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PROPOSAL OF METHODOLOGY FOR PRACTICAL APPLICATION OF NONPARAMETRIC CONTROL CHARTS

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Abstract: This paper deals with the methodology for practical application of nonparametric control charts. This topic is very important for two reasons: firstly nonparametric control charts are very effective instruments for the realization of the statistical process monitoring phase I due to their robustness against various deviations from the data assumptions that must be met when applying model-based control charts. Secondly nonparametric control charts have very weak SW support and also they are not taught in the frame of training courses not even of the university study programmes. For that reason the practitioners do not know them and do not use them. The paper offers the proposal how to practically apply these control charts which is based on the complex simulation study of various nonparametric control charts performance when various data assumptions have not been met. The study has covered these nonparametric control charts: Shewhart sign control chart, nonparametric EWMA and nonparametric CUSUM control charts, nonparametric progressive mean control chart, control chart based on Mood statistics and robust median absolute deviation control chart. All charts have been studied in condition of not normally distributed data, autocorrelated data and data with nonconstant distribution parameters. The simulations were realized for statistically stable (IC – in control) and also statistically unstable (OC – out of control) processes. For the evaluation of the control charts performance median run length, 0.05-quantile, and 0.95-quantile were used.
Keywords: statistical process monitoring, nonparametric control charts, median run length, simulation, quantiles

1. INTRODUCTION

This paper deals with the methodology for practical application of nonparametric control charts as a tool of statistical process monitoring (SPM). Various researches recommend to replace well-known SPC (statistical process control) term with SPM that is becoming more and more obvious (Capizzi, 2015; De Ketelaere et al., 2016); Woodal, 2017). The reasons are that the term SPM better expresses the core of these

methods and that many users think that SPC covers only simple control charts such as charts for the average and range (Woodal, 2017).
The problem of nonparametric control charts is very important for two reasons: firstly nonparametric control charts are very effective instruments for the realization of the phase I of statistical process monitoring due to their robustness against various deviations from the data assumptions that must be met when applying model-based control charts. As it is stated in (Capizzi, 2015) the full knowledge of the mathematical IC model in practice can happen seldom and obviously the process parameters and control limits of control charts must be estimated using data collected sequentially from the process during the phase I of SPM which forms basis for the effective process monitoring during the phase II. The precision of this estimation is influenced by the correct specification of the underlying IC model and wrong setting model will influence correctness of the decision making in the frame of SPM. This problem can be solved using nonparametric control charts the main advantage of which is the fact that the user does not need to assume any parametric probability distribution for the underlying process to set up the chart (Chakraborti, 2001). As compared to the parametric methods the IC run length distribution of nonparametric control charts stays the same at least for all continuous distributions (Chakraborti, 2001; Capizzi, 2015; Coelho, 2015). Secondly nonparametric control charts have very weak SW support and also they are not taught nearly at all in the frame of training courses not even of the university study programmes. For that reason the practitioners do not know them and do not use them. The first hypothesis is supported by Table 1. In this table it can be seen that no SW offers any nonparametric control chart.

Table 1
Analysis of SW products according to their menu of control charts

SW **Type of control chart**	**A**	**B**	**C**	**D**	**E**	**F**	**G**	**H**	**I**	**J**
x-bar, R	x	x	x	x	x	x	x	x	x	x
x-bar, s	x	x		x	x		x	x	x	x
Me, R									x	
x_j, MR	x	x		x	x	x	x	x	x	x
p	x	x	x	x	x	x	x	x	x	x
np	x	x	x	x	x	x	x	x	x	x
u	x	x	x	x	x	x	x	x	x	x
c	x	x	x	x	x	x	x	x	x	x
CUSUM	x	x	x		x	x			x	x
EWMA	x	x	x		x				x	x
Hotelling		x			x				x	
Acceptance									x	
Target short run										x
Standardized short run		x								x

Source: (Smajdorová, 2019)

Legend: A = JPM, B = Minitab, C = NCSS, D = Palstat, E = QC Expert, F = SAS/QC,
G = SigmaXL, H =SPSS, I = Statgraphics Centurion, J = Statististica

In spite of this practical situation there has been written a lot of papers and articles relating to nonparametric control charts (see for instance Chakraborti et al., 2001; Das, 2008; Chakraborti and Van de Wiel, 2008; Bush et al., 2010; Murakami and Matsuki, 2010; Graham et al., 2011; Yang et al., 2011; Lu, 2015; Oprime et al., 2016; Wang et al., 2016). Due to the nonparametric methods properties this gap between practice and theory must be removed and nonparametric methods must be made available to practitioners to stay the obvious methods for effective statistical process monitoring especially in the phase I. For that reason we realized the analysis of the performance of various nonparametric control charts through simulation using Excel and based on that we proposed the methodology for practical application of nonparametric control charts. Inevitable part of this experiment the SW support for construction and analysis of studied nonparametric control charts was created (see Smajdorová, 2019).

