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# Predicting Specific Stress of Cotton Staple Ring Spun Yarns: Experimental and Theoretical Results

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**Abstract**
*The aim of this research is to predict the yarn specific stress from fiber specific stress and fiber stress utilisation. In this paper a new approach is introduced to predict the specific stress-strain curves of cotton carded and combed yarns. The force on single fiber is worked out and these fiber forces are combined together to obtain forces acting on yarn. The theoretical model introduces the utilisation of fiber stress on the basis of the fiber specific stress-strain curve, twist angle, fiber directional distribution parameter C and contraction ratio. A comparison of experimental results suggests that the specific stress-strain curves predicted have reasonable agreement with the experimental yarn specific stress-strain curves for all types of yarns. Thus this model is valid to predict the specific stress-strain curves for carded and combed cotton ring spun yarns.*

**Key words:** *fiber stress utilisation, yarn specific stress, fiber specific stress, carded and combed cotton yarns.*

## Introduction

The tensile strength, being an important quality parameter of yarn, has been widely studied and still comprises an extensive part of literature on textile yam mechanics. Staple spun yarn study is considered important because of their bulk application in service fabrics. Therefore to understand the mechanical behaviour and predict the strength of staple spun yarns are very important from a technology point of view.

However, study of the influence of yarn geometry on yarn performance is becoming exceedingly complicated due to difficulties in the investigation of fiber to fiber friction as well as the variation in internal pressure, fiber strength, length and crimp. Although staple spun yarns have a simple structure, they still possess very complex geometry, prominently influenced by the method of manufacture.

The prediction of yarn properties from fiber properties and process parameters has been attempted by various researchers over the years. Research on the mechanical behavior of textile yams has been popular in the textile community and history of modern textile processing machinery for two hundred years [1]. Nevertheless proper research work was conducted in the early 1900's to establish theoretical relationships among fiber properties, textile structural factors and material behaviour. It can be dated back to the work of Gegauff [2], Gurney [3] and Peirce [4], as the foundation for modern textile mechanics. Sullivan [5] developed relations for yam strength, fiber properties and yam twist, but his theory was only applicable for low twist yarns. Platt [6] developed an equation for the strength of filament yarns and then modified it for staple yarns. Furthermore Gregory [7-9] modified Sullivan's formula for calculation of the maximum strength. Hearle [10, 11] published a theory considering the lateral pressure among fibers, where he assumed small strains, and that fiber deformation obeys Hooke's law and there is no lateral contraction. Later on he proved that it is possible to study the mechanics of staple yarns considering the effect of twist, fiber migration and discontinuities at fiber ends. Furthermore he modified his theoretical model for the strength of staple yarns and considered lateral contraction and large strains. Treloar [12] applied the rubber model and concluded that migration has a minor effect on the tensile strength of yarn. Zurek et al [13] developed a theoretical model for predicting yarn strength and strain from fiber fineness, length, stress and strain. Pan [14-16] developed a constitutive theory for short fiber twisted yarns with and without slippage and developed a relation for the strength of staple spun yarns. Gosh et al. [17] developed a mathematical model to predict the strength of spun yarns based on the failure mechanism of spun yarns. Most of these researchers established a mathematical approach to study the yarn structure from the fiber properties and process parameters.

The main aim of this research is the prediction of yarn specific stress-strain

### *List of abbreviations:*

| | |
|---|---|
| $\varepsilon_f$ | *Fiber strain* |
| $\varepsilon_Y$ | *Yarn strain* |
| $\beta_D$ | *Maximum twist angle* |
| $\beta$ | *Twist angle* |
| $\eta$ | *Contraction ratio* |
| $\varphi_g$ | *Fiber stress utilisation from Gegauff* |
| $s$ | *Fiber cross section area* |
| $F_Y$ | *Component of fiber force in yarn direction* |
| $dr$ | *Radius of differential annulus of yarn at elemental radius r.* |
| $D$ | *Yarn diameter* |
| $Z$ | *Yarn twist* |
| $P_Y$ | *Yarn axial force* |
| $P_f$ | *Fiber axial force* |
| $\mu$ | *Yarn packing density* |

