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Simplified steady-state analytical model for manoeuvring and control of ASD tug in escort push operations

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Abstract
This paper evaluates an ASD tug's main control parameters in terms of: the propeller thrust, a direction thereof (the thruster angle), and the hull drift angle for given escort speed and required push force. Such a relation is sometimes referred to as the tug performance diagram. A simplified model of tug hydrodynamics is used to arrive at the qualitative and most representative relations of the above variables. This model is rather generic that can also suit any type of tug hydrodynamics, including even that related to a conventional tug.

Introduction

For safe and efficient ship-tug operation from the viewpoint of a tug's master (towmaster) we need to have exact knowledge and understanding of the complex relations between multiple input control variables and the output performance. The output performance is mostly indicated by the force applied on the towed ship, either in pull or push mode.

In steering / manoeuvring a tug (of any type of propulsion) there are direct and inverse control problems. In the direct problem we are interested in the tow force (the effective force applied on the towed ship) excited under given speed and control parameters. On the contrary, the inverse problem consists in establishing necessary control parameters for the assumed tow force and escort speed. The direct and inverse problems are sometimes called passive and active ones accordingly. In both tasks, a convenient mathematical description of the above mentioned input/output dependence is absolutely essential in reaching an efficient solution to the control problem.

A good understanding of control principles ensures i.a. a self-confidence of the towmaster and especially his right response in rapid and emergency situations, involving many different hydrodynamic and mechanical effects. Such skills or competences will certainly supplement those outlined e.g. in [1] or [2].

The required control parameters for a tug essentially change with the speed of the assisted ship. ASD tugs are specially capable of executing the tow assistance with high speed, much better (in wider limits) than conventional tugs.

Some research centres claim they developed software for computing tow forces, as well as necessary control parameter values on a tug in steady state situations. However, appropriate results concerning both an applied mathematical model and a detailed, well documented tug performance output in the form of charts are practically not published. Such data are needed for the training process and many optimisation studies on tug control and design as recently undertaken in Poland in view of the new LNG import terminal in Świnoujście. For an ASD tug of 50 t BP (bollard pull) in pushing mode [3, page 58] presents only a single and not complete chart of control, which rather originates in [4]. Similarly, for a VSP (Voith-Schneider Propeller) tractor tug while pushing stern-first, as compatible to an ASD tug operation (bow-first), one can also refer to [5]. In addition, seemingly a very valuable reference [6] provides control variables in a difficult, implicit format that is hard to be handled.

Under such somehow negative background, the objectives of this paper are:

- to prepare an initial platform / framework for computing the control parameters of a tug in the escort (non-zero speed) steady-state conditions of movement, which as far as possible avoids numerical solving of nonlinear equations, thus attempting to reach a direct / explicit analytical formulation;
- to test and evaluate at present this model with simple, analytically given hydrodynamic data, but only for the push mode of towing, as being relatively easier.

All these efforts shall bring to light the mechanism of equilibrium for a tug and her limits of operation. The explicit formulation ensures a parameterisation of the model, with which we can easily investigate various hydrodynamic designs of the tug, quickly conduct sensitivity analyses, and validate any simulation (numerical) results as based on more sophisticated models.

Though it is further indirectly assumed that azimuth thruster(-s) is installed, the obtained solution is general enough for any propulsor type.

Basic formulation of force equilibrium

The mutual ship-tug arrangement during the so-called indirect pushing operation, together with forces, is presented in figure 1. The indirect towing involves taking advantage of a tug's underwater hull hydrodynamic force while rendering assistance at significant escort speed. The tug-fixed coordinate system *Mxy* is positioned for convenience at the intersection of her centre plane and midship section, with *x* axis pointing forward and *y* axis to starboard side.

It shall be further kept in mind that escorting on starboard side of the ship is related to negative drift angles β and positive thruster angles δ. The thruster angle means the angle of the resulting thrust in tug's body axes. Though we generically assume a single thruster, this thrust force can also be equally distributed to dual propulsors (as usual in modern tugs), if such exist and are steered parallel. If the thrust force is provided by the classical rudder-propeller complex, the positive thrust angle is provided by deflecting the rudder to portside. Though both angles for the latter case – the resulting thrust angle and the rudder angle – are by convention positive in the ship manoeuvring hydrodynamics, there is no easy relation between them as compared to azimuth thrusters. In the case of azimuth thrusters, at least for a stationary tug or ship, both angles are identical.