The rest of the paper is divided into two parts. In the first part the simulation study on the performance of analysed control charts which is the basis for the proposed methodology of the practical application of the nonparametric control charts will be presented. In the second part will be described the proposed methodology.

2. SIMULATION STUDY

The complex simulation study of various nonparametric control charts performance when various data assumptions have not been met was made for these control charts: Shewhart sign control chart, nonparametric EWMA and nonparametric CUSUM control chart, nonparametric progressive mean control chart, control chart based on Mood statistics and robust median absolute deviation control chart. The simulations were realized for statistically stable (IC) and also unstable (OC) processes with simulated sustainable mean shift of 1,5σ and 2σ (δ =1,5; 2). The goal of these studies was to define which nonparametric control chart or control charts are the best in condition of various deviations from the data assumptions and what type of the control chart is the most universal (having the best performance for all deviations).

All charts have been studied in condition of not normally distributed data, autocorrelated data and data with nonconstant distribution parameters.

Table 2 The relation between deviation from the data assumption and a type of distribution

Type of deviation from the data assumption	**Applied probability distribution**
No deviation	Normal distribution N (0,1)
Deviation from normality (higher kurtosis)	Student distribution t_3 with 3 degrees of freedom
Deviation from normality (lower kurtosis)	Uniform distribution with parameters $a = -\sqrt{3}\,; b = \sqrt{3}$
Deviation from normality (skewed distribution)	Pearson distribution χ_3^2 with 3 degrees of freedom
Nonconstant mean and dispersion	Mixed distribution - 50 % N (0,1) + 50 % N (2,1) Mixed distribution - 50 % N (0,1) + 50 % N (0,4)
Data are not independent	Data from the model AR (1): $x_i = 0{,}5x_{i-1} + a_i$,

Source: (Smajdorová, 2019)

In Table 2 the relation between the type of deviation from the data assumption and the type of probability distribution from which data were generated to simulate a particular deviation is summarized.
The simulations were repeated 10 000 times and from every simulation repetition the indicator run length RL was set (RL is a number of subgroups taken before there is a signal (point beyond the control limits in a control chart). For IC processes RL(0) is set (RL(0) is defined as a number of samples until a control chart signals - point is beyond the control limits), given that the process is statistically stable. We want RL(0) to be as great as possible. For OC processes RL(δ) is set (RL(δ) is defined as a number of samples until a control chart signals - point is beyond the control limits), given the process is not statistically stable (the mean has shifted). We want RL(δ) to be as short as possible. Values of RL were the basis for the computation of IC performance indicators average run length ARL(0), median run length MRL(0) and 0.05-quantile and OC performance indicators ARL(δ), MRL(δ) and 0.95-quantile. As the RL distribution is highly skewed it is not a typical RL and it can led to misleading conclusions. On the other hand MRL is more trustworthy robust indicator (Chin and Khoo, 2012). For this reason we used for the comparison of the control charts performance MRL and quantiles. As the best nonparametric control charts seem to be nonparametric CUSUM (Graham et al., 2014) and nonparametric Shewhart sign control chart SSCC (Bakir et al., 2015).
The test statistic for nonparametric CUSUM can be computed as follows:

$$S_i = \max\{0\,; S_{i-1} + SMW_i - k\} \tag{1}$$

where Mann-Whitney statistic *SMW* can be computed using the formula (2):

$$SMW_i = \frac{MW_i - E_0(MW_i)}{\sqrt{var_0(MW_i)}} \tag{2}$$

$$S_0 = 0 \tag{3}$$

$$k = \frac{\Delta}{2} \tag{4}$$

Control limits are done by the value *H:*

$$H = h.\sigma \tag{5}$$

It is recommended to use *h* = 4 or 5 (Graham et al., 2014).

