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TOTAL INERTIAL TOLERANCING, A NEW WAY TO DRIVE PRODUCTION

Inertial tolerancing is a new concept of tolerancing which has done a first standardization in France (NF XP E 04-008 (2009). The paper presents a generalization of the inertial tolerancing: total inertia. The goal of total inertial tolerancing is to use the information include in the numeric description of the product. The Total inertial tolerancing defined "consistent functional subset" and different coordinate systems. For each of these subsets, we defined the maximum variability accepted (maximum inertia) from digital target. Inertia is the mean square deviation of the differences between the actual part and the target, measured in accordance with normal to the surface. Each functional subset will be identified by different colors.
The purpose of the production is to produce parts with the least variability compared to the numerical shape. The production problem can be represented by two vectors: The vector of the deviations from the target on all measured points, and the vector of the control factors. Thereof, the question is: What is the value to apply on each corrector to minimize the vector of deviations? A reply is given by the total inertial tolerancing of which the link between the maximum inertia and the production is strong. Thus, the problem consists into compute the pseudo-inverse matrix of the relation between the deviation and impact vectors. This pseudo-inverse matrix allows minimizing the least squares deviation, in other words, minimizing the inertia. In this paper, we will present an example of inertial tolerancing specification of a complex part and we will show how to adjust a production with its new approach.

1. INTRODUCTION

Inertial tolerancing presented for the first time in 2001 [1] allows the utilization of a statistical partitioning of dispersions without functional risk. This novel concept reaching maturity at the present time has become the subject matter of a standard for the first time [2] The inertia of a characteristic is an indication of the magnitude of variation about the desired target. It is defined by the following relation (eq1):

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$$I = \sqrt{\sigma^2 + \delta^2} \quad (1)$$

With σ the standard deviation and δ the deviation from the target

Inertial tolerancing consists in defining the maximum allowable value of this parameter as a tolerance on the characteristic. It is simple to show that:

– inertia is equal to the standard deviation of the distribution whenever production is centered on the target ($\delta = 0$)
– the maximum allowable off-centering ($\sigma = 0$) is equal to the inertia
– the acceptance region in a graph of δ, σ has the shape of a semicircle (Fig. 1). Therefore, inertia allows for some excursion of the mean about the target related to the value of the production standard deviation.

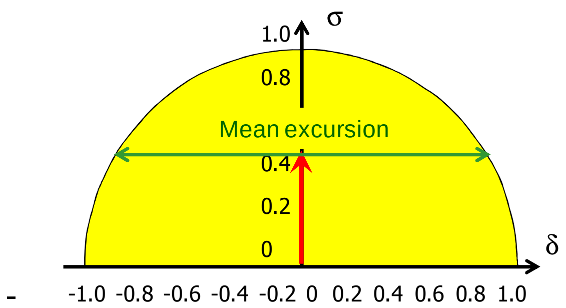


Fig. 1. Inertia in a δ, σ chart

If there is a relationship between a functional characteristic Y and multiple characteristics x, such that $Y = f(x_1 \ldots x_k$ inertial tolerancing allows to ensure a desired non-conformity rates on Y giving more margin to production [3]. The inertial tolerancing is an evolution of the statistical tolerancing that eliminates the well known risks of this mode of tolerancing with the decentred lots.

From a global standpoint, tolerancing is a language used to describe the geometry of a component that we want to achieve. This language has evolved significantly these past few years and has achieved a level of complexity [4,5,6] which makes it difficult to utilize in production. Whereas current geometric tolerancing is capable of representing functional limits on characteristics well [7], its drawback is that under its current format it produces specifications that are fairly difficult to meet in production because they are divorced from machine settings. To illustrate this point, let us consider the location of a bore hole. From a functional standpoint, this location is representative indeed but in manufacturing, control of production machinery frequently involves two movements in *X* and in *Y*, for instance. Therefore, there necessarily is a transfer of characteristics to transition from a product functional specification to a manufacturing specification. This translation inevitably involves a reduction in allowable variation and a considerable time in production setup. The other drawback of current tolerancing practices is that they use zone tolerancing. This method of limiting variation is problematic with respect to compatibility with a statistical approach to tolerancing unlike the inertial representation which is statistical by nature.

