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THERMODYNAMIC CALCULATION OF A ROTARY ENGINE WITH EXTERNAL HEAT SUPPLY BASED ON THE IDEAL RALLIS CYCLE

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Abstract: The design and kinematic scheme of the operation of a rotary external combustion engine with offset shafts have been developed. Expressions are obtained that make it possible to calculate the values of the increasing and decreasing functions of the working volume of the hot and cold cavities with a change in the angle of rotation of the rotor. An expression is obtained for calculating the compression ratio in the cold cavity of a rotary heat engine with an external heat supply. An expression has been determined that makes it possible to calculate the total torque of a rotary external combustion engine. A comparative analysis of the torque values of a rotary heat engine with an external heat supply and a Wankel engine is carried out. An assessment of the efficiency of an external combustion engine with offset shafts is carried out. Based on the thermodynamic calculations using ideal Erickson and Rallis cycles for a rotary external combustion engine, the processes occurring inside the hot and cold cavities of a heat engine are described. The thermodynamic condition parameters at the characteristic points of the cycle are determined and expressions are obtained that determine the thermal efficiency of the ideal Erickson and Rallis cycles in relation to the considered external combustion engine. A method for calculating the ideal cycle for an external combustion engine with offset shafts is presented.

Key words: thermodynamic calculation, ideal cycle, work, heat, external combustion engine

1. INTRODUCTION

Nowadays, there is a strong demand for a heat engine with a higher thermal efficiency than that of existing power plants. Thus, this paper investigates the possibility of creating an alternative setup design, which is called a rotary external combustion engine.

The paper [1] considers Stirling piston engines with heat recovery, which operate according to the closed thermodynamic Stirling cycle, and piston heat engines with heat recovery, operating according to the Erickson open thermodynamic cycle. The design and operation of Stirling piston engines are discussed by Kruglov [2]. They are compared with existing designs of internal combustion engines. In the studies by Myshinsky and Ryzhkov-Dudonov [3] and Brodyansky [4], the principles of operation of various designs of a Stirling piston engine of external combustion are presented, and the main features of its thermodynamic cycle are considered. Campos et al. [5] consider the configuration of the Stirling engine, consisting of two cylinders, a regenerator and a sliding disk drive mechanism.

The paper presents a mathematical model that combines fundamental and empirical correlations. The feasibility of the Stirling engine operation using any external heat source is considered in Chen Duan et al. [6].

The results of studying the Stirling engine using three different methods, including the non-ideal adiabatic method, are presented in Toghyani et al. [7].

As shown in the literature [1–7], currently, the thermal efficiency of piston-type heat engines with an internal heat supply has reached its limit (50–60%). Achieving a higher thermal efficiency is not possible due to the large amount of heat loss in a running engine.

The analysis of literary sources described above made it possible to draw the following conclusions:

- there is a need to create an alternative power plant, such as an external combustion engine;
- the existing designs of external combustion piston engines with a crank mechanism are very cumbersome and involve significant losses of mechanical energy; and
- new design options for external combustion engines are needed, such as a rotary heat engine.

In Khafizov et al. [8], the design of a rotary external combustion engine is presented; its operation is described using the ideal Stirling cycle.

In the Ali Shufat et al. [9], an analytical simulation of a beta-type Stirling engine powered by solar energy was carried out, as well as a simulation study based on the model equations. Pressure, power and engine speed were considered as the main parameters.

In Paul and Hoffmann [10], a method is presented for optimising the trajectories of pistons in an operating model of a Stirling engine and a comparison is made between the optimised trajectories of piston movement and harmonic trajectories to increase power and efficiency.

Hasanah et al. [11] investigate the influence of the distance to the power crank of the Stirling engine on the angular speed of rotation of the wheels and the amount of generated electrical energy.

In Sirsath et al. [12], the influence of the temperature difference between the heat source and the radiator on the efficiency of three types of Stirling engines was established. It is shown that an increase in the temperature difference increases the efficiency of the engine.

In the study by Ladas and Ibrahim [13], a thermodynamic analysis of the cycle of a Stirling engine for a finite time is given, based on differential equations for the balance of mass and energy with the corresponding equations for the rate of heat transfer. The influence of the time of contact with the heating agent and regeneration on the output power and the efficiency of the Stirling engine is established.

In the study by Jana and Marekb [14], a mathematical model of time discretisation is presented, assuming that the cylinders are adiabatic spaces. The model makes it possible to optimise the dimensions of the main elements of the Stirling engine, such as heat exchanger, regenerator and cylinder and piston.

In Zhao et al. [15], a 3D simulation of fluid dynamics and heat transfer in an improved free piston Stirling engine was carried out. The turbulence model with improved wall treatment provides reasonable accuracy and stable convergence.

Podešva and Poruba [16] consider three types of mechanisms that analyse the movements of the piston and their behaviour. Special attention is given to the piston movement mode.

García et al. [17] compares the simulation results obtained on the basis of various thermodynamic models of Stirling engines, including the characteristics of both instantaneous and specified operating parameters.

Toghyani et al. [18] present the results of studies of the Stirling engine using three different methods, including the use of the non-ideal adiabatic method. The applied methods were compared, and the best results obtained were compared for similarity with the method of making decisions about the ideal solution.