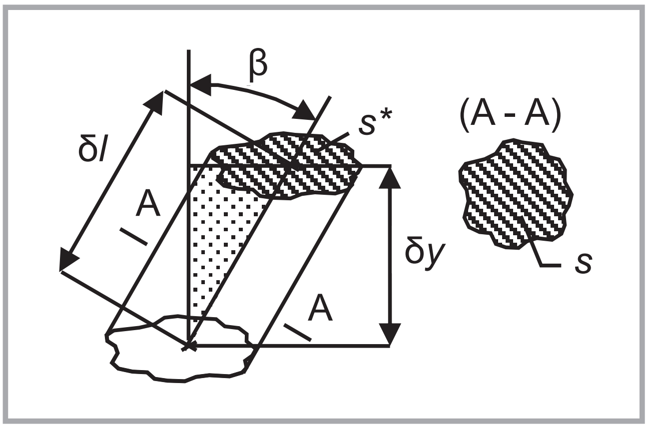


***Figure 1.*** *One fiber element.*

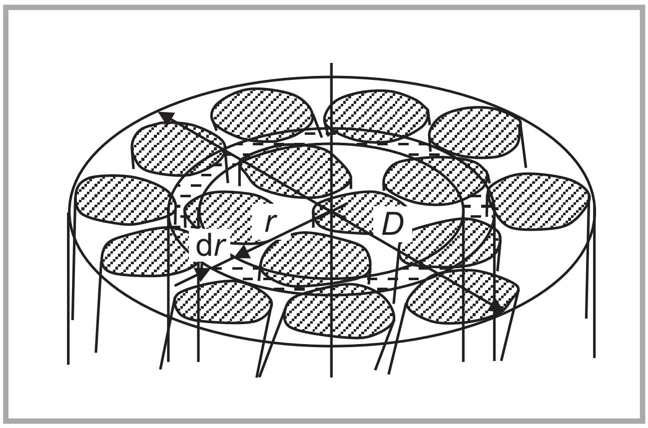


***Figure 2.*** *Differential layer in the yarn cross-section.*

curves and their validation with experimental results for staple spun carded and combed cotton yarns. In the theoretical part, Gegauff's ***Equation (3)*** is modified to determine the utilisation of fiber stress at each value of strain. Furthermore the direct delta function was introduced in the derived ***Equation (16)*** to minimise the effect of the directional distribution of fiber in the yarn. A rapid and scientific approach has been introduced for the prediction of specific stress-strain curves for staple spun yarn before the process of breaking from knowledge of the fiber stress-strain curve and twist angle using Matlab software.

## Theoretical model

Fiber strain ($\varepsilon_f$) in an assembly of fibers twisted at some arbitrary angle can be calculated using the following relation [2]:

$$(1 + \varepsilon_f)^2 = 1 + 2\varepsilon_Y (\cos^2\beta - \eta\sin^2\beta) + \varepsilon_Y{}^2 (\cos^2\beta + \eta^2\sin^2\beta) \quad (1)$$

On simplification:

$$\varepsilon_f + \frac{\varepsilon_f{}^2}{2} = \varepsilon_Y (\cos^2\beta - \eta\sin^2\beta) + \frac{\varepsilon_Y{}^2}{2} (\cos^2\beta + \eta^2\sin^2\beta)$$

It can be shown that at very low yarn strains (*ca.* 0 – 0.1), the second term on the right and left hand side in the expression above becomes negligible, while the first term is little affected. Then the expression above can be written as follows:

$$\varepsilon_f = \varepsilon_Y (\cos^2\beta - \eta\sin^2\beta) \quad (2)$$

Then, if we know the maximum twist angle ($\beta_D$) in yarns, the fiber stress utilisation ($\varphi_g$) can be calculated using the following, well known, Gegauff's equation:

$$\varphi_g = (1 + \eta)\cos^2\beta_D + \frac{\eta(\ln\cos^2\beta_D)}{\tan^2\beta_D} \quad (3)$$

At η = 0, the equation above is reduced to

$$\varphi_g = \cos^2\beta_D \quad (4)$$

***Equations (3)*** and ***(4)*** can be used to calculate fiber stress utilisation at the maximum twist level when the strains are small. To calculate the fiber stress utilisation at different strains, the theory has been modified as below. According to Hooke's law, stress is directly proportional to stain i.e.