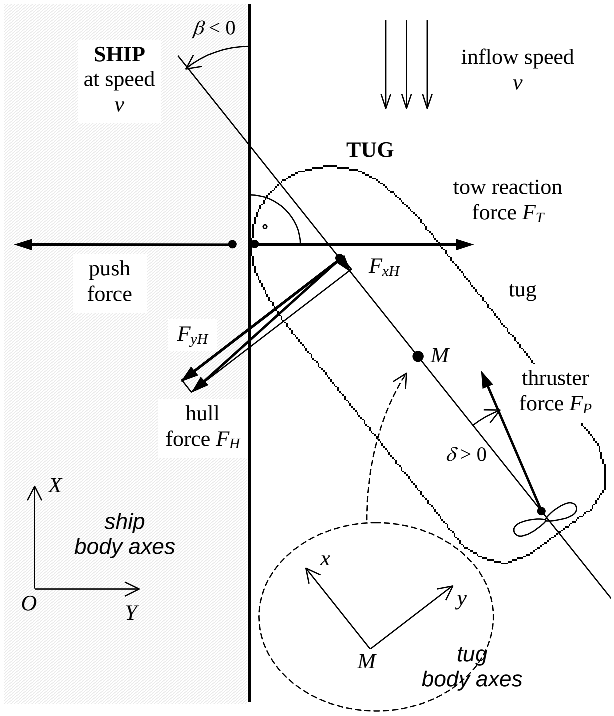

Fig. 1. Steady-state condition of indirect pushing operation

On the contrary, for the rudder-propeller system based on the Schilling rudder the maximum rudder angle of 65–70°, while running the propeller ahead, diverts the effective thrust to 90°. The Becker flapped rudder at its nominal 45° gives here slightly lower value of the thrust force angle, likely around 80–85°. For the conventional stern rudder such data are mostly not analysed and published – the propeller and rudder forces are described and computed (or measured) quite independently and such substituted to motion equations. The direction of the combined (thrust) force against the rudder angle is here not revealed due to a lack of interest. In general, the rudder-propeller system involves very complex propeller-rudder interactions, varying with the hull speed much more than that for azimuth thrusters. In addition, the helm angle for the conventional rudder is traditionally/historically limited to 35°. It also shall be remembered that for the propeller running astern the rudder is not so effective and the resulting force, almost entirely dominated by the propeller negative thrust and to some extent by its lateral component (well known to navigators), is almost constant and does not provide any means of control. Such a significant limitation in performance for the rudder-propeller system, and thus in the towing performance, is often shown in the so-called polar vector diagrams of the effective thrust as published for conventional tugs, see for example [3]. However, the relation between the rudder angle and the resulting thrust is almost omitted there.

The tow elastic reaction force F_T, hereafter briefly called also the tow force, is considered normal to the ship's hull in that no friction exists at the contact point. The tow force is equal in the absolute value to the effective push force applied on the ship. However, the latter force by the earlier definition is the proper tow force. The lack of tangential forces at the interacting surfaces, here arising from friction, is also equivalent in our model to the absence of mooring line(-s) for making fast the tug's bow alongside. This line, either applied as spring line or bow-line from the tug's viewpoint, is sometimes used to support escorting. Adopting a longitudinal component (in the ship body axes) that the tug would exert on the ship will certainly make the following mathematical model little complicated. The real need for such additional assumptions and possible improvements to the present formulation will be examined in next reports.

The equilibrium condition between the hull (H), thruster/propeller (P), and tow (T) forces in the tug's coordinates take the form:

$$\begin{cases} F_{xH} + F_{xP} + F_{xT} = 0 \\ F_{yH} + F_{yP} + F_{yT} = 0 \\ M_H + M_P + M_T = 0 \end{cases} \tag{1}$$

where:

- F_x, F_y – longitudinal and lateral components of each force;
- M – moment developed by particular force (however, in the case of hull this could incorporate also the effect of the hydrodynamic couple of forces that results in the zero force).

The tug's hull hydrodynamic forces are commonly written as follows:

$$\begin{bmatrix} F_{xH} \\ F_{yH} \\ M_{zH} \end{bmatrix} = 0.5\rho L T v^2 \cdot \begin{bmatrix} c_{fxh}(\beta) \\ c_{fyh}(\beta) \\ L \cdot c_{mzh}(\beta) \end{bmatrix} \tag{2}$$

where:

- ρ – water density [kg/m^3];
- L, T – tug's length (between perpendiculars) and draft (extreme) in [m];
- v – absolute inflow speed [m/s];
- c_{fxh}, c_{fyh}, c_{mzh} – nondimensional hydrodynamic coefficients [–].

For further calculations we adopt the following conditions of the tug and the environment: water density 1000 kg/m^3, L = 30.5 m, T = 5 m. However, most of obtained numerical and graphical data are nondimensional and thus independent of these quantities.

The product LT constitutes the reference or representative area for defining the nondimensional hydrodynamic coefficients and thus the resulting hydrodynamic forces. Since the tug is considered in the steady-state oblique movement, i.e. without turning, the hydrodynamic coefficients are functions of the drift angle β solely. Otherwise, we have to incorporate the other input variable of the hull hydrodynamic forces and moment – namely the nondimensional yaw velocity. Data on the yaw effect in hull hydrodynamics, especially over full range of the drift angle, are considerably missing in the literature. This might be contributed to some extent to difficulties in measurements in the model scale. On the other hand, the hydrodynamic coefficients as pure functions of the drift angle are frequently and quite accurately provided. The drift angle in ship hydrodynamics is adopted negative when the inflow comes from starboard side of the ship, as in our case of the tug, giving the hydrodynamic force to portside.

For the objectives of the present paper – development and initial appraisal of the mathematical model, and getting some reference numerical (quantitative) results – simple trigonometric functions will be used to approximate the hull hydrodynamic coefficients at even keel:

$$\begin{aligned} c_{fxh}(\beta) &= -0.03\cos\beta \\ c_{fyh}(\beta) &= +0.5\sin\beta \\ c_{mzh}(\beta) &= +0.1\sin 2\beta \end{aligned} \tag{3}$$

where $\beta \in \left(-180°, +180°\right\rangle$.

These rough but practically sufficient and very handy relations are illustrated in figure 2. For the assumptions of the present paper, we will look here for an equilibrium solution in the negative range of drift angles: from 0° up to the practical limiting value –90° and combined with positive thruster angles δ (to starboard side). A validation of these first choice expressions (3) was undertaken in [7].

Let's define at this point for subsequent derivations the following useful ratios with the hull nondimensional lateral force in the denominator (as practically avoiding the division by zero):

$$c'_{fxh}(\beta) = \frac{c_{fxh}(\beta)}{c_{fyh}(\beta)}, \qquad c'_{mzh}(\beta) = \frac{c_{mzh}(\beta)}{c_{fyh}(\beta)} \tag{4}$$

The relations (4) for data given by (3) are graphically demonstrated in figure 3. The functions (4) are fundamental in the process of getting the balance of forces as will be shown later.