The ideal Stirling and Erickson cycles are special cases of the ideal Rallis cycle, and therefore this cycle can also be used to describe the operation of a rotary heat engine with an external heat supply.

Currently, there is a problem of increasing the specific power of existing heat engines. To solve this problem, it is necessary to develop a new power plant, i.e. a rotary engine of external continuous combustion. Patents for one such plant are available in the literature [19, 20]. Stirling, Erickson and Rallis cycles can be used to describe the operation of a rotary external combustion engine. It should be noted that until now there has been no single methodology for the mathematical calculation of the mentioned cycles to describe the operation of heat engines.

There is a small number of studies that describe the operation of heat engines, which are based on the Rallis thermodynamic cycle. A patent describing one such heat engine is available in the literature [21].

The purpose of the study is a thermodynamic calculation of the ideal Rallis cycle, which quite accurately describes the thermal processes occurring in an external combustion engine.

2. DESCRIPTION OF THE HEAT ENGINE

Patents are available in the literature showing one of the options for a rotary external combustion engine [19, 20]. The diagram of the heat engine is shown in Fig. 1.

The technical result is achieved due to the fact that a rotary external combustion engine contains a stator and a profiled rotor installed in it with the formation of cold 2 and hot 1 working cavities of variable volume 3. The rotor has a constant length of the largest section (a chord) in its various positions on two supporting working motor shafts.

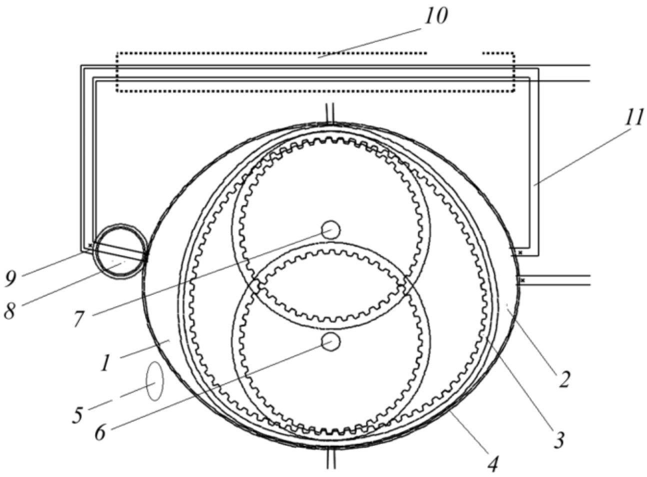

Fig. 1. Diagram of a rotary heat engine with an external heat supply: 1 is the hot cavity; 2 is the cold cavity; 3 is the rotor; 4 is the case; 5 is the external heat source; 6 is the first power take-off shaft; 7 is the second power take-off shaft; 8 is spool valve; 9 is the bypass pipelines; 10 is the recuperative heat exchanger; 11 is the inlet pipeline

The cold cavity is provided with inlet openings communicated with the inlet pipelines 11, as well as outlet openings that are communicated with the inlet openings of the hot cavity by the bypass pipelines 9.

The hot cavity is also provided with outlet openings connected with the exhaust pipelines, and the exhaust and bypass pipelines are located in the recuperative heat exchanger 10 with the possibility of transferring heat from the heated outlet pipelines of the next portion of the working fluid in the bypass pipelines.

The engine contains a spool cylindrical body 8, which is connected kinematically with the working shafts of the rotor and containing a gas distribution shaft, where, through radial bypass and outlet, openings are made to facilitate the possibility of connecting the bypass pipelines with the inlet openings of the hot cavity through bypass channels, as well as of connecting the hot cavity with the exhaust pipelines using outlet channels. In this case, the spool member 8 is installed with the possibility of connection between the holes and the pipelines during its rotation, synchronised with the rotation of the working rotor shafts 6 and 7, in the corresponding angular position of the rotor 3 relative to the stator.

Before entering the hot cavity, self-acting valves are installed with the ability to prevent the bypass of the working fluid from the hot cavity into the bypass pipeline.

The spool body 8 can be located around the hot cavity of the engine. The camshaft of the spool valve and the supporting rotor shafts are located in parallel. The spool valve shaft is connected to the rotor shafts with the ability to change its angular position relative to the angular position of the rotor shaft, depending on the engine operating mode.

Self-acting valves can be installed directly in the gas distribution shaft of the spool valve or in the bypass line in front of the

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spool valve.

The design of the presented heat engine can be characterised as a rotary planetary engine with an external heat supply. This engine has hot 1 and cold 2 cavities of variable volume, which are formed by the working surfaces of the rotor 3 and the case 4. In these cavities, thermodynamic processes are implemented forming a direct Rallis cycle.

Air is supplied to the cold cavity through the inlet pipeline 11, and the air is removed from the hot cavity into the environment through the exhaust pipeline. Also, the air gives up its heat to the working fluid in the recuperative heat exchanger 10.

