukcję suwnicy pomostowej, przy założeniu pewnych uproszczeń. Sprowadzono model obiektu rzeczywistego do zagadnienia dwuwymiarowego, w którym rozważa się możliwość przemieszczania wciągnika wzdłuż dźwigara, a ruch wzdłużny suwnicy nie występuje. Sformułowano zagadnienie brzegowe odnośnie do drgań układu na podstawie zasady Hamiltona. Przedstawiono symulacje numeryczne określające wpływ położenia elementu masowego (który reprezentuje układ wciągnika wraz z ładunkiem) na częstości drgań własnych układu. Przeprowadzono badania eksperymentalne odnośnie do drgań rozważanego układu, pozwalające na weryfikację przyjętego modelu matematycznego.

Słowa kluczowe: suwnica, drgania swobodne, belka.

Introduction

The free vibrations of the beams are one of the fundamental problem considered in the analysis of the vibration of structural elements. The vibrations of the beam can be considered as a discrete or continuous system, depending on the description [11, 12] of the problem. To the analysis of the beam systems the Bernoulli-Euler theory, in which is assumed that the segment before and after deformation is straight and perpendicular to the axis of the beam, is applied. The second method to formulation and solution of the free vibration problem of the beams is Timoshenko theory. This theory additionally takes into account the influence of lateral forces on deflection and rotational inertia [16].

The overhead cranes are one of the types of the cranes operating in intermittent mode, which are equipped with a mechanism to hoisting and lowering the load. The hoist used to displacement of the load vertically and horizontally. Moreover the overhead crane can move along its bridge deck (the work of the structure in three-dimensional space) [1, 14, 15]. Hoisting, lowering, and operation of electric/hydraulic motors, mechanical transmissions, etc. can affect on the vibration of the structure and in a consequence can influence on the strength of the construction and operator comfort (if the crane handling station is directly connected to its structure). For this reason, during the designing of this type structures the vibration analyses are important. The problem of vibration of the overhead cranes was analyzed in the papers [7, 8, 13]. In the paper [7] the impact of hoisting (hoist system) on crane vibration was presented. Results of experimental and numerical vibration of bridge cranes have been included. In the papers [8, 13], the vibration studies of the overhead cranes in which the dynamical properties has been taken account were carried out.

In this paper, the beam with additional mass element has been considered. The Bernoulli-Euler theory has been used to formulation and solution of the vibration of the continuous-discrete system. Presented beam-mass system were analyzed by many researchers. Results of numerical calculations concerning free vibrations analysis of beam carrying mass element under different boundary conditions were presented in works [2-4, 10]. Two theories of beam vibrations were compared in [6]. Also non-linear vibrations were analyzed for different boundary conditions and results of numerical simulations and experimental analyzes were presented in papers [9, 10]. In this work presented model can be treated as the simplified bridge crane. The influence of the position of the mass element (hoist and mass of the load) on the free vibrations of the system was analyzed. Numerical simulations and experimental studies were performed for verification of accepted mathematical model. Also the nu-numerical results were presented for sample parameters of bridge cranes.

Theoretical formulation and solution of the free vibration problem

Scheme of the analyzed single-girder overhead gantry system is shown in Fig. 1. This system consists of a pivotally mounted on both sides girder (l_1+l_2) and a hoist (a body mass m and a mass moment of inertia I_b).

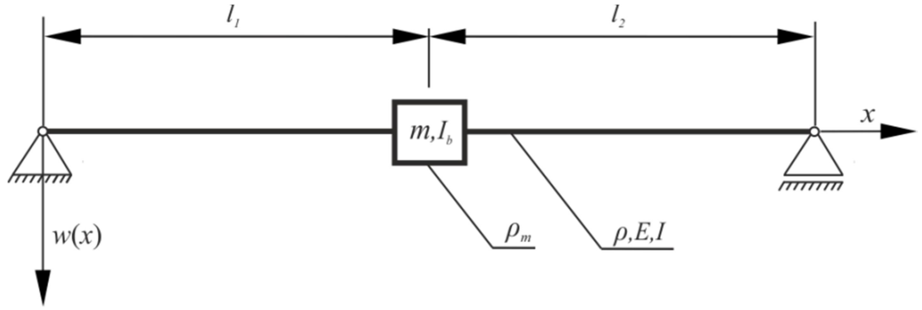


Fig 1. Scheme of the overhead crane

The formulation of the boundary problem with respect to the system's own vibrations was based on the Hamilton principle [4,5]:

$$\int_{t1}^{t2}(T-V)dt=0. \tag{1}$$

Potential and kinetic energy can be written as:

$$V=\frac{1}{2}EI\sum_{i=1}\int_0^{l_i}\left(\frac{\partial^2 w_i\left(x_i\right)}{\partial x_i^{\,2}}\right)^2 dx_i\,, \tag{2a}$$

$$T=\frac{1}{2}\rho A\sum_{i=1}\int_0^{l_i}\left(\frac{\partial w_i\left(x_i,t\right)}{\partial t}\right)^2 dx_i+\frac{1}{2}m\left(\frac{\partial w_1\left(x_1,t\right)}{\partial t}\bigg|^{x_1=l_1}\right)^2+\frac{1}{2}I_b\left(\frac{\partial^2 w_1\left(x_1,t\right)}{\partial x_1\partial t}\bigg|^{x_1=l_1}\right)^2$$

(2b)

Geometrical boundary conditions are presented as follow:

$$w_1(0,t)=0\ ;\ w_2(l_2,t)=0\ ;\ w_1(l_1,t)=w_2(0,t) \tag{3a-d}$$

$$\left.\frac{\partial w_1(x_1,t)}{\partial x_1}\right|^{x_1=l_1} = \left.\frac{\partial w_2(x_2,t)}{\partial x_2}\right|^{x_2=0}$$

Taking into account equations (2a, b) and (3a, b) in Hamilton principle, after completing the necessary mathematical transformations, the equations of motion and the natural boundary conditions have been obtained. After separation of variables, the equations of motion are presented as follow:

$$-EI\frac{d^4Y_i(x_i)}{dx_i^4} + \rho A\omega^2 Y_i(x_i) = 0. \tag{4}$$

Natural boundary conditions after separation of variables are:

$$EI\left.\frac{d^2Y_1(x_1)}{\partial x_1^2}\right|^{x_1=0} = 0 \;;\; -EI\left.\frac{d^2Y_2(x_2)}{\partial x_2^2}\right|^{x_2=l_2} = 0,$$

$$EI\left.\frac{d^3Y_1(x_1)}{dx_1^3}\right|^{x_1=l_1} - EI\left.\frac{d^3Y_2(x_2)}{dx_2^3}\right|^{x_2=0} + m\omega^2 \left.Y_1(x_1)\right|^{x_1=l_1} = 0, \tag{5a-d}$$

$$-EI\left.\frac{d^2Y_1(x_1)}{dx_1^2}\right|^{x_1=l_1} + EI\left.\frac{d^2Y_2(x_2)}{dx_2^2}\right|^{x_2=0} + I_b\omega^2\left.\frac{dY_1(x_1)}{dx_1}\right|^{x_1=l_1} = 0.$$

The solution of equations (4) can be written as:

$$Y(x) = A\sin(kx) + B\cos(kx) + C\sinh(kx) + D\cosh(kx), \tag{6}$$

where: $k = \sqrt[4]{\dfrac{\rho A\omega^2}{EI}}$.

Taking into account the solution (6) in the boundary conditions, the system of equations, which the matrix determinant of coefficients equated to zero leads to the transcendental equation and the natural frequency of the system can be determined, is obtained.

Block diagram of the operation of the numeric program

In this paper, program for the initial calculation of free vibrations of bridge cranes is presented as a block diagram (Fig. 2). The first action is to run the program. The next step is entering the data in form of input parameters. These parameters are: E - Young modulus, ρ - density of girder material, l - girder length, l_1 - hoist distance from crane road, ρ_m - hoist material density, dimensions of girder cross-sections (for typical crane girders cross-sections), m – hoist weight and mass of the load, I_b - mass moment of inertia of the hoist and the load. In the next stage the program calculates: I - geometrical moment of inertia of the girder, A - cross-sectional area of the girder. On the basis of Hamilton's principle, matrix of coefficients is created by the program. This matrix is called matrix **C**, whose elements are filled on the basis of boundary conditions for ω = 0. By the implementation of the variable ω, the determinant of matrix **C** is calculated. When the value of the determinant of the matrix **C** equals 0, the following natural frequencies are obtained. In order to better present the operations performed in the program, the block diagram is presented (see Fig. 2).