$$\sigma = E . \varepsilon_f \quad (5)$$

Since the above relationship holds up to the elastic limit only, a general relation that relates fiber stress to strain over the whole range of strains can be written as:

$$\sigma_f = f(\varepsilon_f) \quad (6)$$

If the yarn is subjected to a tensile force (F), the force on the constituting individual fibers of cross section area *s* is:

$$F = \sigma_f . s = s . f(\varepsilon_f) \quad (7)$$

If the fibers lie at some angle (β), the component of force in the direction of the yarn axis is:

$$F_Y = F\cos\beta = f(\varepsilon_f) . s . \cos\beta \quad (8)$$

Since the effective fiber cross-sectional area $s^*$ of one fiber in the yarn cross-section, illustrated in the ***Figure 1***, is:

$$s^* = s / \cos\beta \quad (9)$$

The normal fiber stress in yarn in this vector area may be calculated using the following equation:

$$\sigma_Y = \frac{F_Y}{s^*} = \frac{f(\varepsilon_f) s . \cos\beta}{s / \cos\beta} = f(\varepsilon_f) . \cos^2\beta \quad (10)$$

The area of differential annulus of yarn, as shown in ***Figure 2***, is $2\pi r\mathrm{d}r$. Then the packing density of yarn, μ, can be determined as follows:

$$\mu = \frac{\text{total area of fiber sections}}{\text{total area of yarn section}} = \frac{\mathrm{d}S}{2\pi r\mathrm{d}r}$$

Where dS is the area of the fiber section in the differential annulus of the yarn. Therefore we can rearrange the equation above as follows:

$$\mathrm{d}S = (2\pi r\mathrm{d}r)\,\mu \quad (11)$$

The resultant yarn axial force is then a sum of the forces experienced by individual fibers integrated over the whole yarn diameter:

$$P_Y = \int_{r=0}^{r=\frac{D}{2}} \sigma_Y . \mathrm{d}S = \int_0^{\frac{D}{2}} f(\varepsilon_f) . \cos^2\beta . 2\pi r\mathrm{d}r\mu \quad (12)$$

The *r* and d*r* may be defined in terms of the twist angle (β) as below:

$$r = \frac{2\pi rZ}{2\pi Z} = \frac{D\tan\beta}{2\pi DZ} = \frac{D\tan\beta}{2\tan\beta_D},$$

$$\text{and } \mathrm{d}r = \frac{D}{2\tan\beta_D}\frac{\mathrm{d}\beta}{\cos^2\beta}$$

Substitution of *r* and d*r* into ***Equation (12)*** and subsequent rearrangement yields the following:

$$P_Y = \int_0^{\beta_D} f(\varepsilon_f) . \cos^2\beta . 2\pi\mu \left[\frac{D}{2\tan\beta_D}\right]^2 \frac{\sin\beta}{\cos^3\beta}\,\mathrm{d}\beta \quad (13)$$

On simplification and rearrangement, the resultant yarn axial force is:

$$P_Y = 2\pi\mu \left[\frac{D}{2\tan\beta_D}\right]^2 \int_0^{\beta_D} \tan\beta . f(\varepsilon_f)\,\mathrm{d}\beta \quad (14)$$

Although tanβ is analytical, the stress-strain function is not so; therefore the equation above can only be solved using numerical methods. The fiber stress f ($\varepsilon_f$)

can be determined at any strain from the fiber stress-strain curve using interpolation techniques. At a certain yarn strain ($\varepsilon_Y$), $\varepsilon_f$ can be calculated from zero to maximum twist angles using ***Equation (2)***.

On substitution of $\varepsilon_f$ in ***Equation (13)***, the resultant axial force in the twisted-fiber-bundle i.e. yarn is calculated over the whole spectrum of strain values.