The working fluid (a portion of air) enters the cold cavity 2 through the inlet pipeline 11 and the inlet hole. Next, inside the cold cavity 2, due to the rotation of the rotor 3, the working fluid is compressed with the removal of heat from heated air through the wall of the case into the environment. Then a portion of air from the cold cavity enters the pipeline connecting the two cavities. The working fluid receives heat from the air leaving the hot cavity inside the recuperative heat exchanger 10.

In the hot cavity, the working fluid expands due to the supply of heat through the wall of the engine case from an external heat source. In this case, a certain torque is imparted to the rotor, due to which the air moves through the exhaust pipe from the hot cavity to the recuperative heat exchanger, and then is expelled into the environment. In this case, along the route, the air relinquishes its heat to the working fluid moving from the cold cavity 2 to the hot cavity 1 in the recuperative heat exchanger. Due to the rotation of the rotor 3, two power take-off shafts acquire a certain torque. Next, the processes taking place inside the engine are repeated.

The kinematic scheme of a rotary engine with an external heat supply is shown in Fig. 2.

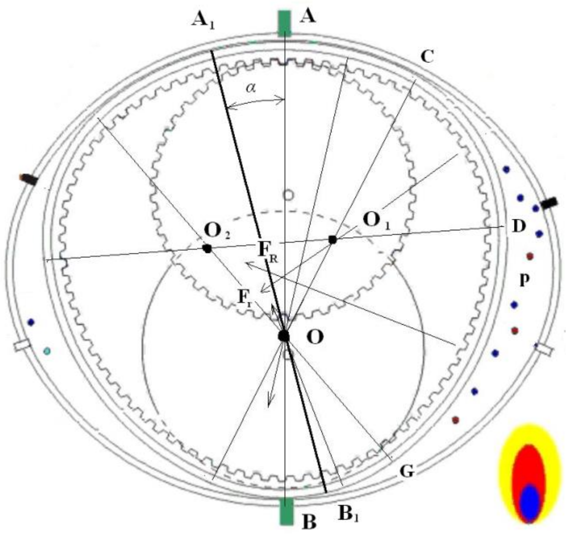

Fig. 2. Kinematic and dynamic analysis of the operation of a rotary heat engine with an external heat supply

The analysis of the diagram (Fig. 2) allows us to conclude that the instantaneous centre of rotation *O*, which coincides at the initial moment of time with one of the centres of curvature of the regular polycircle, is motionless during the stroke of changing the working volume and is located on the diametrical line *AB*. A line connects the points of contact of the rotor with the radial seal plates.

The full stroke of the rotary heat engine is carried out when the angle of rotation of the rotor changes from 0 to $\frac{\pi}{k}$ (where k is the number of “angles” of the polycircle).

In this case, the volumes of the hot and cold cavities change synchronously, continuously and monotonously in antiphase. If the working volume of the hot cavity increases from the minimum value V_{min} to the maximum value V_{max}, then the working volume of the cold cavity decreases from the maximum value V_{max} to the minimum value V_{min}. In the next cycle, the nature of the change in the working volumes in the cold and hot cavities is reversed.

At the end of the stroke, the instantaneous centre of rotation of the rotor abruptly moves along the diametrical line *AB* to the opposite position located at a distance *r* from the point *A*.

Thus, in the developed heat engine, there is a change in the position of the instantaneous axis of rotation of the rotor. In this case, the working contours of the rotor are regular polycircles.

From Fig. 2 it follows that the increment in the area of the contour of the working cavity between the points of contact of the rotor with the radial sealing plates when the rotor is turned through an angle α is equal to the difference in the surface areas of the sectors A_1OA and B_1OB.

Expressions were obtained that make it possible to calculate the values of the increasing V_1 and decreasing V_2 functions of the working volume of the hot and cold cavities when changing the angle α from 0 to $\frac{\pi}{k}$:

$$V_1 = H(R^2 - r^2)\frac{\alpha}{2}, \quad (1)$$

$$V_2 = H(R^2 - r^2)\frac{\frac{\pi}{k} - \alpha}{2}, \quad (2)$$

where: *H* is rotor width or distance between closing end planes; *R* is the radius of the large segment of the outer envelope of the rotor; *r* is the radius of the smaller segment of the outer envelope of the rotor; and *k* is the number of “angles” of the polycircle.

An expression was determined that allows calculating the working volumes of the hot and cold cavities depending on the change in the angle of rotation of the rotor:

$$V_{max} = V_{min} + H(R^2 - r^2)\frac{\pi}{2k}. \quad (3)$$

A relation was obtained for calculating the compression ratio in the cavity of a rotary heat engine with an external heat supply:

$$\varepsilon = 1 + \frac{H(R^2 + r^2)\pi}{2kV_{min}}. \quad (4)$$

The number of strokes per one complete revolution of the rotor *j* is determined by the angular length $\frac{\pi}{k}$ of one stroke and the number of working cavities in a rotary heat machine:

$$j = 6k, \quad (5)$$

where *k* is the number of “angles” of the polycircle.

With the number of “angles” of the polycircle of $k = 3$, the number of strokes per one complete revolution of the rotor is $j = 18$. Thus, during one complete revolution of the rotor shaft, 18 strokes occur in the hot and cold cavities of a rotary heat machine with an external heat supply, or in fact 18 thermodynamic processes are implemented.