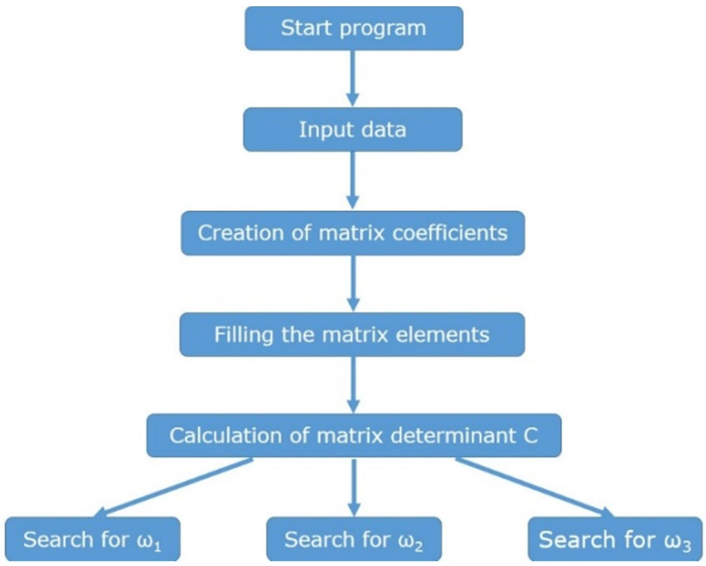


Fig 2. Block diagram for free vibration calculation

Experimental stand

The experimental studies of free vibration of the analyzed system were conducted on a testing rig WP120 (Fig. 3 – on the left) to stability analysis of slender systems manufactured by GUNT and Brüel & Kjaer vibration analyzer with LABScope software (Fig. 3 – on the right). This stand allows to attach the beams with a certain cross-section in a manner that has been included in the mathematical model.

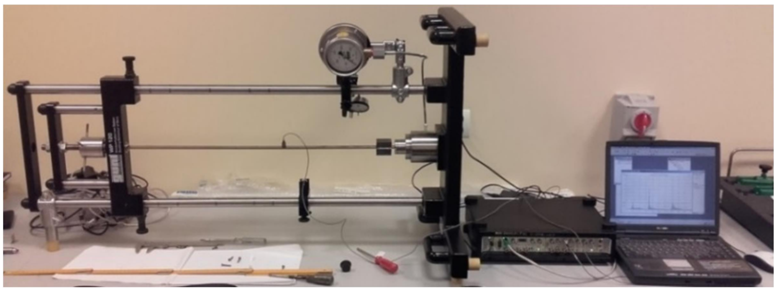


Fig 3. Testing rig for free vibration research

Two different beams and three different mass elements have been analyzed in the experimental studies. The intersection of the beam and mass element is shown in Fig. 4 and the dimensions are given in Table 1.

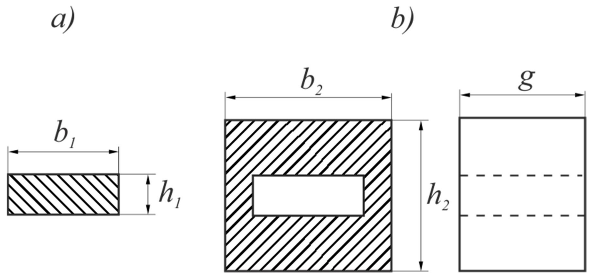


Fig 4. Sections of analyzed a) beam, b) masses

Table 1. Dimensions, properties of beams and masses

Beams					
	l **[m]**	b_1 **[m]**	h_1 **[m]**	E **[Pa]**	ρ **[kg/m^3]**
B1	0.7	20	4	$2.1 \cdot 10^{11}$	7800
B2	0.5	20	4	$2.1 \cdot 10^{11}$	7800
Mass elements					
	g **[m]**	b_2 **[m]**	h_2 **[m]**	E **[Pa]**	ρ **[kg/m^3]**
M1	0.04	0.04	0.14	$2.1 \cdot 10^{11}$	7800
M2	0.04	0.04	0.24	$2.1 \cdot 10^{11}$	7800
M3	0.04	0.04	0.34	$2.1 \cdot 10^{11}$	7800

Numerical and experimental results

On the basis of presented mathematical model and with the help of experimental stand the numerical and experimental studies have been performed. The experimental results confirmed the correctness of theoretical model. The verified mathematical model has been also used to calculate of free vibration frequencies of the model, which can substitute the bridge crane.