At the same yarn strain $\varepsilon_Y = \varepsilon_f$, a yarn of the same fineness has the same substantial cross-sectional area, and hence the axial force in the parallel fiber is as below:

$$P_f = \sigma_f . S = f(\varepsilon_Y) . \pi\mu \left[\frac{D}{2}\right]^2 \qquad (15)$$

From ***Equations (14)*** and ***(15)***, the fiber stress utilisation, $\varphi_{c1}(\varepsilon_Y)$, of the twisted yarn at a certain twist angle can be calculated over the whole range of strains:

$$\varphi_{c1}(\varepsilon_Y) = \frac{2}{\sigma_f(\varepsilon_Y)\tan^2\beta_D}\int_0^{\beta_D}\sigma_f(\varepsilon_f)\tan\beta d\beta \qquad (16)$$

Where $\sigma_f(\varepsilon_f)$ represents fiber stress as a function of the fiber strain and $\sigma_f(\varepsilon_f)$ stands for the fiber stress as a function of the yarn strain.

The direct delta function was introduced in ***Equation (16)*** for the directional distribution of fibers in the yarn and ***Equation (13)*** is modified for fiber stress utilisation, $\varphi_{c2}(\varepsilon_Y)$ as follows:

$$\varphi_{c2}(\varepsilon_Y) = \frac{\sigma_Y(\varepsilon_Y)}{\sigma_f(\varepsilon_Y)} = \frac{2}{\sigma_f(\varepsilon_Y)\tan^2\beta_D}\int_0^{\beta_D}\left[\int_0^{\theta_u}\sigma_f(\varepsilon_f)\cos^2\theta\, U(\theta)d\theta\right]\frac{\sin\beta}{\cos^3\beta}d\beta \qquad (17)$$

Where

$$U(\theta) = \frac{1}{\pi}\frac{C}{C^2-(C^2-1)\cos^2(\theta+\beta)} + \frac{1}{\pi}\frac{C}{C^2-(C^2-1)\cos^2(\theta-\beta)} \qquad (18)$$

C is a measure of the directional distribution of fiber angles in the yarn.

The yarn strain $\varepsilon_Y$ and fiber strain $\varepsilon_f$ are small and are related to the equation as follows:

$$\varepsilon_f = \varepsilon_Y(\cos^2\theta - \eta\sin^2\theta), \quad \theta \in (0, \pi/2) \qquad (19)$$

The unimodel probability density function, $U(\theta)$, of non-oriented angles $\theta \in (0, \frac{\pi}{2})$ was derived earlier by Neckar et al. [18, 19]. Fibers with angles θ approaching $\frac{\pi}{2}$ are found in the yarn structure only very seldomly, however, they were identified by experimental analysis of the internal structure of yarns.

There exists an upper limit of angle $\theta = \theta_u$ by which the fiber strain is equal to zero, i.e. $\varepsilon_f = 0$. Thus it is valid to write

$$0 = \varepsilon_Y(\cos^2\theta_u - \eta\sin^2\theta_u)$$

$$\theta_u = \arcsin\frac{1}{\sqrt{(1+\eta)}}$$

If $\theta \in (\theta_u, \pi/2)$ then the fiber strain is negative according to ***Equation (19)***, where the fiber should be axially compressed and will not be active, and consequently the range of such angles is neglected. The calculation scheme for the utilisation of yarn stress is given in ***Figure 3***.

