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44-100 Gliwice, ul. Krzywoustego 7, POLAND
phone: (+48 32) 237 26 58

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ON ORTHOGONAL PROJECTION OF RIGHT ANGLE

Edwin KOŹNIEWSKI

Bialystok University of Technology, Faculty of Civil and Environmental Engineering,
ul. Wiejska 45E, 15-351 Białystok, POLAND
e-mail: e.kozniewski@pb.edu.pl

Abstract: In some textbooks Descriptive Geometry characteristic invariant of orthogonal projection is used in the opposite sense than is formulated. The paper presents the complete formulation of the theorem and its simple proof suitable for presentation to students.

Keywords: invariant, orthogonal projection, orthogonal projection of right angle

1 Introduction

It happens that in the Descriptive Geometry textbooks invariant characteristic of rectangular projection is used in the opposite sense to its formulation.

In the textbook by B. Grochowski we can find: "The orthogonal projection of right angle, of which one arm is parallel to the projection plane is a right angle." (Grochowski, 1995, p. 18). Later (p. 91), the author, characterizing the straight line perpendicular to the plane, states the sufficiency of the perpendicularity condition, but for two straight lines: horizontal and frontal ones. F. Otto and E. Otto state: "Consider any straight line *a* lying on the projection plane or parallel to the projection plane and skew line *b*. Assume that they are perpendicular to each other. One can easily prove, based on elementary stereometry, that the projections *a'* and *b'* of these lines are perpendicular to each other. This theorem can be reversed, ie. we can prove that if $a' \perp b'$ and $a = a'$ (or only $a || a'$), then already must be $a \perp b$." (Otto & Otto, 1994, p. 38). Other authors confine themselves to formulating this invariant as a conditional phrase. Comprehensive (though without proofs) discussions can be found in the monograph: Geometria wykreślna by St. Polański (editor). We have got two theorems: "If two lines are perpendicular and one of them is a horizontal line, and the remaining one is not perpendicular to projection planes, then the orthogonal projections of these lines are perpendicular." and "If the orthogonal projections of two straight lines *t* and *u* are perpendicular and at least one of these lines is a horizontal line, then the lines *t* and *u* are perpendicular."(Polański et al., 1975, 55-56). In practice of teaching the basics of orthogonal projections there is a fear that the invariant formulated directly (as a sentence $... \rightarrow u \rightarrow v$) will be used as the reverse ($... \rightarrow v \rightarrow u$) without further explanation. In this contribution we give the full proposal, concise perspective on this important property.

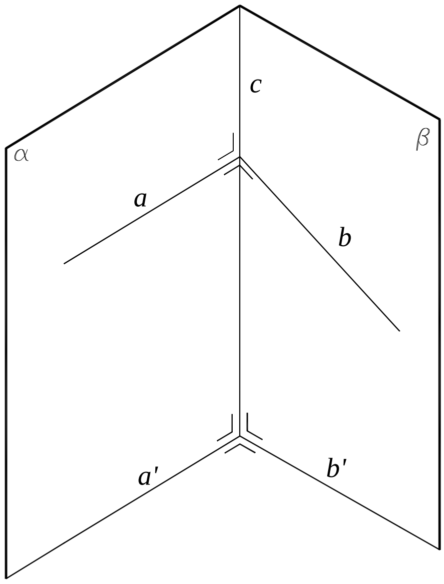

Figure 1: Perpendicularity $a' \perp b'$ and obvious perpendicularity $c \perp a'$ together with $a \| a'$ induce $a \perp b$

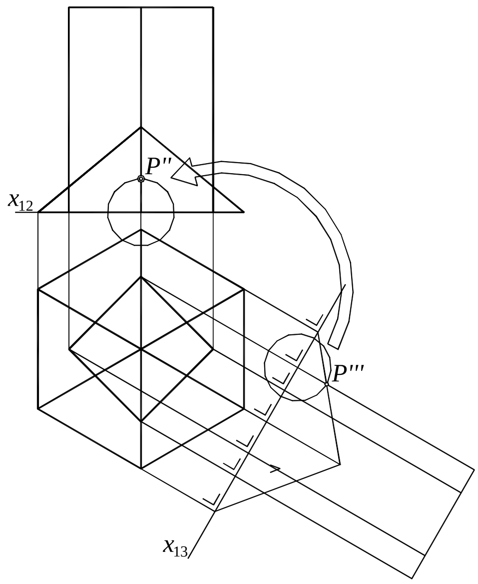

Figure 2: Application of the invariant in construction of an auxiliary view: the projections of projection rays (projectors) are perpendicular to x_{13}, therefore the projection rays are perpendicular to π_3, hence π_3 plane is perpendicular to two desired sides of the base of pyramid (hexagon)

2 The proposal to formulate an invariant on right angle in the orthogonal projection

The present invariant (Characteristic Invariant of Orthogonal Projection - CIOP) is formulated as follows.

CIOP. If one of the arms of an angle is parallel and another is not perpendicular to the projection plane, then the angle is right if and only if its orthographic projection is a right angle.

Proof: Let π be any projection plane; a and b are the lines containing the arms of a given angle; a' and b' are the lines containing the arms of an orthographic projection of this angle, α and β are the edge planes of the arms of the considered angle, and $c = \alpha \cap \beta$ (Fig. 1). From the orthographic projection conditions, regardless of what is the angle, we have the following sentence (from the assumption):

p: $a, a' \subset \alpha, b, b' \subset \beta, c \perp \pi, \alpha \perp \pi, \beta \perp \pi, c \perp a', b'$ and without loss of generality of the second sentence

q: $a \parallel \pi, b \neg \perp \pi$. ($\neg$ denote negative relation)

Then

r: $a \perp b$, s: $a' \perp b'$.

After such denotations, formally, theorem CIOP has the following form:

$p \wedge q \rightarrow (r \leftrightarrow s)$.

We have to prove two theorems:

'$\rightarrow$': $p \wedge q \rightarrow (r \rightarrow s)$;

'$\leftarrow$': $p \wedge q \rightarrow (s \rightarrow r)$.

'$\rightarrow$': We have $a \perp b$ and $a \| a'$. Therefore $a' \perp b$ (it does not matter if a' and b are skew). But $c \perp a', b'$. Hence

$a' \perp b, c$, i.e. $a' \perp \beta(b, c)$, so $a' \perp \beta(b', c)$. Hence $a' \perp b'$.

'$\leftarrow$': We have $a' \perp b'$ i $a \| a'$. Therefore $a \perp b'$ (it does not matter if a and b' are skew) i $a \perp c$. So we have $a \perp \beta(b',c)$, in other word $a \perp \beta(b,c)$. Therefore $a \perp b$.

3 Conclusion

The formulated and proved (in a simple way) property can be successfully presented in the academic lecture or in other classes. It is an opportunity to repeat and/or supplement information about the relationship of perpendicularity in three-dimensional Euclidean space, as well as show an example of logical analysis of geometric theorems. So, we have a small sample of shaping spatial imagination of exercise in mathematical logic.

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O NIEZMIENNIKU CHARAKTERYSTYCZNYM RZUTOWANIA PROSTOKĄTNEGO

W niektórych podręcznikach geometrii wykreślnej niezmiennik charakterystyczny rzutowania prostokątnego jest stosowany w odwrotnym znaczeniu niż jest formułowany. W pracy przedstawiono pełne sformułowanie twierdzenia i jego prosty dowód, nadający się do zaprezentowania studentom.

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