Experimental verification of the mathematical model

The influence of the change of the position of the mass element (mass and mass moment of inertia) attached to the beam (Fig. 1, 2) on the free vibration frequencies has been shown in Fig. 5 and 6.

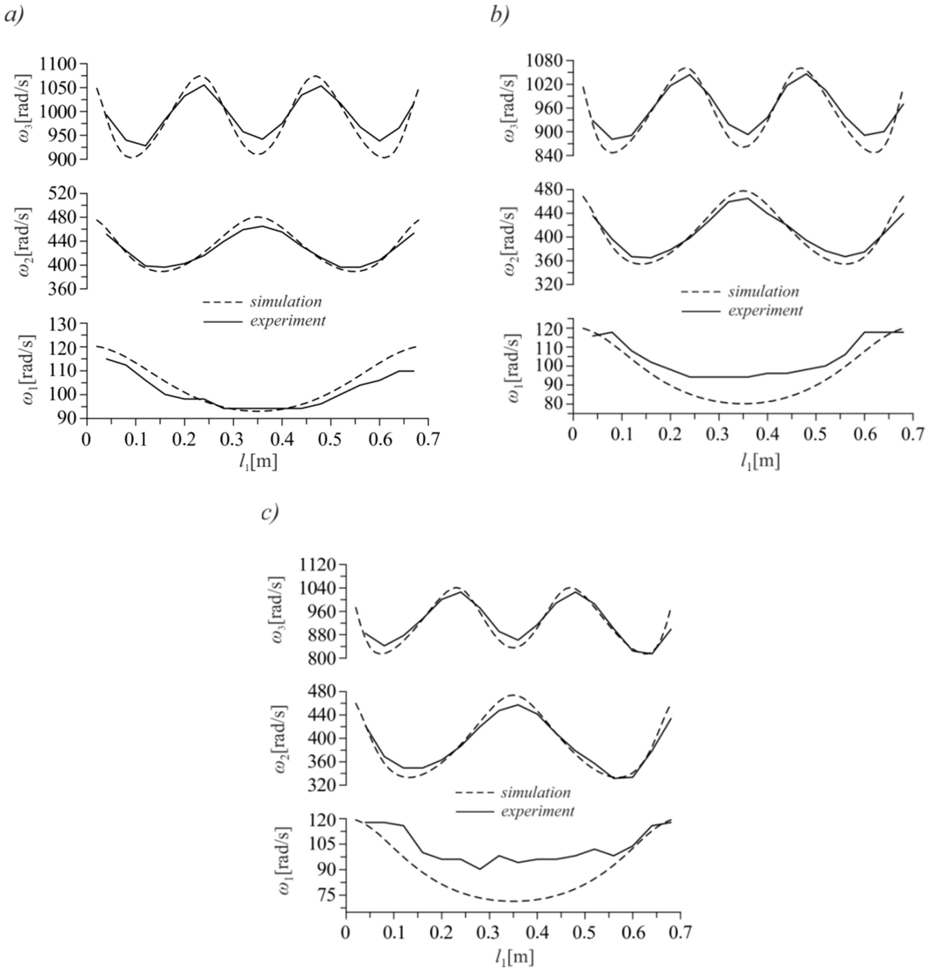


Fig. 5. Influence of change of position of the mass element on values of 1, 2, 3 of the free vibration frequencies for beam B1: a) M1, b) M2, c) M3