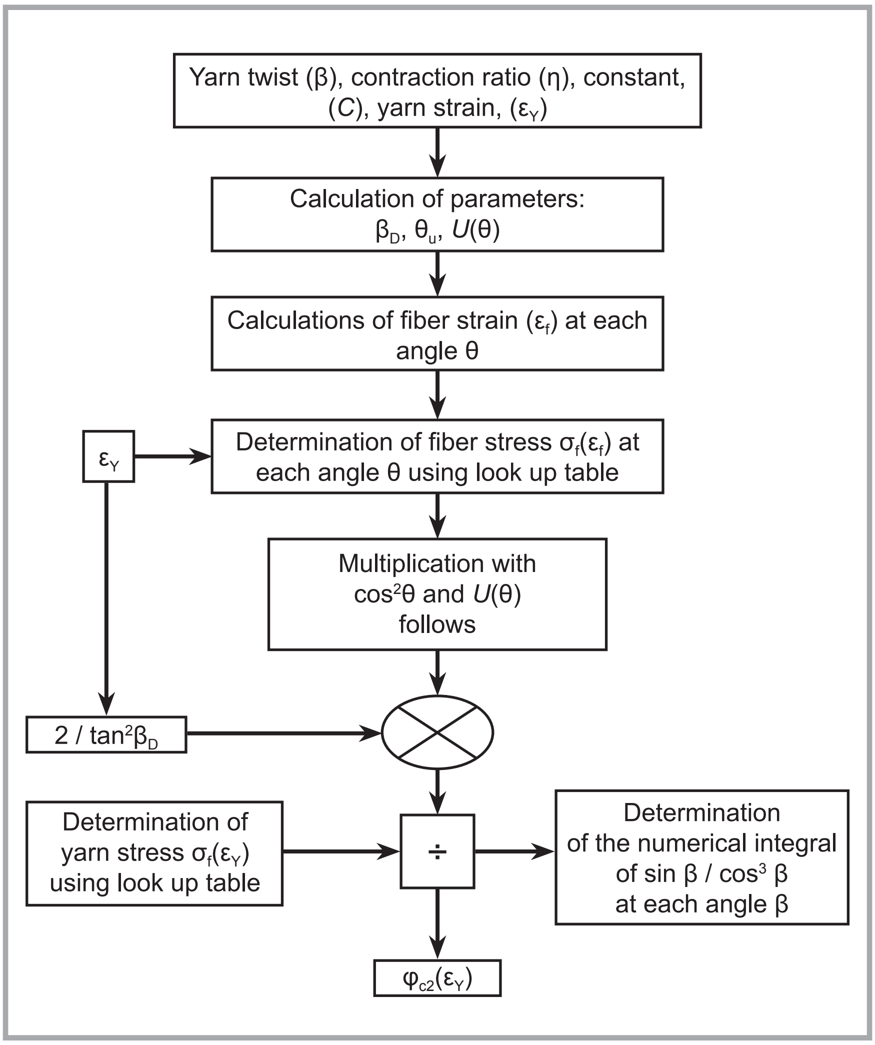


***Figure 3***. *Calculation scheme for utilisation of yarn stress.*

The fiber stress utilisation, $\varphi_{c2}(\varepsilon_Y)$, evaluated from ***Equation (17)*** was finally used to determine the yarn specific stress as a function of yarn strain, $\delta_Y(\varepsilon_Y)$ as follows:

$$\delta_Y(\varepsilon_Y) = \varphi_{c2}(\varepsilon_Y).\delta_f(\varepsilon_f) \qquad (20)$$

Where $\delta_f(\varepsilon_f)$ is the fiber specific stress as a function of the strain.

## Experimental results

### Material

Two types of cotton roving (one from carded and the other – combed sliver produced at Masood Textile Mills Ltd., Faisalabad Pakistan) were used to produce yarn with two linear densities. The same type of cotton was used in both rovings. The yarn was produced on a sample ring frame at the National Textile University, Faisalabad. Cotton fibers of 1.5 dtex fineness with a staple length of 25~30 mm were taken randomly from these rovings to measure the fiber tenacity and elongation.

### Method

Fibers from random positions of roving were tested on a Vibrodyn-400 (Lenzing Instruments, Austria) to measure the fiber strength, fineness and elongation according to the standard test procedure CSN-ENISO 1973. Stress-strain curves were also obtained from fifty cotton fibers taken randomly from these rovings.