The proposal being made in this paper extends the inertial principle to the product in its entirety. We are proposing a development in the method of establishing tolerances for product properties based on existing standards [8,9,10,11,12] which would not compromise expected functionality. We proceed from the assumption that it is possible considerably to simplify the language of tolerancing if it were taken into account that information intrinsic to the desired target shape of the component is available from its CAD digital image. None of the current methods factor in recent developments in technology to a sufficient degree. At the present time, we have digital models providing images of desired targets, CNC machines, and 3D measurement instrumentation; however, the development of tolerancing methods has not completely caught up to this digital situation.

In addition to simplifying the method of representing expected functionality, total inertial tolerancing being the best model of statistical combinations of tolerances, enables tolerances to be specified in a manner that is fully compatible with production control thus eliminating the need to transfer characteristics and limiting the amount of production setup.

2. ESTABLISHING CONFORMITY BASED ON TOTAL INERTIA

2.1. DEFINITION OF TOTAL INERTIA

In the absence of production variability, tolerancing is useless because it is sufficient to specify the desired target values. Where in a paper layout the support does not contain any reliable information, in the case of a digital layout the CAD model contains information that must be taken into account. Thus, under the hypothesis that there is no production variability, the target digital model is by itself the repository of all the information on the desired target, and there is no need even for minimal notes on the layout.

Unfortunately, this no-variability hypothesis is not realistic, and acceptable variability about the desired target value needs to be specified. Functional analysis helps identify the effect of surface variations with respect to another surface [13,14,15] on the expected functionality of a product. To limit such variations, current tolerancing practice dictates that the geometry be described based on a parameterization of the product's geometry. A product is accepted as conformant if the actual surface is located between two geometric zones resulting from product parameterization.

In the case of a complex geometry, such as a skew surface, many companies no longer undertake an exhaustive description of a part's geometry, and instead indicate by way of a specification only a form tolerance with respect to the ideal geometry as contained in a digital model. Thus, a following question may be asked: if this practice produces good results for complex surfaces, can it be extended to the case of simpler geometry?

The logic of total inertial tolerancing is consistent with this extension. We proceed from the hypothesis that geometry defined by a digital model is the ideal case and that the functional need is to limit deviations from this ideal geometry. The first response could be provided with the help of existing geometric approaches by generalizing default form tolerancing. However, in the 3D case same as in 1D, zone tolerancing is not compatible with the statistical approach to tolerances. Our proposal is then to extend the principle of inertial tolerancing and to impose tolerances on deviations from the target by setting a maximum threshold for the square root of the quadratic mean of the deviations from the target over a finite number of points randomly distributed throughout the entire surface.

For surface i, inertia I is computed using the following relation:

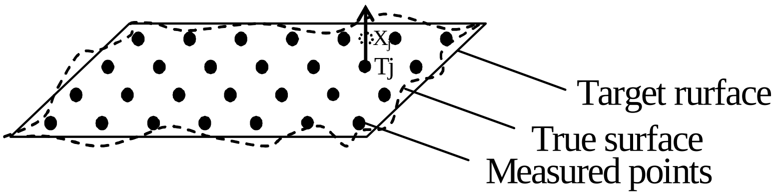


Fig. 2. Definition of total inertia

$$I_i = \sqrt{\frac{1}{n}\sum_{j=1}^{n}\left(X_{ij} - T_j\right)^2} = \sqrt{\sigma_i^2 + \left(\overline{X_i} - T\right)^2} \tag{2}$$

with

X_{ij} : measure j on the surface i

σ_i : standard deviation of the measures on the surface i

T : Target point considered

n : number of measurement on the surface

I_i : surface inertia

We have demonstrated [16,17] that this measure of variability about the target provides a better representation of statistical behavior during assembly than the conventional zone specification.

2.2. DESCRIBING A COMPONENT IN TERMS OF TOTAL INERTIA

2.2.1. TOLERANCING OF COHERENT FUNCTIONAL SUB-ASSEMBLIES

Total inertial tolerancing defines "coherent functional sub-assemblies." For each of such sub-assemblies, acceptable maximum variability (maximum inertia) with respect to a digital target is defined. Then, tolerancing is simply the identification of these functional sub-assemblies represented by different colors, the definition of a product reference, and toler ancing of maximum inertia for each of the sub-assemblies.