From Fig. 2 it follows that the value M_1 of the rotor torque relative to the centre of rotation *O* is equal to the sum of the two values of the torques M_R and M_r, generated by the forces F_R and F_r, respectively. The resulting action of the forces determines the magnitude of the pressure p with which the working fluid acts on the corresponding sections of the working cylindrical surface of the rotor, limited by the generators at the points *C*, *D* and *G*.

The action of the force F_R is directed along the bisector of the angle GO_2D:

$$|F_R| = pH\,|DG| = pHR\frac{\pi}{3}. \quad (6)$$

Let us determine the ratio for calculating the arm of the force F_R relative to the centre of rotation *O* and obtain an expression for determining the magnitude of the torque M_R:

$$\frac{OO_1}{2} = \frac{R-r}{2} \Rightarrow M_R = pH(R - r)\frac{\pi R}{6}. \quad (7)$$

The force F_r is directed along the bisector of the angle CO_1D:

$$|F_r| = pH\,|CD| = pHr\frac{\pi}{3}. \quad (8)$$

Let us determine the ratio for calculating the arm of the force F_r and obtain an expression for determining the magnitude of the torque M_r:

$$\frac{OO_2}{2} = \frac{R-r}{2} \Rightarrow M_r = pH(R - r)\frac{\pi r}{6} \quad (9)$$

It should be noted that the torques $M_{AC} = M_{BG} = 0$, since the direction of action of the resulting forces F_{AC} and F_{BG} pass through the centre of rotation *O*.

Let us determine the total torque of a rotary heat machine with an external heat supply

$$M_1 = M_R + M_r = pH(R^2 - r^2)\frac{\pi}{6} = p(V_{max} - V_{min}) = pV, \quad (10)$$

where *H* is the rotor width or distance between closing end planes; *R* is the radius of the large segment of the outer envelope of the rotor; *r* is the radius of the smaller segment of the outer envelope of the rotor; and *p* is the value of pressure with which the working fluid acts on the corresponding sections of the working cylindrical surface of the rotor.

We present the results of a comparative analysis of the torque values of a rotary heat engine with an external heat supply M_1 and a Wankel engine M_2.

Let us assume that the initial conditions for the generation of the working fluid, the working volumes $V = V_{max} - V_{min}$, and the dynamics of the pressure change p for the two compared engines are identical.

For a Wankel engine with a triangular rotor, the working volume V is determined as:

$$V = \sqrt{3}arH = \sqrt{3}nRH, \quad (11)$$

where *R* is the radius of the rolling circle in the procedure for synthesising the epitrochoid of the stator working contour;
r is the radius of the circle being rolled in the procedure for synthesising the epitrochoid of the stator working contour;
H is the width of the rotor along the generatrix of the cylinder;
$a = c \cdot R$ is the producing radius of the epitrochoid of the stator working contour; and
c is the coefficient of elongation of the epitrochoid of the stator working contour.

Thus, the torque of the Wankel engine will be determined as:

$$M_2 = p\sqrt{3}arH|\sin 2\psi| = p\sqrt{3}nRH|\sin 2\psi| == p\sqrt{3}V\left(\frac{r}{R}\right)|\sin 2\psi|, \quad (12)$$

where ψ is the angle between the line of action of the resulting driving force of the working fluid pressure and the crank of the eccentric shaft.

To carry out a comparative analysis of the values of the torques M_1 and M_2 for the two engines, it is necessary to fix the values of the simplex $\frac{r}{R}$, as well as the value $\sin 2\psi$.

Supposing that $\frac{r}{R} = \frac{2}{3}$, and the value $\sin 2\psi = \frac{1}{2}$, then the expression in Eq. (12) will take the following form:

$$M_2 = \frac{2\sqrt{3}}{3} \cdot \frac{1}{2} \cdot pV = 0.577pV. \quad (13)$$

To compare the values of the torque of a rotary heat engine with an external heat supply M_1 and a Wankel engine M_2, we divide the expression in Eq. (10) by the relation in Eq. (13) and obtain:

$$\frac{M_1}{M_2} = \frac{pV}{0{,}577pV} = 1.73. \quad (14)$$

Thus, the torque of the proposed rotary heat engine with an external heat supply is 1.73 times higher than the torque of the Wankel engine.

The originality of the design of the proposed rotary heat machine with an external heat supply lies in the fact that the rotor moves not on gears, but on solid roller wheels.

The roller wheels are installed in parallel to the gears and roll on the smooth inner surface of the rotor with less friction than the gear transmission. Moreover, the radius of the roller wheels and the gears is less than the radius of the small segment of the rotor gear rim, as a result of which, when the instantaneous axis of rotation of the rotor is changed, a complete "head-on" blow of the small segment against the gears does not occur, and the transfer of the rotor torque to the shafts will be quite smooth.

For the supply and removal of the working body in a rotary heat engine with external fuel combustion, there are inlet and outlet pipelines. On the inner surface of the profiled rotor, shaped as a curved figure, there is a gear rim and a synchronisation mechanism made in the form of gears mounted on two shafts with the possibility of their interaction with the gear rim of the rotor.