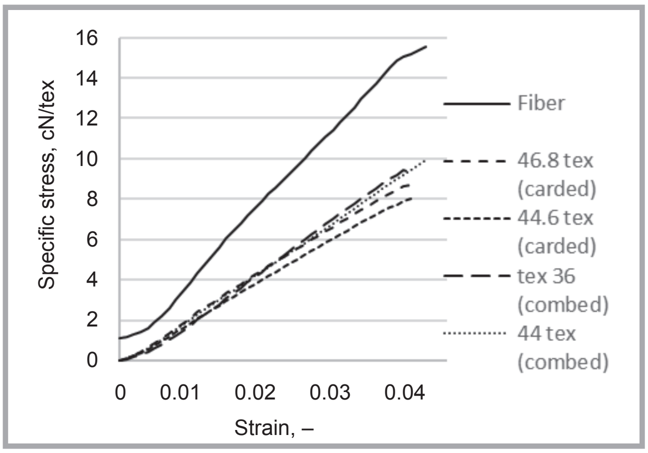


***Figure 4**. Specific stress-strain curves for cotton fiber and yarns.*

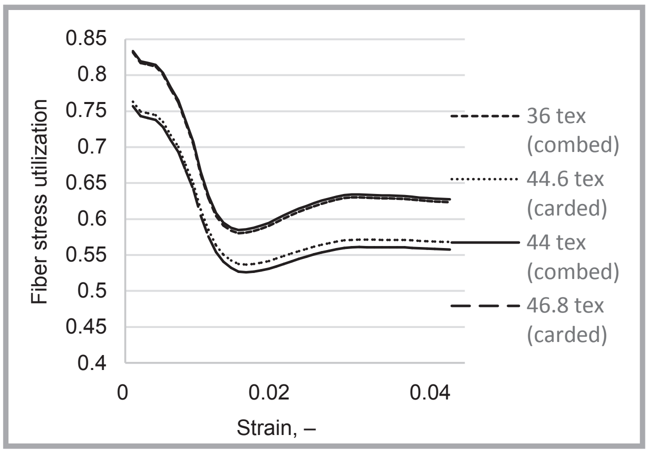


***Figure 5.** Predicted fiber stress utilisation in cotton yarns.*

Each type of carded and combed cotton yarn was tested on as Instron-4411 (USA) according to the standard method CSN-ENISO 2060 to measure force and elongation. A total of fifty tests were conducted to obtain the average stress-strain curves for each type of carded and combed yarns.

The yarn fineness was measured according to standard test method CSN 80 00 50. The yarn diameter was measured according to the standard test method interni norma C.22.102-01/01, and the yarn twist was measured according to the standard test method CSN 80 07 01. The yarn linear densities, yarn twist and twist angle ($\tan\beta_D = \pi DZ$) for both types of carded and combed cotton yarns are presented in ***Table 1***.

## Results and discussions

### Average curves

The average specific stress-strain curves for fiber and yarns from fifty samples were plotted using Matlab software and linear interpolation. The average curves for fiber as well as carded and combed yarn are shown in ***Figure 4***. The shape of specific stress-strain curves for cotton fiber and yarns was similar. All carded and combed yarn curves are under the fiber specific stress-strain curves because of discontinuities in the staple spun yarn. The combed cotton yarn curves lie at a slightly higher position as compared with the carded yarn due to improved fiber length and migration in that yarn.

### Fiber stress utilisation

The fiber stress utilization for each type of yarn was determined from the data given in ***Table 1***. The contraction ratio η was taken as 0.5. ***Equation (17)***, following the calculation scheme in ***Figure 3***, was solved numerically using Matlab software to obtain the fiber stress utilisation for each type of yarn. The parameter $C = 4.9$ was taken for each carded cotton yarn, while for combed the higher value of $C = 10$ was used.

The higher value of the directional distribution parameter for combed yarns was due to improved fiber length and migration in that yarn due to the combing process. The fiber stress utilization curves predicted for each type of yarn are shown in ***Figure 5***.The higher fiber stress utilisation in the region of lower strain might be the contribution of all fibers in the yarn. As the yarn strain is increased, the number of contributing fibers is reduced due to the slippage and straightening of the fibers. At the same time, the fiber which is in a proper grip of the yarn structure from both ends starts to extend. When the strain is further increased, the majority of fibers slip, which reduces fiber stress utilization and the gripped fibers start bearing the stress. After this stage the polymer chains of the fiber start to be oriented and thus bear better stress. Later on fiber polymer chains start breaking due to higher strain near the point of break.

### Yarn specific stress

Using ***Equation (20)***, the yarn specific stress, $\delta_Y(\varepsilon_Y)$ was predicted from the fiber stress utilisation for each cotton yarn, $\varphi_{c2}(\varepsilon_Y)$ and the average fiber specific stress-strain, $\delta_f(\varepsilon_f)$. The predicted and experimental yarn specific stress-strain curves before breaking for each type of carded cotton yarn are shown in ***Figure 6 (a)*** and ***(b)***.

The predicted yarn specific stress-strain curve agreed well with experimental yarn specific stress-strain curves both in shape and position for carded and combed cotton yarns. Liu et al. [20] reported a similar conclusion for the measurement of fiber and yarn strains, and they studied the stress-strain relationship beyond a strain of 1%. Therefore the theoretical relations are not fully valid in the region of very small strains due to relative variation in the process of measurement or fiber crimp.