Figure 3 shows an example of a component completely specified in terms of total inertia. In this example, the component reference is defined by the ordering:

Plane A /Axis B / Plane C for the final rotation

These three surfaces make up the first functional sub-assembly. The inertia for this zone is 0.0132. To compute deviations from the target, the ideal shape of the component is balanced by least squares successively with respect to Surface A in the first three degrees of freedom, then with respect to Axis B, and with respect to Plane C.

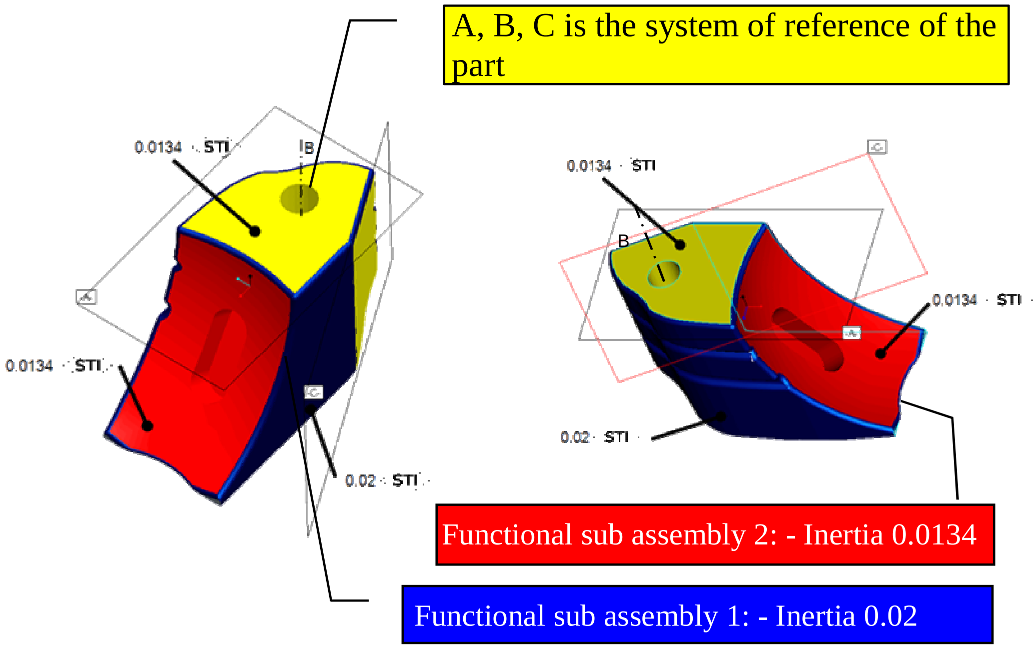


Fig. 3. Example of a component specified in total inertia

With the target component in position, the value 0.0134 signifies that the square root of the quadratic mean of the deviations of the actual component from the target component as measured along a normal to the surface should not exceed 0.0134. For the other two functional sub-assemblies, the blue and the red, maximum acceptable inertia is also specified. Given the greater functional weight of the red surface, its inertia is lower. In this paper, we are not going to address detailed computation of maximum acceptable inertia which has been presented in multiple works [18,19] for the 1D case. Research for the 3D case is ongoing.

2.2.2. SURFACE REFERENCES

In the example in Fig. 3, the component has only a single reference; however, it is quite possible to define multiple references for the component and for each functional sub-assembly to specify the reference that helps position the target surface.

2.2.3. PREFERRED DIRECTION

In some cases, it is desirable to accept different variabilities in different directions. This is the situation in Fig. 4.

In this case, it is necessary to define inertia along a preferred axis. Acceptable total inertia is also defined for the entire surface as well as along a preferred axis. In this case, the local target is an axis parallel to the reference minimizing inertia. In the Fig. 4. example, inertia is defined with the reference A (plan) by the specification 0.1 and with the reference B (a preferred direction) by the specification 0.05.