The rotary heat engine is equipped with round roller wheels, and the rotor itself contains smooth grooves for them on the inner surface, with the support of which the rotor rolls inside the case with the least friction. Roller wheels have a smaller radius than the one of the inner envelope of the small segment of the rotor, and they can be installed symmetrically to the gears on both shafts or on their axle shafts attached to the end walls of the rotary heat engine.

The gears contain at least one tooth less than six times the number of teeth on the small rotor segment. For permanent engagement of the gears with the rotor gear rim, both shafts are parallelly displaced in opposite directions.

When the position of the rotor changes inside the case, the volumes of the hot cavity and the cold cavity change. The outer and inner envelopes of the rotor have a profile, which is formed by three pairs of opposite large and small segments (sectors of 60°) of circles with centres at the vertices of an equilateral triangle.

Such a profile of the outer and inner envelope, when the rotor moves inside the case, ensures the immobility of the compression plates, which are made of fluoroplastic, and creates a constant engagement of the gear rim with two gears of the power take-off shafts.

3. THERMODYNAMIC ANALYSIS

In the presented article, a specific thermodynamic cycle is determined, which most accurately describes the processes occurring inside a rotary engine with an external heat supply. For this, a

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thermodynamic analysis of the processes occurring inside the heat engine was carried out using the Erickson and Rallis ideal cycles.

When carrying out thermodynamic analysis, it is assumed that thermodynamic cycles are reversible and closed and that the working body is 1 kg of ideal gas.

Let us carry out a thermodynamic analysis of the operation of a rotary heat engine using the Erickson ideal cycle. It consists of two isotherms and two isobars (Fig. 3).

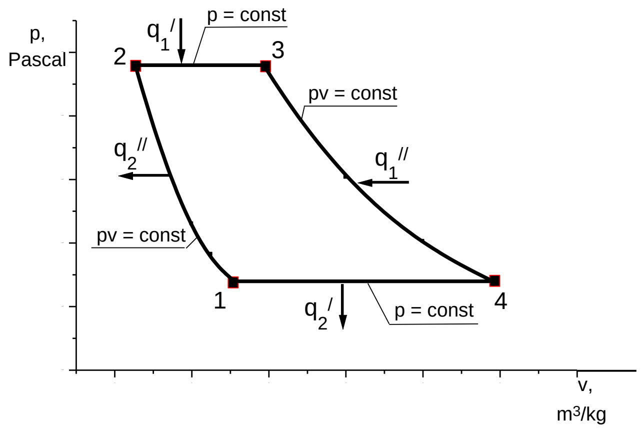

Fig. 3. Diagram of the ideal Erickson cycle in p, v – coordinates

An ideal gas with initial parameters p_1, v_1 and T_2 is compressed along the isotherm 1 – 2 to point 2 with heat removal of $q_2'' = RT_2 \ln\frac{v_1}{v_2}$ to a cold source. The isobar 2 – 3 imparts the amount of heat of $q_1' = c_p(T_1 - T_2)$ from an external hot source to the working fluid. From the point 3, the working fluid expands along the isotherm 3 – 4 with the heat supply of $q_1'' = RT_1 \ln\frac{v_4}{v_3} = RT_1 \ln\frac{v_1}{v_2}$ from an external hot source. Finally, along the isobar 4 – 1, the working fluid returns to its original state, while the amount of heat of $q_2' = c_p(T_1 - T_2)$ is removed to the cold source.

The characteristics of the cycle are compression ratio $\varepsilon = \frac{v_1}{v_2}$, and preliminary expansion ratio $\rho = \frac{v_3}{v_2}$.

Let us determine the parameters of the working fluid at all characteristic points of the Erickson cycle:

at point 1:

– pressure p_1 is given;
– absolute temperature T_2 is given;
– specific volume v_1:

$$v_1 = \frac{RT_2}{p_1}; \tag{15}$$

at point 2:

– ● specific volume v2:

$$\varepsilon = \frac{v_1}{v_2} \Rightarrow v_2 = \frac{v_1}{\varepsilon}; \tag{16}$$

– absolute temperature T_2 is known and does not change, since the process 1 – 2 is isothermal;
– pressure p_2:

$$p_2 v_2 = RT_2 \Rightarrow p_2 = \frac{RT_2}{v_2}; \tag{17}$$

at point 3:

– pressure p_3=p_2, since the process 2 – 3 is isobaric;
– specific volume v_3:

$$\frac{v_3}{v_2} = \rho \Rightarrow v_3 = v_2\rho = \frac{v_1}{\varepsilon}\rho; \tag{18}$$

– absolute temperature T_1:

$$p_3 v_3 = RT_1 \Rightarrow T_1 = \frac{p_3 v_3}{R}; \tag{19}$$

at point 4:

– absolute temperature T_1 is known and does not change, since the process 3 – 4 is isothermal;
– pressure $p_4 = p_1$, since the process 4 – 1 is isobaric;
– specific volume v_4:

$$p_1 v_4 = RT_1 \Rightarrow v_4 = \frac{RT_1}{p_1}. \tag{20}$$

The expression that determines the thermal efficiency of the Erickson cycle takes on the following appearance:

$$\eta_t = 1 - \frac{q_2}{q_1} = 1 - \frac{c_p(T_1 - T_2) + RT_2 \ln\frac{v_1}{v_2}}{c_p(T_1 - T_2) + RT_1 \ln\frac{v_4}{v_3}}. \tag{21}$$