The first ratio of (4) can be interpreted as a deflection angle, strictly its tangent, of the resultant

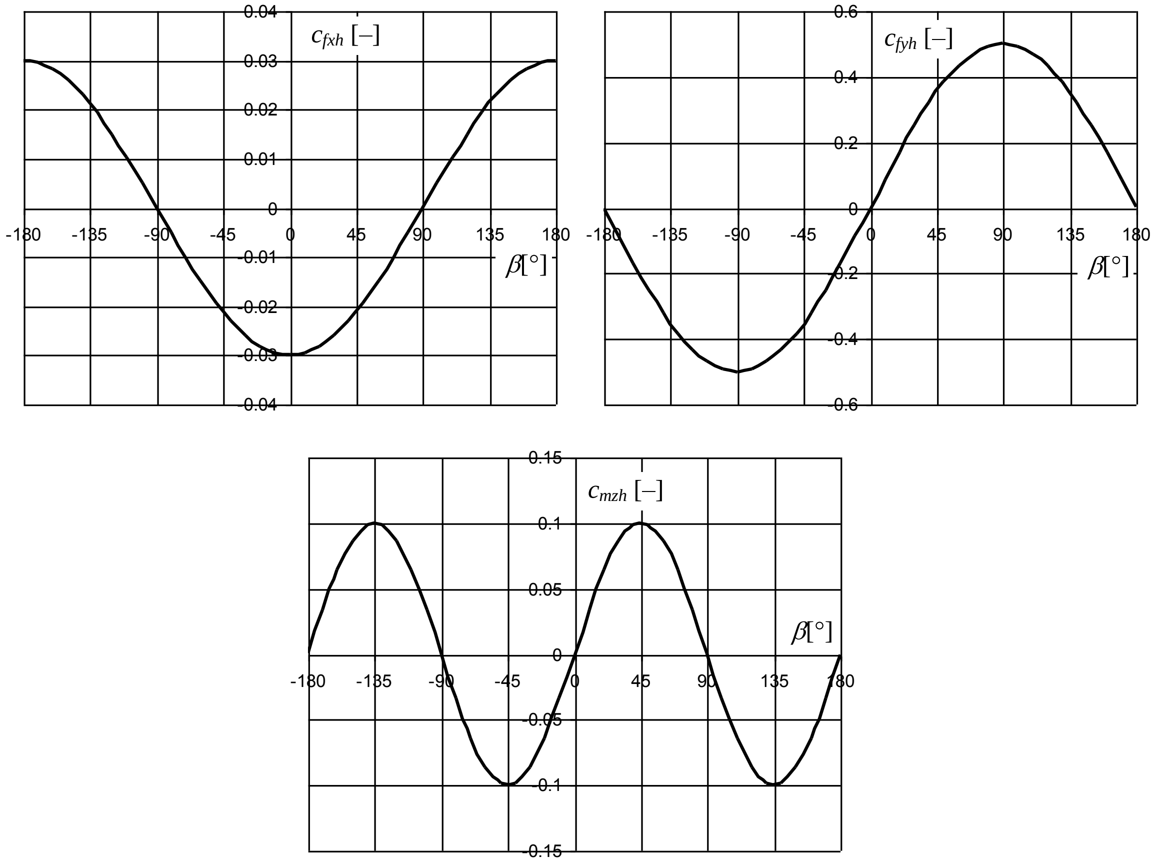

Fig. 2. Exemplary simple analytical approximations to the hull hydrodynamic coefficients

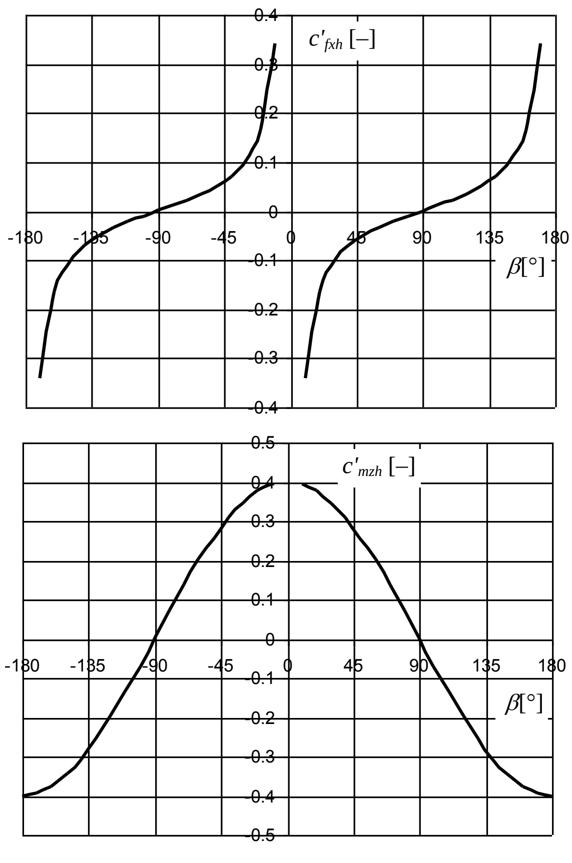

Fig. 3. Relative hull longitudinal force and yaw moment coefficients

hull force from the normal (perpendicular) direction to the hull. This direction is normal only in ideal fluid. However, in real (viscous) fluids and ship flows there are significant tangential stresses at the ship's hull surface. This deflection angle could reach even 20° and more.

The second ratio of (4) can be explained as the nondimensional arm, in ship's length units, counted from the midship section (the M origin). This could be quite abstract if it assumes values beyond the ship physical limits in that a certain hydrodynamic couple of forces, as aforementioned in the comments to equation (2), exists in the ship flow.

The thruster forces and moment in (1) read:

$$\begin{bmatrix} F_{xP} \\ F_{yP} \\ M_{zP} \end{bmatrix} = F_P \begin{bmatrix} \cos\delta \\ \sin\delta \\ \sin\delta \cdot x_P \end{bmatrix} \approx F_P \begin{bmatrix} \cos\delta \\ \sin\delta \\ -0.5L \cdot \sin\delta \end{bmatrix} \tag{5}$$

where:

- F_P – absolute value of thrust (always positive);
- x_P – thruster position (negative in aft direction).