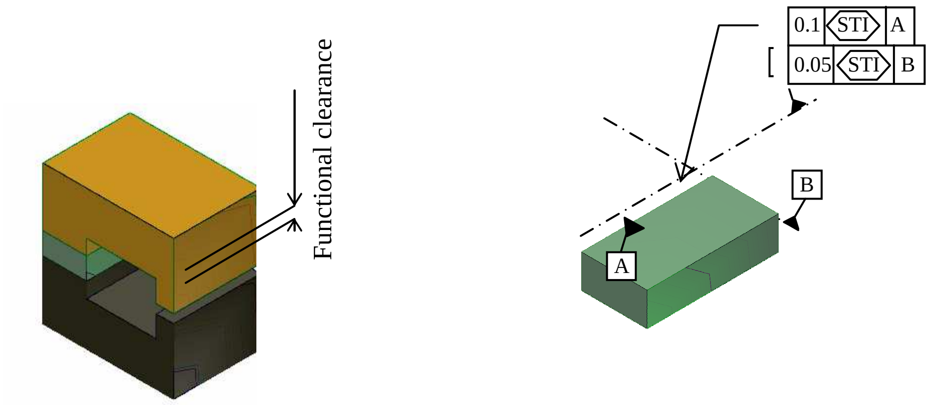


Fig. 4. Example of a prefered direction

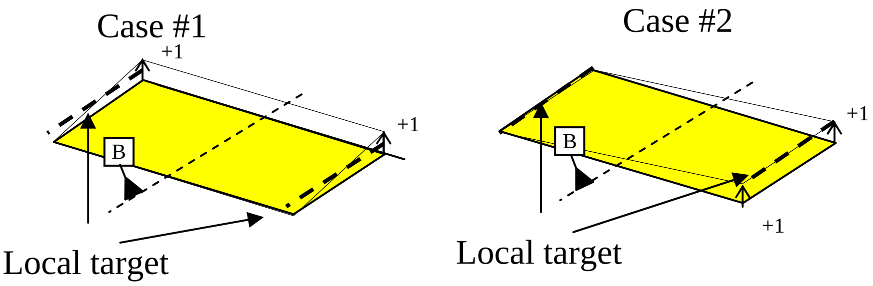


Fig. 5. Example of a prefered direction

The Fig. 5. described two situations very different in a functional point of view. In both case inertia with the reference A is equal. The difference between the two cases becomes apparent from the inertia computation in the preferred direction B.

In Cases 1 and 2, total inertia is computed using the relation (eq2), we find

$$I = \frac{1}{4}\sum \delta_i^2 = 0.5$$

The inertia computation in the preferred direction B is computed using the deviation from the local target parallel to the reference B minimizing inertia.

In case #1 we find: $I = \frac{1}{4}\sum \delta_i^2 = 0.25$

In case #2 we find: $I = \frac{1}{4}\sum \delta_i^2 = 0$

In the case of a total inertial tolerance of 0.5 and a tolerance in the preferred direction B of 0.2, Case #1 would be rejected whereas Case #2 would be accepted.

2.3. MOTIVATION FOR SPECIFICATIONS IN TERMS OF TOTAL INERTIA

The approach of total inertial tolerancing is not incompatible with the existing GPS standards. Like simple inertia which was described in a standard [2], total inertial may easily be used within the framework of GPS. The motivation for selecting a representation in terms of total inertia rather than the conventional geometric specifications is based on several points.

Speed of Compiling a Specification

A single specified criterion applies to all the surfaces of a functional subassembly. This eliminates the need to decompose these surfaces into geometric elements and to lay out such geometric elements.

Ease of Reading and Understanding a Layout

When shown in terms of total inertia, each functional subassembly is color coded. Total inertia of a subassembly specified using a single parameter provides a very simple way of placing each subassembly into an hierarchy based on its relative weight.

Accounting for Statistical Assembly Constraints

Inertia provides a better compromise than assembly by zone in the worst case or by statistical analysis.

Ease of Measurement Using 3D Instruments

Inertia is extremely well adapted to the way 3D measurement instruments are used. When controlling by inertia, it is sufficient to feel points distributed uniformly over the entire surface and to measure the distance between the ideal and the actual component.

Consistency with Machine Control

One of the frequently quoted disadvantages of GPS specifications is the problem manufacturers have in making the transition from layout to machine control. GPS geometric tolerancing describes the conformity of a product well but does not lend itself to machine control. We will demonstrate below the ease of control and the consistency between specification and production that exists when working in terms of total inertial tolerancing.

3. RUNNING PRODUCTION IN TERMS OF TOTAL INERTIA

3.1. APPROACH TO INERTIAL CONTROL

The objective of manufacturing is to produce parts with a minimum of variation with respect to the ideal part represented by the digital target. Given that every production process inevitably causes deviations, machine control is required to reduce these deviations to a minimum.

To run a production process, there is a vector of action variables *C* which in general is of a fairly small size. For a numerically controlled machine, for example, these action variables are tool adjustments whereas for an injection molding machine these are process variables such as holding pressure, mold temperature.