The presented algorithm made it possible to thermodynamically calculate the ideal cycle of a rotary heat engine with external fuel combustion in accordance with the following technical specifications: for the ideal Erickson cycle, the thermodynamic condition parameters p_i, v_i and T_i are determined at all characteristic points of the cycle, together with considering the specific supplied q_1 and removed q_2 heat, thermal efficiency of the cycle η_t and construction of this cycle in p, v – coordinates, given that the pressure is set at $p_1 = 1.7 \cdot 10^5$ Pa, the absolute temperature is T_2 = 300 K, the compression ratio is $\varepsilon = \frac{v_1}{v_2} = 2$ and the preliminary expansion ratio is $\rho = \frac{v_3}{v_2} = 2.34$. The working fluid is air with the gas constant of $R = 287\ \frac{\mathrm{J}}{\mathrm{kg \cdot K}}$. We assume the heat capacity of the working fluid constant: $c_p = 1010\ \frac{\mathrm{J}}{\mathrm{kg \cdot K}}$, $c_V = 721\ \frac{\mathrm{J}}{\mathrm{kg \cdot K}}$.

The results of the calculation are presented in Tab. 1.

Tab. 1. Thermodynamic condition parameters of the ideal Erickson cycle

Condition parameters	Point 1	Point 2	Point 3	Point 4
p_i, Pa	$1.7 \cdot 10^5$	$3.4 \cdot 10^5$	$3.4 \cdot 10^5$	$1.7 \cdot 10^5$
v_i, m^3/kg	0.506	0.253	0.592	1.185
T_i, K	300	300	701.9	701.9

Supplied specific heat, q_1, J/kg:

$$q_1 = q_1' + q_1'' = c_p(T_1 - T_2) + RT_1 \ln\frac{v_4}{v_3}$$
$$= 1010 \cdot (701.9 - 300) + 287 \cdot 701.9 \ln\frac{1.185}{0.592} = 5.45 \cdot 10^5\ \frac{\mathrm{J}}{\mathrm{kg}}.$$

Removed specific heat, q_2, J/kg:

$$q_2 = q_2' + q_2'' = c_p(T_1 - T_2) + RT_2 \ln\frac{v_1}{v_2} = 1010 \cdot (701.9 - 300) + 287 \cdot 300 \ln\frac{0.506}{0.253} = 4.65 \cdot 10^5\ \frac{\mathrm{J}}{\mathrm{kg}}.$$

Useful specific heat q, J/kg:

$$q = q_1 - q_2 = 8 \cdot 10^4\ \frac{\mathrm{J}}{\mathrm{kg}}.$$

Thermal efficiency of the Erickson cycle:

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$\eta_t = \frac{q_1 - q_2}{q_1} \cdot 100\ \% = 14.7\ \%.$

The efficiency of any engine can be assessed using the thermal efficiency of the direct Carnot cycle.

The efficiency of any engine can be assessed using the thermal efficiency of the direct Carnot cycle.

$\eta_t = \frac{T_1 - T_2}{T_1} \cdot 100\ \% = \frac{701.9 - 300}{701.9} \cdot 100\ \% = 57.25\ \%.$

We now proceed to represent this cycle as a graph using points in the *p*, *v* – coordinates (Fig. 4).

As a result, the overall thermal efficiency of an engine with an external heat supply operating according to the ideal Erickson cycle for one full rotor revolution will be 44.1 %.

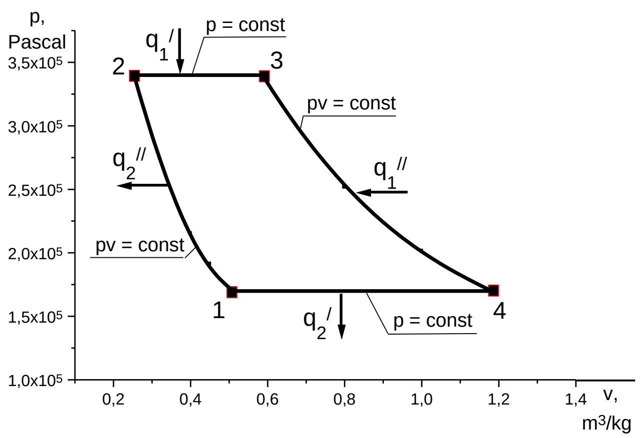

Fig. 4. Results of calculating the ideal Erickson cycle in *p*, *v* – coordinates

We then carry out a thermodynamic analysis of the heat engine operation using the ideal Rallis cycle. It consists of two isotherms, two isochores and two isobars (Fig. 5).