***Table 1.** Yarn specifications.*

| Yarn type | | Actual yarn linear density T, tex | Yarn diameter D, mm | Yarn twist Z, $m^{-1}$ | Twist angle $\beta_D$ |
|---|---|---|---|---|---|
| Carded yarn | | 46.80 | 0.270 | 532 | 24.2 |
| | | 44.64 | 0.262 | 520 | 23.2 |
| Combed yarn | | 36.00 | 0.244 | 580 | 23.4 |
| | | 43.98 | 0.260 | 520 | 23.2 |

## Conclusion

The experimental yarn specific stress-strain curves for carded cotton yarn are low when compared with combed cotton yarn due to improved fiber properties during the combing process. A higher value of parameter C for combed cotton yarn was used to predict fiber stress utilisation, which might be due to good fiber directional distribution, better fiber migration and longer fiber in combed yarn.

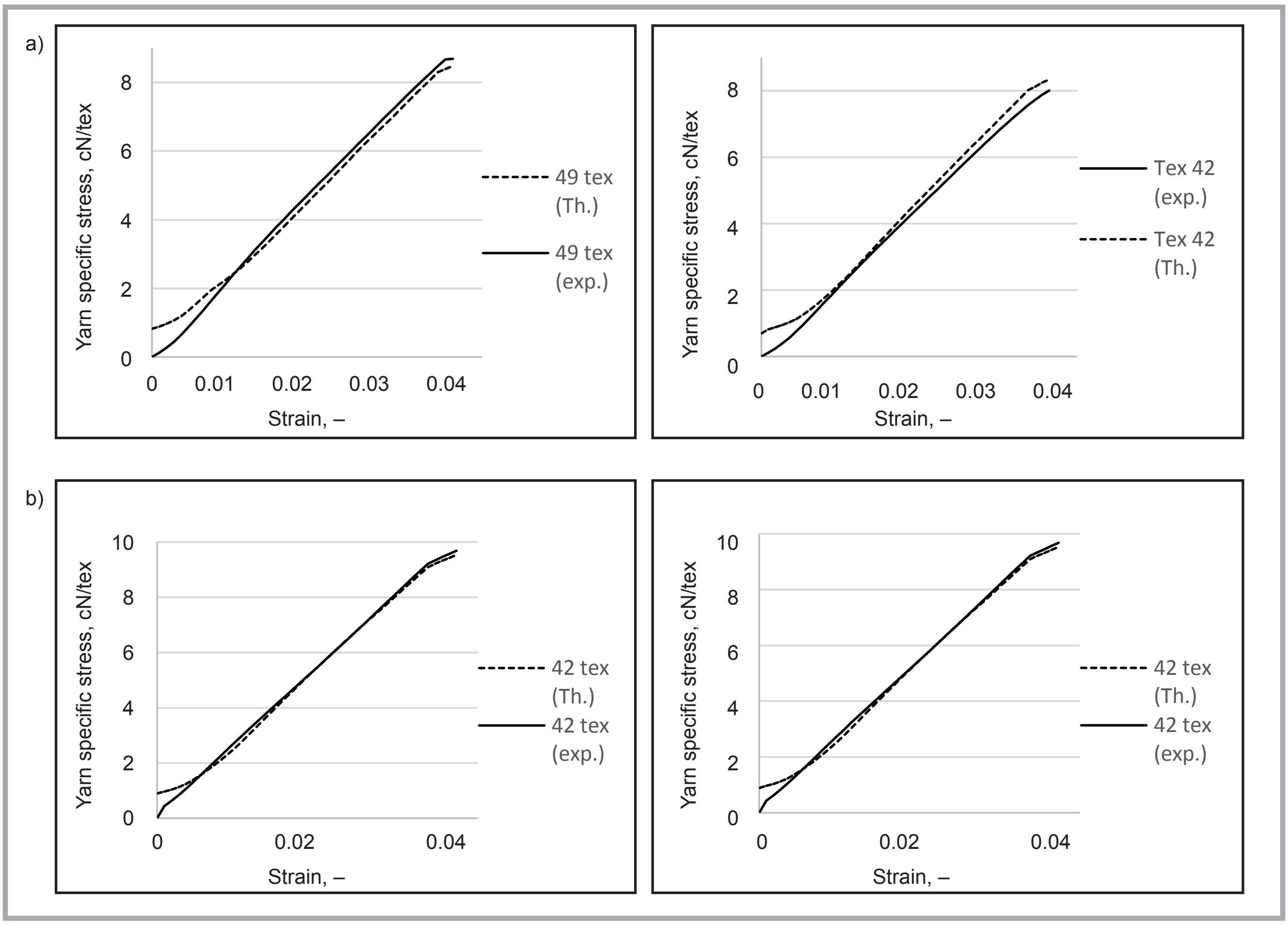


***Figure 6. a)** Yarn specific stress-strain curves for carded cotton yarns, **b)** Yarn specific stress-strain curves for combed cotton yarns.*

The yarn specific stress-strain curves predicted agreed well with the experimental yarn specific stress for both carded and combed yarns. The model predicts the yarn specific stress before the process of breaking when all the fibers in the yarn are working. At the moment we are not able to predict the yarn specific stress up to the point of break. It might be possible in the future to evaluate the yarn specific stress up to the breaking point considering the slippage effect due to friction among the fibers, variability in the breaking stress and strain in fibers as well as the fiber crimp.

## References


1. English W. *The textile Industry*. Longmans London: Green, 1967.
2. Geaguff C.M. Strength and elasticity of cotton threads. *Bull. Soc. Ind. Mulhouse* 1907; 77: 153-176.
3. Gurney H.P. The distribution of stresses in cotton products. *J. Text. Inst.*1925; 16; T269-T289.
4. Peirce F.T. Tensile tests for cotton yarns;V. the weakest link theorems on the strength of long composite specimen. *J. Text. Inst* 1926; 17: T355-T368.
5. Sullivan R.R. A theoretical approach to the problem of yarn strength. *J. Appl. Phys* 1942; 13:157-167.