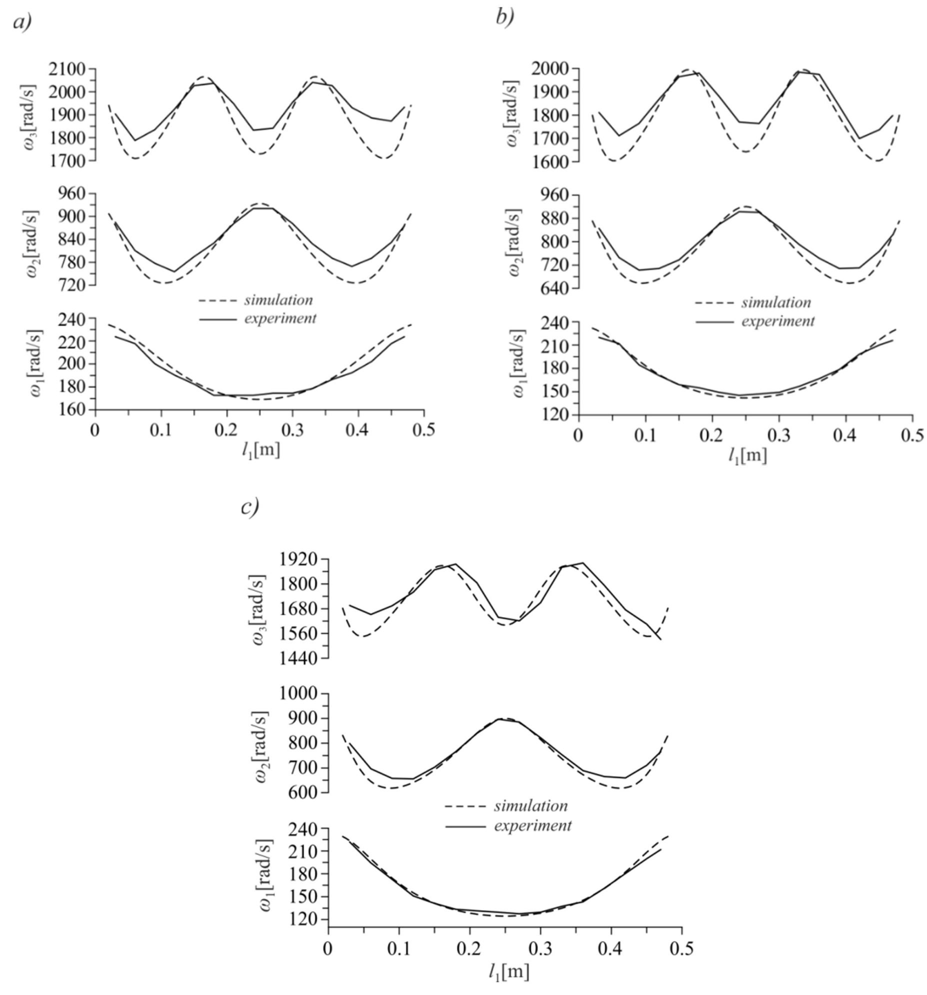


Fig. 6. Influence of the change of position of the mass element to the values of 1,2, 3 of the free vibration frequencies for beam B2: a) M1, b) M2, c) M3

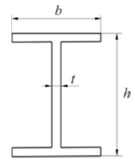
On the basis of presented results, it has been stated that the correlation between numerical and experimental results for the first eigenfrequency is higher for B2. For the second and third frequencies the correlation is similar. The correlation between mathematical model and the real construction was higher for system with smaller slenderness ratio.

Presented mathematical model and prepared numerical program can be used for different vibration problems, for instance the vibrations of the bridge crane can be analyzed.

Free vibration frequency of the bridge crane

The verified mathematical model has been used to solution of vibration problem of the real bridge crane (the parameters are shown in Table 2).

Table 2. Dimensions, properties of beams in real case

Beams							
	l **[m]**	***b*** **[m]**	***h*** **[m]**	***t*** **[m]**	***E*** **[Pa]**	$\boldsymbol{\rho}$ **[kg/m^3]**	**Section**
C1	15	0.2	0.5	0.016	$2.1 \cdot 10^{11}$	7800	
C2	7.5	0.1	0.2	0.0085	$2.1 \cdot 10^{11}$	7800	

Results for the real parameters of the girders (C1 and C2) are shown in Figures 8–9.