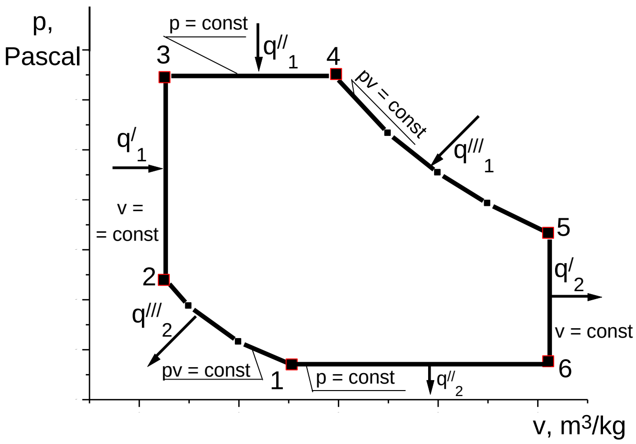

Fig. 5. Diagram of the ideal Rallis cycle in *p*, *v* – coordinates

The presented *p*, *v* – diagram of the ideal Rallis cycle allows one to determine the thermodynamic condition parameters at all reference points of the presented direct cycle.

An ideal gas with initial parameters p_1, v_1 and T_2 is compressed along the isotherm 1 – 2 to point 2 with heat removal of $q_2''' = RT_2 \ln\frac{v_1}{v_2}$ to a cold source. The isochor 2 – 3 imparts the amount of heat of $q_1' = c_v(T_3 - T_2)$ from exhaust gases in a recuperative heat exchanger to the working fluid. The isobar 3 – 4 imparts the amount of heat of $q_1'' = c_p(T_1 - T_3)$ from an external hot source to the working fluid. From the point 4, the working fluid expands along the isotherm 4 – 5 with the heat supply of $q_1''' = RT_1 \ln\frac{v_5}{v_4}$ from an external hot source. Along the isochore 5 – 6, the working fluid moves to the point 6 and at the same time the amount of heat of $q_2' = c_v(T_1 - T_6)$ is removed from the exhaust gases to the working fluid in the recuperative heat exchanger. Finally, along the isobar 6 – 1, the working fluid returns to its original state, while the amount of heat of $q_2'' = c_p(T_6 - T_2)$ is removed to the cold source.

The characteristics of the cycle are compression ratio $\varepsilon = \frac{v_1}{v_2}$, pressurisation ratio $\lambda = \frac{p_3}{p_2}$, preliminary expansion ratio $\rho = \frac{v_4}{v_3}$, and expansion ratio $\sigma = \frac{v_5}{v_4}$.

Let us determine the parameters of the working fluid at all characteristic points of the Rallis cycle:

at point 1:
- pressure p_1 is given;
- absolute temperature T_2 is given;
- specific volume v_1:

$$v_1 = \frac{RT_2}{p_1}; \quad (22)$$

at point 2:
- specific volume v_2:

$$\varepsilon = \frac{v_1}{v_2} \Rightarrow v_2 = \frac{v_1}{\varepsilon}; \quad (23)$$

- absolute temperature T_2 is known and does not change, since the process 1 – 2 is isothermal;
- pressure p_2:

$$p_2 v_2 = RT_2 \Rightarrow p_2 = \frac{RT_2}{v_2}; \quad (24)$$

at point 3:
- specific volume v_3 equals $v_3 = v_2 = \frac{v_1}{\varepsilon}$, since the process 2 – 3 is isochoric;
- pressure p_3:

$$\frac{p_3}{p_2} = \lambda \Rightarrow p_3 = p_2\lambda; \quad (25)$$

- absolute temperature T_3:

$$p_3 v_2 = RT_3 \Rightarrow T_3 = \frac{p_3 v_2}{R}; \quad (26)$$

at point 4:
- pressure $p_4 = p_3$, since the process 3 – 4 is isobaric;
- specific volume v_4:

$$\frac{v_4}{v_3} = \rho \Rightarrow v_4 = v_3\rho; \quad (27)$$

- absolute temperature T_1:

$$p_3 v_4 = RT_1 \Rightarrow T_1 = \frac{p_3 v_4}{R}; \quad (28)$$

at point 5:
- absolute temperature T_1 is known and does not change, since the process 4 – 5 is isothermal;
- specific volume v_5:

$$\frac{v_5}{v_4} = \sigma \Rightarrow v_5 = v_4\sigma; \quad (29)$$

- pressure p_5:

$$p_5 v_5 = RT_1 \Rightarrow p_5 = \frac{RT_1}{v_5}; \quad (30)$$

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at point 6:

– pressure p_6=p_1, since the process 6 – 1 is isobaric;
– absolute temperature T_6:

$$\frac{p_5}{p_1} = \frac{T_1}{T_6} \Rightarrow T_6 = \frac{p_1 T_1}{p_5}; \quad (31)$$

– specific volume v_6=v_5, since the process 5 – 6 is isochoric.