6. Platt M.M. Mechanics of Elastic Performance of Textile Materials: Part IV. Some Aspects of Stress Analysis of Textile Structures-staple-FiberYarns. *Text Res J.* 1950; 20: 519-538.
7. Gregory J. The relation between strength measurements made on fiber, fiber bundles, yarns and cloth. *J. Text Inst* 41: T515.
8. Gregory J. The strength of artificial yarn elements in relation to bulk and twist. *J Text Inst* 1953; 41: T30-T52.
9. Gregory J. The strength of twisted yarn elements in relation to the properties of the constituent fibers. *J Text Inst* 1953; 41:T499-T512.
10. Hearle JWS. The mechanics of twisted yarns: The influenceof transverse forces on tensile behaviour. *J Text Inst* 1958; 49: T389-T407.
11. Hearle J.W.S. Theoretical Analysis of the Mechanics of Twisted Staple Fiber Yarns. *Text Res J* 1965; 35: 1060-1071.
12. Treloar L. R. G and Riding G. A theory of the stress-strain properties of continuous filament yarns. *J Text Inst* 1963; 54: T156.
13. Zurek W, Frydrych I and Zakrzewski S. A Method of Predicting the Strength and Breaking Strain of Cotton Yarn. *Text Res* 1987; 57: 439-444.
14. Pan N. Prediction of statistical strength of twisted fiber structures. *J Mater Sci* 1993; 28: 6107-6114.
15. Pan N, Hua T, and Qiu Y. Relationship Between Fiber and Yarn Strength. *Text Res J* 2001; 71: 960-964.
16. Pan N. Develpment of a Constitutive Theory for Short Fiber Yarns: Mechanics of staple yarn with out slippage effect. *Text Res J* 1992; 62: 749-765.
17. Gosh A, Ishtiaque S.M and Rengasamy R.S. Analysis of spun yarn failure. Part II: Translation of strength from fibers bundle to different spun yarns. *Text Res J* 2005; 75(10): 741-744
18. Neckář B and Das D. *Theory of Structure and Mechanics of Fibrous Assemblies.* New Delhi: Woodhead Publishing India, 2012.
19. Neckář B and Soni M. K. Modeling of Radial Fiber Migration in Yarn. *Text Res J* 2006; 76: 486-491.
20. Liu T, Choi K. F and Li Y. Mechanical modeling of single yarns. *Text Res J* 2007; 77: 123-130.


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