On the other hand, the manufactured product has deviations from the ideal product. These deviations are represented by a vector of deviations between the target and the real value which we will refer to as *E*. This second vector contains all the measurements made on the product. Generally, this is a vector of a large dimension. It may contain thousands of points for a scanned part.

Finally, it is frequently possible either through computation or experimentation to derive a matrix allowing to associate the vector C with the vector E by the relation

$$E= X.C \quad (3)$$

In the case of numeric control, for instance, it is very easy to compute the effect of an adjustment on the deviation of a measured point, and in the case of an injection molding machine, a test plan could be made to vary the vector E, thereby generating the matrix X empirically.

Following correction, we obtain

$$E1 = E - X.C + \varepsilon$$

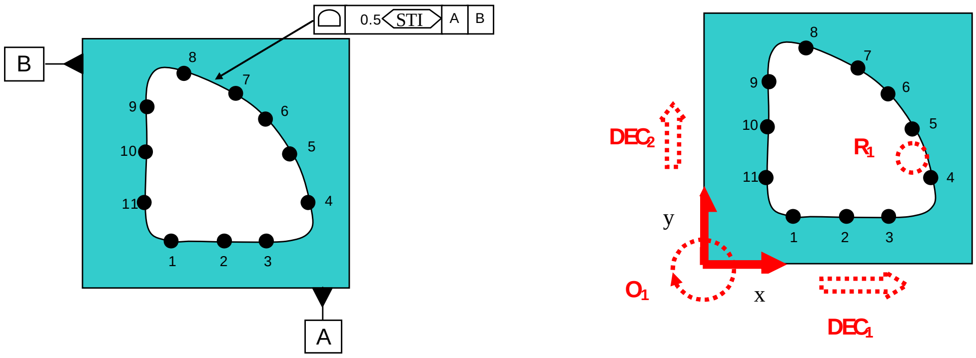
The question of production control may then be formulated in the following terms: What are the components of the vector *C* which would allow me to minimize the residuals following correction ε, or even $\varepsilon'\varepsilon$ which is but predictable adjusted inertia. The answer to that question is to minimize the least squares using the Gaussian pseudo-inverse:

$$C = (X'X)^{-1}X'E \quad (4)$$

We also note the consistency of the specification in terms of total inertia in the plan and the issues of process control. In fact, minimization of the least squares is nothing else but the minimization of total inertia! Unlike conventional geometric tolerancing, there is no transfer of parameters to perform between the schematic wherein a part is defined and production control. The total inertial approach is an approach that is consistent from the description of a functional need all the way through production.

3.2. SIMPLE EXAMPLE OF CONTROL IN TERM OF TOTAL INERTIA

Figure 6 (a) shows the reference part built from the datum A, B and C which is the normal datum to AB. The form is defined by a set of 11 points. Each measured point is following the normal of the target surface. The geometry of the profile can be adjusted by two locating correctors (DEC1, DEC2), rotation corrector (O1) and radius corrector (R1). We note that O1 corresponds to the rotation of the base where is positioned the part.



(a) (b)

Fig. 6. Example of control in total inertia

Incidence matrix X

From the coordinate of the 11 points, the incidence matrix X is defined with using the relation of small displacements (eq 5):

$$X_{ij} = \left(\overrightarrow{T_i} + \overrightarrow{\zeta_0 M_j} \wedge \overrightarrow{R_i}\right)\overrightarrow{n_j} \qquad (5)$$

With

X_{ij}: incidence of the corrector i on the point j

T_i : vector translation of the correction

ζ_0: point of application of the rotation

R_i: Vector rotation

n_i: the normal vector at the surface at the point i

The dimension of the incidence matrix is composed by eleven line and four columns which correspond to the number of measured point and the number of corrector to adjust the deviation of the vector *E*.

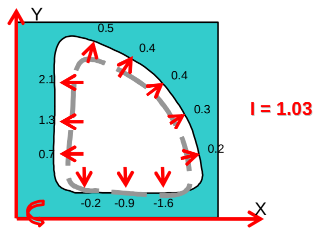
$$X = \begin{bmatrix} 0 & 1 & 1 & -3.5 \\ 0 & 1 & 1 & -6.75 \\ 0 & 1 & 1 & -10.65 \\ -0.98 & -0.17 & 1 & -1.62 \\ -0.86 & -0.5 & 1 & 0.73 \\ -0.71 & 0.71 & 1 & 0 \\ -0.5 & -0.86 & 1 & 0.73 \\ -0.17 & 0.95 & 1 & 1.62 \\ 1 & 0 & 1 & 10.65 \\ 1 & 0 & 1 & 6.75 \\ 1 & 0 & 1 & 3.5 \end{bmatrix}.$$

The following table is the matrix $X^* = (X'X)^{-1}X'$.