In SSCC chart every measurement is compared to the target value $\theta_0$and then it is transformed as follows:

$$sign(x) = x_{ij} - \theta_0 = \begin{cases} 1, & x_{ij} > \theta_0 \\ 0, & x_{ij} = \theta_0 \\ -1, & x_{ij} < \theta_0 \end{cases} \tag{6}$$

Statistic, recorded to the control chart SN_i , is computed as follows:

$$SN_i = \sum_{j=1}^{n} \left(x_{ij} - \theta_0\right) \tag{7}$$

Central line and control charts are computed using the following formulas:

$$UCL = c \quad (8)$$

$$CL = 0 \quad (9)$$

$$LCL = -c \quad (10)$$

$$c = 2t - n \quad (11)$$

where *n* is the rational subgroup size and *t* is constant that can be found for instance in (Graham, 2008).

3. PROPOSAL OF THE METHODOLOGY FOR THE PRACTICAL APPLICATION OF NONPARAMETRIC CONTROL CHARTS

Based on the results of the simulation studies the methodology for the practical application of nonparametric control charts was proposed. It consists of 4 phases (see the Fig. 1).

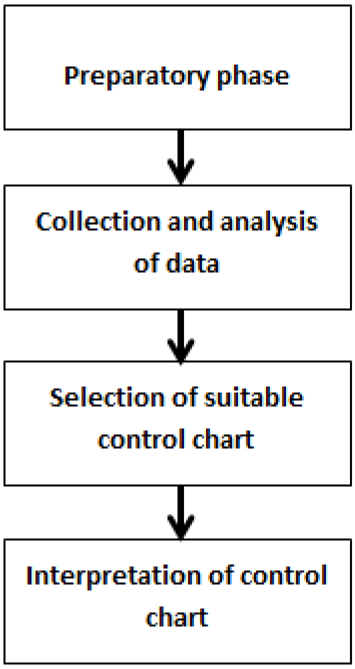

Fig. 1. Phases of proposed methodology

In the preparatory phase the main activities are as follows: setting the quality characteristic or the process parameter; selection of suitable inspection location, defining the form of data collection a recording, definition of the control interval length, defining the form of the rational subgroups creation. The second phase is very important for the following decisions in the third phase. There must be verified data assumptions. According to the results it must be decided if to select some Shewhart control chart or to select some nonparametric method. Verification should be done using statistical hypothesis testing supplemented with graphical methods. The results from this phase enter into the third phase. How to select the best nonparametric control chart can be found in Table 3.

Table 3
Results of simulation studies

Type of data	**Suitable nonparametric control chart**
Data are skewed	CUSUM SSCC (for *n* = 10)
Nonnormally distributed data with high kurtosis	CUSUM (for *n* = 10) SSCC (for *n* = 5)
Nonnormally distributed data with lower kurtosis	CUSUM SSCC
Autocorrelated data	SSCC
Conconstant mean	CUSUM SSCC (for *n* = 10)
Nonconstant dispersion	CUSUM

Source: (Smajdorová, 2019)

The last phase is devoted to the statistical stability evaluation. The interpretation of the nonparametric control charts is the same as for the Shewhart or other model-based control charts.

4. CONCLUSION

The paper dealt with the proposal of the practical application of the nonparametric control charts suitable especially for the phase I of SPM. The methodology is based on the complex simulation study of various nonparametric control charts performance when various data assumptions have not been met. The study has covered these nonparametric control charts: Shewhart sign control chart, nonparametric EWMA and nonparametric CUSUM control chart, nonparametric progressive mean control chart, control chart based on Mood statistics and robust median absolute deviation control chart. All charts have been studied in condition of not normally distributed data, autocorrelated data and data with nonconstant distribution parameters. The simulations were realized for statistically stable (IC – in control) and also statistically unstable (OC – out of control) processes. For the evaluation of the control charts performance median run length MRL, 0.05-quantile, and 0.95-quantile were used. The simulation led to the conclusion that the best control charts for various deviations from normality, for dependent data and also nonconstant mean or dispersion are very simple Shewhart sign control chart (SSCC) and nonparametric CUSUM chart. At the end very simple methodology for the practical application of nonparametric control charts has been designed in four phases: preparatory phase, phase of collection and analysis of data, phase of suitable control chart selection and phase of control chart interpretation.

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REFERENCES

Abbas, N., Zafar, R. F., Riaz, M., Hussain, Z., 2013. *Progressive Mean Control Chart for Monitoring Process Location Parameter*, Quality and Reliability Engineering International, 29(3), 357-367, DOI: 10.1002/qre.1386. ISSN 07488017.