In (5) we have already adopted the thruster location in the centre plane and its fore and aft coordinate at the aft perpendicular ($x_P \approx -0.5L$). The real values for x_P in the case of harbour or escort tugs might go towards the midship section up to even $-0.3L$.

The thruster (or thrust) angle δ takes value from the range (–180°, +180°⟩.

One should also be aware that assuming in (5) the constant value of the thrust force means its independence of other variables, particularly of the inflow speed v. This is a crucial item that needs our further investigation in the future and a possible improvement made to the mathematical model, however, being paid for with an additional sophistication of the model and difficulties in analysing the simulation results. Over the last years, there has been a tremendous progress in the research on the advance speed effect, particularly for azimuth thrusters, and such data are available: [8, 9] and many others. Also, some inspiration can be taken from [10]. Although the latter directly deals with VSP, but with respect to thrust and torque coefficients (as function of the advance coefficient and pitch ratio), this unconventional propeller reveals very similar hydrodynamic modelling properties to those well known for conventional propellers and thus for azimuth thrusters, as essentially consisting of a conventional yet rotatable propeller.

The tow (push) force in tug's coordinates is described by:

$$\begin{bmatrix} F_{xT} \\ F_{yT} \\ M_{zT} \end{bmatrix} = F_T \begin{bmatrix} \sin\beta \\ \cos\beta \\ \cos\beta \cdot x_T \end{bmatrix} \approx F_T \begin{bmatrix} \sin\beta \\ \cos\beta \\ +0.5L\cdot\cos\delta \end{bmatrix} \quad (6)$$

where:

- F_T – absolute value of tow force (always positive);
- x_P – tow point position (positive in forward direction).

In (6) we have just assumed that the moment of the tow force F_T comes only from the longitudinal abscissa of the contact point in that the tug's beam is completely disregarded. In addition, the tow position is at the forward perpendicular. Both assumptions are rather valid for higher drift angle in the vicinity of ±90° and will be rectified in the future, though it is believed they have no significant influence.

Fundamental derivations

Rearranging the three equations in (1) and dividing side-by-side in pairs: first and second, and third and second, supported by formulations (2), (4), (5), and (6), one finally gets:

$$c'_{fxh}(\beta) = \frac{-\cos\delta - F'_T \sin\beta}{-\sin\delta - F'_T \cos\beta} \quad (7)$$

$$c'_{mzh}(\beta) = \frac{+0.5\sin\delta - 0.5F'_T \cos\beta}{-\sin\delta - F'_T \cos\beta} \quad (8)$$

where:

$$F'_T = \frac{F_T}{F_P} \quad (9)$$

that can be considered as the nondimensional (relative) tow force. Having a brief look at (1), (2), (5), and (6) one can easily conclude that the absolute tow force F_T, as one of the equilibrium variables, is always proportional to the absolute thruster force F_T that is another variable (or parameter) of the these equations. The ratio of both is always constant in the equilibrium solution, if such exists.

A supplement of (7), (8), and (9) to the full equilibrium is the following equation for the tug's hull hydrodynamic lateral force F_{yH}:

$$F_{yH} = 0.5\rho LTv^2 c_{fyh}(\beta) = -F_P(\sin\delta + \cos\beta\cdot F'_T) \quad (10)$$

which can be split into two but more convenient relations:

$$F'_{yH} = -(\sin\delta + \cos\beta \cdot F'_T) \quad (11)$$

$$F'_{yH} = \frac{F_{yH}}{F_P} = \frac{0.5\rho LTv^2 c_{fyh}(\beta)}{F_P} \quad (12)$$

The absolute hull lateral force F_{yH} is the only term in our equations that comprises the speed dependence. So, the expression (12) turns into:

$$v = \sqrt{\frac{F'_{yH}\cdot F_P}{0.5\rho LTc_{fyh}(\beta)}} \quad (13)$$

This way the escort speed, enabling the desired steady-state condition of towing, is linearly related to the square root of the absolute thruster force F_P, and inversely proportional to the water density ρ or the tug's lateral area of the underwater hull LT.

Finally, we have arrived at the four basic relations: (7), (8), (11), (13). The first three directly originate from the equilibrium equations (1) and the last one is a part the hull hydrodynamic force formulation (2). We are going to further look for mutual relations of five variables:

$$\beta,\ \delta,\ F_T,\ F_P,\ \text{and}\ v$$

or

$$\beta,\ \delta,\ F'_T,\ F_P,\ \text{and}\ v.$$

However, the equations (7), (8), and (11) provide unique triples of (β, δ, F'_T) in that the drift angle β shall serve as parameter (the independent variable), since the thruster angle δ and the relative tow force F'_T can then be formulated as <u>direct</u> functions of β thus avoiding the numerical solution of the essentially nonlinear equations (if other variable from the above set is taken for the domain). The drift angle is also one of the important steering parameters for the tug's master.

There are two strategies possible for steering the tug – active (inverse) and passive (direct) – which are defined according to the input parameters (initial assumptions or requests) and the chosen unknowns as steering variables. One can namely select the required absolute tow force F_T at the speed of advance v, in which the absolute thruster force F_P is searched for, together with the other basic steering parameters: β and δ. Such strategy we can call an active steering. A passive steering is obtained if we want to find the effective tow force F_T (apart from β and δ) if the thruster force F_P is known at the speed v.

The solutions provided later in the paper encompass the passive steering in that two distinct absolute thruster force are simulated as corresponding to 10 t and 50 t.