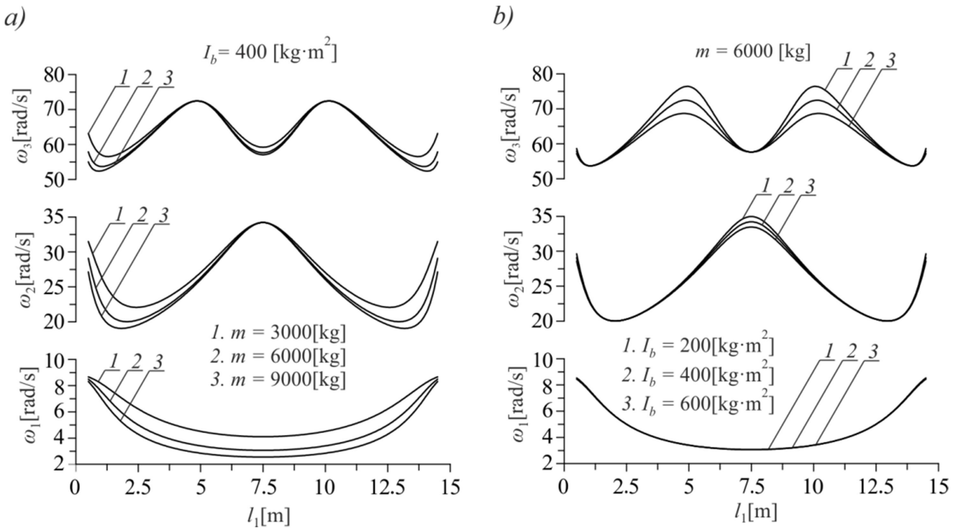


Fig. 8. Influence of change of position of mass element to values of 1, 2, 3 of vibration frequencies for beam C1: a) mass change, b) change of mass moment of inertia

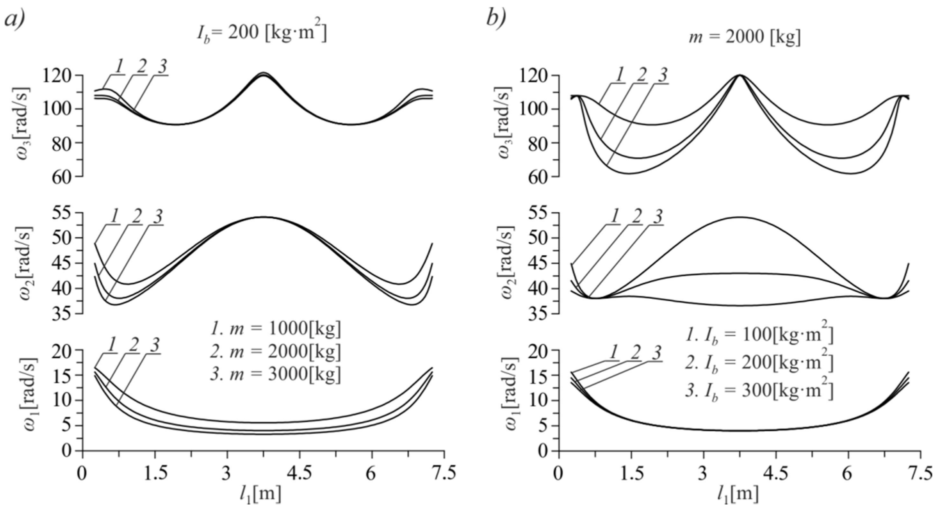


Fig. 9. Influence of change of position mass element to values of 1, 2, 3 of vibration frequencies for beam C2: a) mass change, b) change of mass moment of inertia

It has been observed that the increase of the mass of the element modeling trolley triggers the decrease of the eigenfrequencies, in particular when the trolley is between the nodes of the mode of free vibration frequencies. In the case of the increase of mass moment of inertia, the frequencies decrease, especially in the places close to the mode nodes.

Summary

The free vibrations of the beams are one of the main problems of the mechanical vibration analyses. Within the scope of this work the formulation and solution of the free vibration problem of the beam with additional mass element have been performed. The accepted theoretical model can substitute the real bridge crane. Numerical and experimental studies on vibration of the analyzed system were performed and the mathematical model was verified. Two beams with different lengths were used due to experimental tests. The effect of the change of position, mass and mass moment of inertia of the hoist on the free vibration frequencies of the system was shown. Numerical calculations were also performed for dimensions of real bridge crane. The carried out numerical program can be used by constructors to pre-determine the eigenfrequency of the bridge cranes (or similar structures), especially if they can be exposed to vibration at specified frequencies.

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