Bakir, S., Prater, T., Kiser, S., 2015. *A Simple Nonparametric Quality Control Chart for Monitoring Students' GPAs*. SOP Transactions on Statistics and Analysis, 2015(1), 8-16, DOI: 10.15764/STSA.2015.01002

Bush, H.M., Chongfuangprinya, P., Chen, V. C., Sukchotrat, T., Kim, S. B., 2010. *Nonparametric multivariate control charts based on a linkage ranking algorithm*, Quality and Reliability Engineering International, 26(7), 663-675, DOI: 10.1002/qre.1129.

Capizzi, G., 2015. *Recent advantages in process monitoring: Nonparametric and variable-selection methods for phase I and II*. Quality Engineering, 27, 44-67, DOI: 10.1080/08982112.2015.968046.

Chakraborti, S., Van Der Laan, P., Bakir, S. T., 2001. *Nonparametric control charts : an overview and some results*, Journal of Quality Technology, 33(3), 304-315 [cit. 2017-11-06].

Chakraborti, S., Van De Wiel, M. A., 2008. A nonparametric control chart based on the Mann-Whitney statistic. *Beyond Parametrics in Interdisciplinary Research: Festschrift in Honor of Professor Pranab K. Sen* Beachwood, Ohio, USA: Institute of Mathematical Statistics, 156, DOI: 10.1214/193940307000000112.

Chin, W. S., Khoo, M. B. C., 2012. *A study of the median run length (MRL) Performance of the EWMA T chart for the mean*. The South African Journal of Industrial Engineering, 23(3), 42-55, DOI: 10.7166/23-3-510

Coelho, M., Chakraborti, S., Marien, A., Graham, M. A., 2015. *A comparison of phase i control charts.* The South African Journal of Industrial Engineering, 26(2), 178-190, DOI: 10.7166/26-2-1026

Das, N. *A., 2008. Note on the efficiency of nonparametric control chart for monitoring process variability*, Economic Quality Control, 23(1), 85-93, DOI: 10.1515/EQC.2008.85

De Ketelaere, B. T., Rato, T., Schmitt, E., Hubert, M., 2016. *Statistical process monitoring in data rich environments,* Quality Engineering, 28(1), 127-142.

Graham, M. A., Chakraborti, S., Mukherjee, A., 2014. *Design and implementation of CUSUM exceedance control charts for unknown location.* International Journal of Production Research, 52(18), DOI: 10.1080/00207543.2014.917214.

Graham, M. A., Chakraborti, S., Human, S. W., 2011. *A nonparametric exponentially weighted moving average signed-rank chart for monitoring location.* Computational Statistics & Data Analysis, 55(8), DOI: 10.1016/j.csda.2011.02.013.

Lu, S. L., 2015. *An Extended Nonparametric Exponentially Weighted Moving Average Sign Control Chart.* Quality and Reliability Engineering International, 31(1), 3-13, DOI: 10.1002/qre.1673.

Murakami, H. T. Matsuki, 2010. *A nonparametric control chart based on the Mood statistic for dispersion*, The International Journal of Advanced Manufacturing Technology, 49(5-8), 757-763, DOI: 10.1007/s00170-009-2439-3.

Oprime, P.C., de Toledo, J. C., Gonzáles, M. O. A., Chakraborti, S., 2016. Methods for determining the control limits of nonparametric charts for monitoring location and scale, Gest Prod., 23(1), 146-164.

Smajdorová, T., 2019. *Application of nonparametric methods in statistical process control* (in Czech), Disertation thesis. Ostrava, VŠB-TUO, Czech Republic, 185.

Wang, D., L. Zhang, Q., Xiong, Q., 2016. *A non parametric CUSUM control chart based on the Mann–Whitney statistic*. Communications in Statistics - Theory and Methods, 46(10), DOI: 10.1080/03610926.2015.1073314

Woodal, W. H., 2017. *Bridging the gap between theory and practice in basic statistical process monitoring*, Quality Engineering, 29(1,2-15), 1532-1547, DOI: 10.1080/08982112.2016.1210449

Yang, S. F., Lin, J. S., Cheng, S. W., 2011. *A new nonparametric EWMA sign control chart*, Expert Systems with Applications, 38(5), DOI: 10.1016/j.eswa.2010.11.044