The expressions (7) and (8) can be transformed into:

$$F_T' = \frac{-\cos\delta + \sin\delta \cdot c_{cfxh}'(\beta)}{+\sin\beta - \cos\beta \cdot c_{cfxh}'(\beta)} \tag{14}$$

$$F_T' = \frac{\sin\delta}{\cos\beta} \cdot \frac{0.5 + c_{mzh}'(\beta)}{0.5 - c_{mzh}'(\beta)} \tag{15}$$

and made equal, thus leading to the direct function $\delta = \delta(\beta)$:

$$\frac{1}{\tan\delta} = \frac{1+2c_{mzh}'(\beta)}{1-2c_{mzh}'(\beta)} \cdot \left[-\tan\beta + c_{fxh}'(\beta)\right] + c_{fxh}'(\beta) \tag{16}$$

The next step now is to compute the relative tow force F'_T by means of (14) or (15). The both equations supply the same value.

The third and final stage is to evaluate the relative hull lateral force F'_{yH}, according to (11), for the purpose of obtaining the speed v – refer to (13). Hence the speed v is assigned to particular values of β and δ, dependent of course on the absolute thruster force. The higher is this force, the higher is the speed. If we plot the solved values of F'_T, β and δ versus the speed v for particular thruster force (e.g. 10 t or 50 t), we can state that the input thruster force is responsible for scaling the horizontal axis.

Simplifications and numerical results

Let's suppose the first possible simplification in that the hull hydrodynamic force is normal to the hull centre plane, i.e. the hull resistance (precisely the hull longitudinal force) is negligibly smaller than the lateral force. We are thus assuming:

$$c_{fxh}(\beta) = c_{fxh}'(\beta) \equiv 0 \tag{17}$$

This assumption $c'_{fxh} = 0$, which we call further <u>the single simplification</u> case, lead – see (16) and (14) – to the following final form:

$$\begin{cases} F_{yT}' = -\dfrac{\cos\delta}{\sin\beta} \\ F_{yH}' = -\left(\sin\delta + \cos\beta \cdot F_{yT}'\right) \\ \tan\delta = \dfrac{1 - 2\cdot c_{mzh}'(\beta)}{-\tan\beta \cdot \left(1 + 2\cdot c_{mzh}'(\beta)\right)} \end{cases} \tag{18}$$

For the assumption $c'_{mzh} = 0$ (theoretically justified for the needs of sensitivity analysis, but hardly proved in practice), we have:

$$\begin{cases} \tan\delta = \dfrac{1}{-\tan\beta + 2\cdot c_{fxh}'(\beta)} \\ F_{yH}' = -\left(\sin\delta + \cos\beta \cdot F_{yT}'\right) \\ F_{yT}' = \dfrac{\sin\delta}{\cos\beta} \end{cases} \tag{19}$$

The relations (19) are however not considered later in detail and not simulated in this paper.

For both assumptions together: $c'_{fxh} = 0$ and $c'_{mzh} = 0$, which we will below refer to as <u>the dual simplification</u> case, one can read:

$$\begin{cases} \tan\delta = \dfrac{1}{-\tan\beta} \Rightarrow \delta = 90° + \beta \\ F_{yH}' = -2\cos\beta = -2\sin\delta \\ F_{yT}' = 1 \end{cases} \tag{20}$$

The performance of both simplifications – single (18) and dual (20) one – is presented in figures 4 and 5. The behaviour of the full model, through (11), (14), and (16), is demonstrated in the next figure 6. Thus figures 4 to 6 are arranged from the simplest to the most advanced instance of the model. In all these diagrams the usual range of escort speeds up to about 10 knots (5 m/s) is considered only.

The simplifications (18), (19), (20), and possible other more or less sophisticated approximations to the hydrodynamics of the hull and the thruster as well, are designed of course to provide the limits of operational control variables if the tug is subject to optimisation and innovations.