Table 1. Matrix X and X*

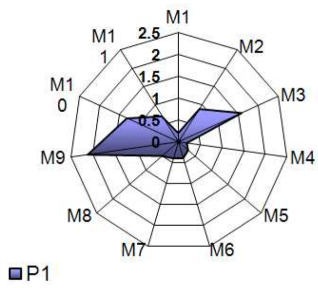
	1	2	3	4	5	6	7	8	9	10	11
DEC_1	-0.23	-0	0.28	-0.41	-0.24	-0.1	0.06	0.259	-0.14	0.145	0.38
DEC_2	0.38	0.14	-0.14	0.26	0.06	-0.1	-0.24	-0.41	0.281	-0	-0.23
R_1	0.09	0.09	0.094	0.088	0.087	0.087	0.087	0.088	0.094	0.094	0.094
O_1	0.03	-0.01	-0.06	0.039	0.018	0	-0.02	-0.04	0.057	0.01	-0.03

From Table1 and eq 4, it is possible to deduce an adjustment of the measured deviation illustrates Fig 7 (b). Fig. 7 (a) shows the target form and the measured form in dashed line. The inertia of the measured form is equal to 1.03. Therefore, it is important to adjust these deviations to assume the specification. The graphic presents Fig 7 (b) and corresponds to the absolute deviation for each point Mi.



(a)

Point Mesuré	P1
M1	-0.2
M2	-0.9
M3	-1.6
M4	0.2
M5	0.3
M6	0.4
M7	0.4
M8	0.5
M9	2.1
M10	1.3
M11	0.7



(b)

Fig. 7. Defect of the measured Part

From the deviation Fig. 7(b), we obtain the values of adjustment (Tab. 2). The proposed adjustment leads to an IEA equal to 97, 67 %, thus an inertia about 0.02.

Table 2.Value of the corrector to adjust the measured deviation

	Corrector
DEC1	0.11
DEC2	-0.22
R1	0.29
O1	0.19

3.3. MOTIVATION FOR CONTROL IN TERMS OF TOTAL INERTIA

Control in terms of total inertia provides a way to control the adjustments directly without the mediation of geometric parameters to associate surfaces. In addition to the simplification which we have already pointed out, the overriding motivation for this method is the precision of computing the corrections. In fact, the very nature of the computation of corrections using the pseudo-inverse makes it applicable to all available information in its entirety. To illustrate this point, let us take the computation of the translation C1. Points 9, 10, and 11 provide information on the position in X but there is also information in Points 4, 5, and 6. Under conventional control methods, the parameterization required to judge conformity with geometric specification causes information loss.

The second point is in the consistency between the functional specification and control. Control in terms of inertia aims to minimize the inertia of functional surfaces thereby producing parts with the smallest possible deviation from the ideal target. This approach produces a lot more quality than the conventional approach which consists in including all the surfaces in a zone.

Finally, let us emphasize the speed of adjustment and correction in the case of complex 3D parts. Once the variables to be adjusted are selected, the matrix X and the pseudo-inverse are computed automatically clearly formalizing the rules of control. With this approach, it is equally easy to control a 5-axis machining center and a machine with but a single axis.

4. CONCLUSION

In this paper we have presented the concept of total inertial tolerancing as well as an approach to process control which must go with this new tolerancing method. This approach has numerous properties desirable for many manufacturing companies:

- simplicity relative to geometric tolerancing;
- ease of comprehension of tolerance layouts in terms of total inertia;
- consistency of specification, measurement method, and production control;
- very significant reduction in the need to expend effort to transfer tolerances when retooling;
- very high quality of product made using total inertial control.

This approach is the object of multiple experiments in Switzerland and France, in particular at the Arve Industrie Mont Blanc pole of competitiveness. All of our results indicate that a breakthrough is achieved in the quality of parts manufactured using this concept. These results point to the possibility of mass-manufacturing new complex products with much lower variability than is possible to achieve currently.

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