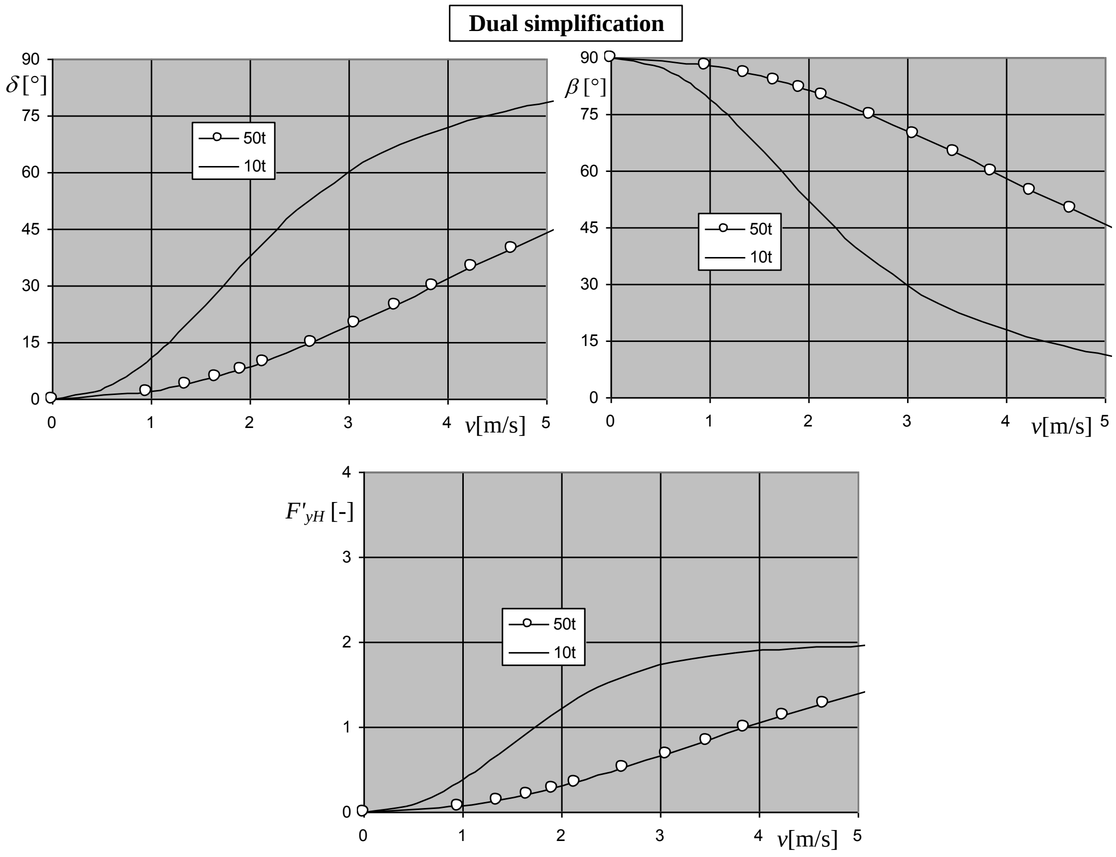

Fig. 4. Dual simplification model performance for practical escort speeds

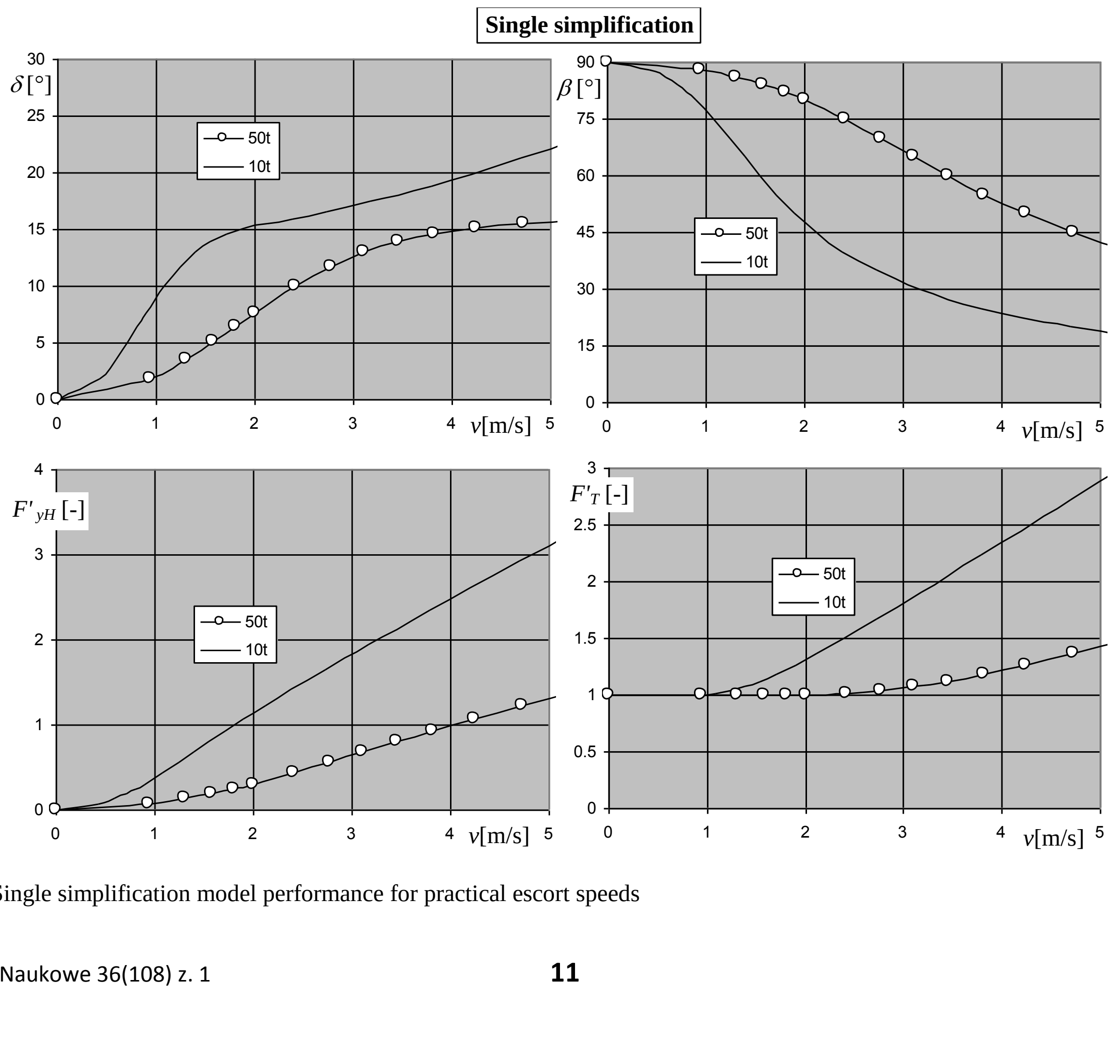

Fig. 5. Single simplification model performance for practical escort speeds

Figure 4 does not contain the relative tow force F'_T since it simply yields unity, i.e. there is no amplification of the propeller force. The efficiency of indirect mode of towing, through advantage of the hull hydrodynamic force for non-zero speed, gets essentially none. The hull lateral force, dominating

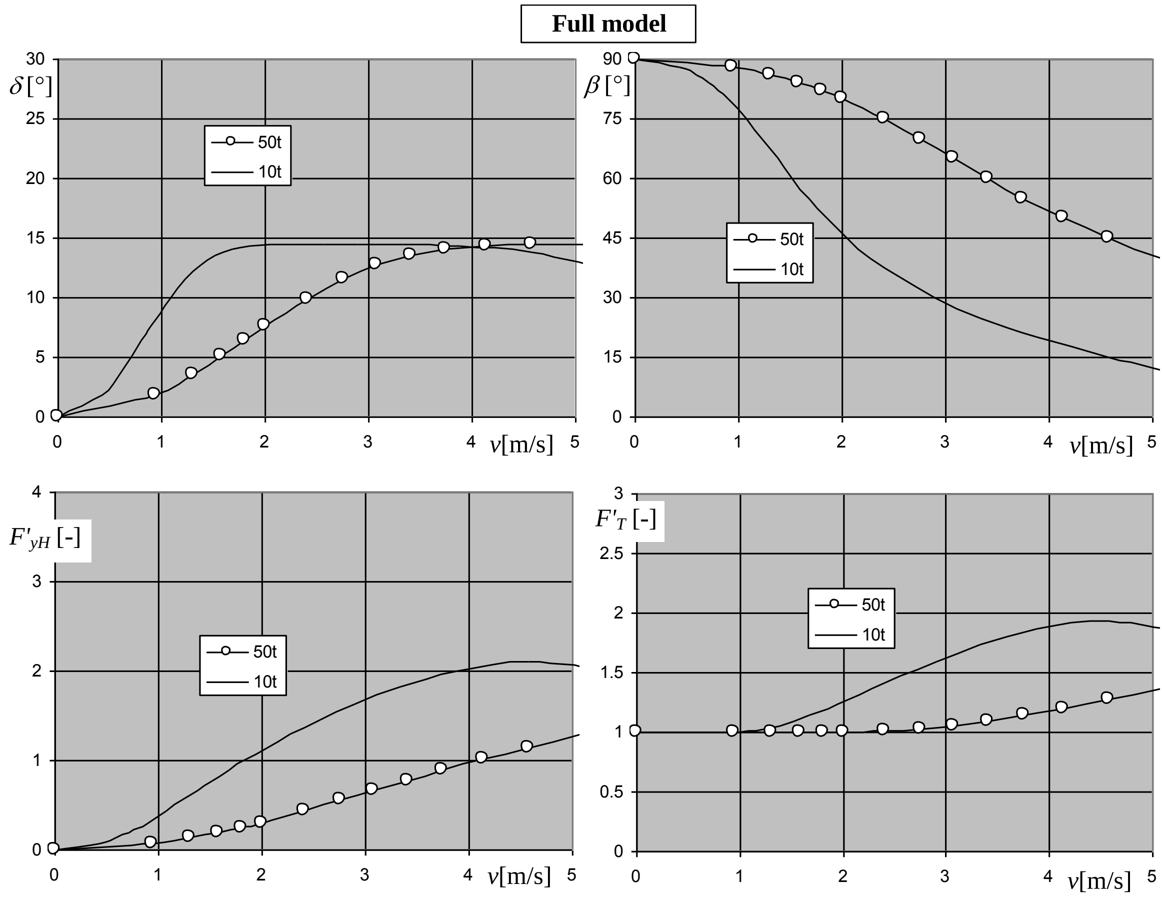

Fig. 6. Full model performance for practical escort speeds

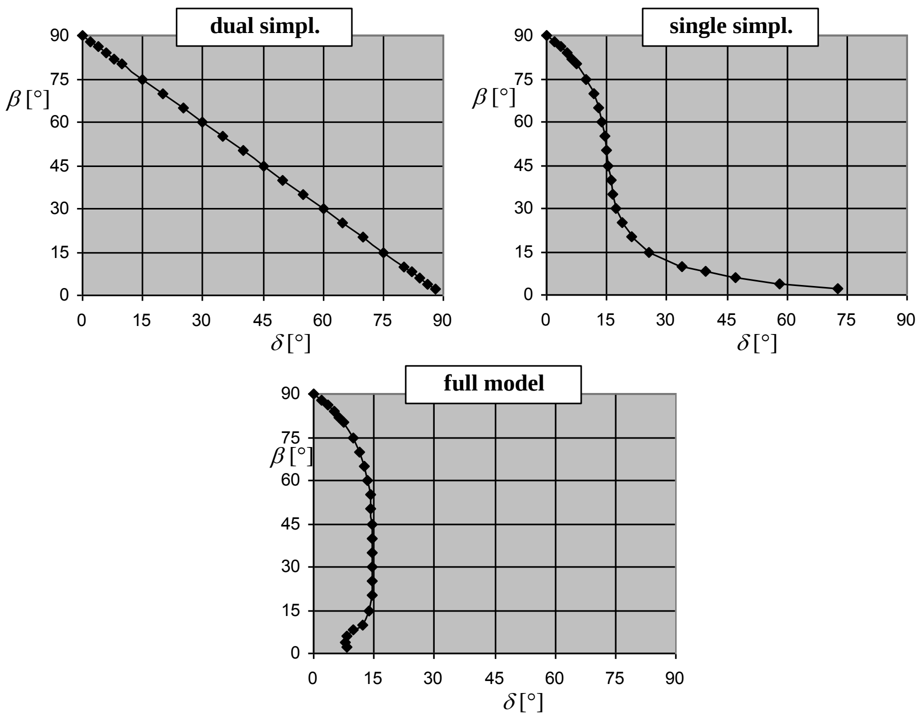

Fig. 7. Drift-thruster angle relation for whole range of drift angle

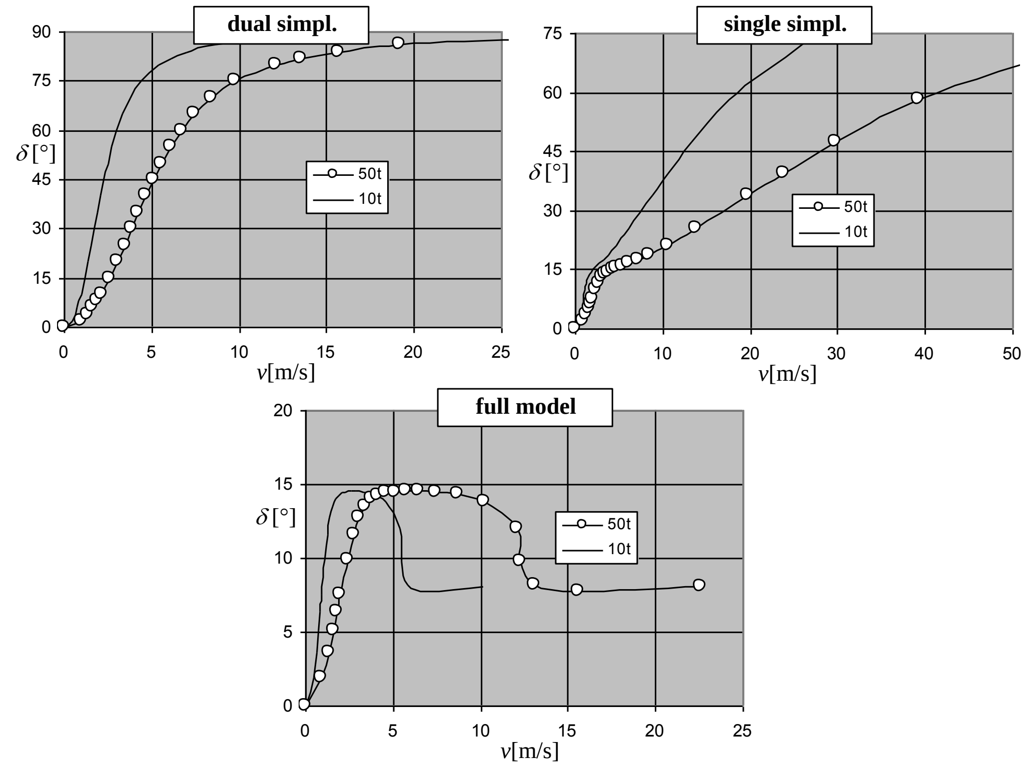

Fig. 8. Performance of all models within the whole range of speed

the hull total force (or contributing in 100% to the latter as in this case of dual simplification), and represented by F'_{yH}, assumes maximum value of 2.

For zero speed, the tug has to push normal to the hull (β±90°), i.e. in the "direct" way. The higher is the speed, the higher is the thruster angle and the lower the drift angle. However, this not specific to the dual simplification. Though there is no amplification of the thruster force for the dual simplification, if escort speed exists, the tug has to be less inclined to the ship's hull to "transfer 1:1" the thruster force to the push force. The thruster angle is surprisingly linearly dependent on the drift angle, which is graphically presented in figure 7, together with the output of other cases of the model.

However, the plots of figure 7 comprise the whole range of the drift angle in that the higher speeds (more than 5 m/s as adopted in figures 4 to 6) must be involved to get the equilibrium. Figure 8 gives the impression of the maximum speed in each case of simplification.

The largest differences between our instances of the model appear in the charts $\delta(v)$ – the top ones in figures 4 to 6, which is additionally confirmed in figure 7. For the dual simplification case the required δ is roughly twice as large as that in the other two cases. This curve shall be treated as the limiting range of the thruster angle.

For the single simplification (connected with rather weak assumption $c_{fxh} = 0$) and full model at speeds higher than 1 m/s (2 knots) we are able to develop nearly twice higher push force than the thruster force – refer to the last charts in figures 5 and 6. Thus the indirect mode has proved its excellence.

However, if the hull longitudinal resistance is omitted in the tug's hydrodynamic model (the single simplification case), the relative tow force F'_T can surprisingly go far beyond the value of 2, but this is coupled with very high speeds.

Both models, the single simplification and full ones, perform quite similarly within practical escort speeds.

Figure 7 proves that the full model is the only model that limits the thruster angle, which is remarkably below 15° in all operational situations. Of course, the full model is also based on some approximations to the tug's hydrodynamics.

Within practical escort speed range the drift-speed relation β–v is the least vulnerable to simplifications made to the model. However, the version of simplification is connected with completely a different speed range, leading even to an enormous speed – see figure 8 – though mathematically correct, rarely to be realised. This huge speed is related to small drift angles in the order of a few degrees.

For the full model, the thruster angle is the same for the wide range of speed, thus it is insensitive of the escort speed. This way some adjustments can be made only by means of the drift angle (the inclination angle to the ship's hull). This is of course a kind of unusual steering guidance for towmasters.

Conclusions

The obtained charts in the paper could serve as rough and clear guidance for towmasters.

Secondly, such data are also very useful for assessing and optimising more sophisticated models, where e.g. hull-thruster and thruster-thruster interactions are included, by providing a basis for further sensitivity analyses. In the latter context, one should be aware that incorporating some special hydrodynamic effects into the model would not always result in a significant change of a tug control parameters as essential from the viewpoint of a towmaster.

Thirdly, one can attempt to design the tug controllers both for fast- and real-time preliminary simulations, when the human input or interaction from a towmaster (to a tug simulator interconnected with the assisted ship simulator) is not yet necessary. In safety studies, this of course only concerns early stages of investigations, but is not so time and human resources consuming. Adequate tug automatic controllers guarantee proper "time constants" of tug response under various environmental and operational circumstances. This is much better than commonly adopted in simulators the so-called vector tugs (in terms of force and its direction, and their rates of change) as activated from the instructor's station by the instructor himself. While constructing control laws for the tug automatic controllers, the research on the steady-state manoeuvring conditions, including that undertaken in the present study, would certainly provide great help.

The presented model of tug's equilibrium of forces is really encouraging. It is worthwhile to next carry out investigations of the effects of the forward contact point abscissa and the thruster aft location. As mentioned before, they have been assumed equal in absolute magnitude – both are half the tug's length and set on both ends of the tug.

The algorithm is very general and the adopted simple formulas for tug's hull hydrodynamics provide exemplary numerical values. However, any definition, including the lookup-table stored data, can be easily linked to the algorithm. Thus the next valuable research steps could also be testing the model with real data of tug's hull hydrodynamic coefficients and improving the thruster submodel.

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