The expression that determines the thermal efficiency of the Rallis cycle assumes the following form:

$$\eta_t = 1 - \frac{q_2}{q_1} = 1 - \frac{c_v(T_1 - T_6) + c_p(T_6 - T_2) + RT_2 \ln\frac{v_1}{v_2}}{c_v(T_3 - T_2) + c_p(T_1 - T_3) + RT_1 \ln\frac{v_5}{v_4}}. \quad (32)$$

The presented algorithm made it possible to thermodynamically calculate the ideal cycle of a rotary heat engine with external fuel combustion in accordance with the technical task: for the ideal Rallis cycle, we determine the thermodynamic condition parameters p_i, v_i and T_i at all characteristic points of the cycle, the specific supplied q_1 and removed q_2 heat and thermal efficiency of the cycle η_t, and construct this cycle in *p*, *v* – coordinates, if the pressure is set at p_1=1.7·10^5 Pa, the absolute temperature is T_2 = 300 K, the compression ratio is $\varepsilon = \frac{v_1}{v_2} = 2$, the pressurisation ratio is $\lambda = \frac{p_3}{p_2} = 2.2$, the preliminary expansion ratio is $\rho = \frac{v_4}{v_3} = 2.34$ and the expansion ratio is $\sigma = \frac{v_5}{v_4} = 1.73$. The working fluid is air with the gas constant of R = 287 $\frac{\text{J}}{\text{kg}\cdot\text{K}}$. We assume the heat capacity of the working fluid constant: $c_p = 1010\ \frac{\text{J}}{\text{kg}\cdot\text{K}}$, c_v = 721 $\frac{\text{J}}{\text{kg}\cdot\text{K}}$.The results of the calculation are presented in Tab. 2.

Tab. 2. Thermodynamic condition parameters of the ideal Rallis cycle

Condition parameters	Point 1	Point 2	Point 3	Point 4	Point 5	Point 6
p_i, Pa	1.7·10^5	3.4·10^5	7.5·10^5	7.5·10^5	4·10^5	1.7·10^5
v_i, m^3/kg	0.506	0.253	0.253	0.592	1.024	1.024
T_i, K	300	300	659.9	1544	1544	606.5

Supplied specific heat, q_1, J/kg:

$$q_1 = q_1' + q_1'' + q_1''' = c_v(T_3 - T_2) + c_p(T_1 - T_3) + RT_1 \ln\frac{v_5}{v_4}$$
$$= 721 \cdot (659.9 - 300) + 1010 \cdot (1544.3 - 659.9) + 287 \cdot 1544.3 \ln\frac{1.024}{0.592} = 1.39 \cdot 10^6\,\frac{\text{J}}{\text{kg}}.$$

Removed specific heat, q_2, J/kg:

$$q_2 = q_2' + q_2'' + q_2''' = c_v(T_1 - T_6) + c_p(T_6 - T_2) + RT_2 \ln\frac{v_1}{v_2} =$$
$$= 721 \cdot (1544.3 - 606.5) + 1010 \cdot (606.5 - 300) + 287 \cdot 300 \ln\frac{0.506}{0.253} = 1.04 \cdot 10^6\,\frac{\text{J}}{\text{kg}}.$$

Useful specific heat q, J/kg:

$$q = q_1 - q_2 = 3.5 \cdot 10^5\,\frac{\text{J}}{\text{kg}}.$$

Thermal efficiency of the Rallis cycle:

$$\eta_t = \frac{q_1 - q_2}{q_1} \cdot 100\,\% = 25\,\%.$$

The efficiency of any engine can be assessed using the thermal efficiency of the direct Carnot cycle.

$$\eta_t = \frac{T_{max} - T_{min}}{T_{max}} \cdot 100\,\% = \frac{1544.3 - 300}{1544.3} \cdot 100\,\% = 80.57\,\%.$$

We make a graph of this cycle by points in *p*, *v* – coordinates (Fig. 6).

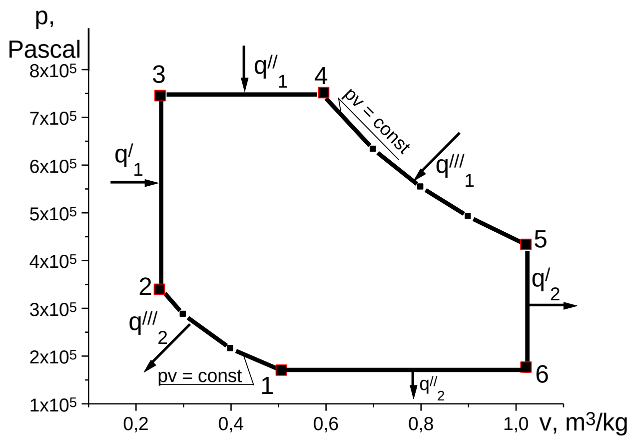

Fig. 6. Results of calculating the ideal Rallis cycle in *p*, *v* – coordinates

Thus, the overall thermal efficiency of an engine with an external heat supply operating according to the ideal Rallis cycle for one full rotor revolution will be 75%. The closer the thermal efficiency of the power plant is to the thermal efficiency of the direct Carnot cycle, the more perfect the power plant. An analysis of thermodynamic processes occurring inside a rotary heat engine with external heat supply made it possible to establish that the cycle that most accurately describes its work is the ideal Rallis cycle.

It should be noted that the ideal Stirling and Erickson cycles are special cases of the ideal Rallis cycle.

4. DISCUSSION

The Rallis cycle is designed to work only with a gaseous working fluid. In order for the dimensions of the engines at a given power to be acceptable, and the external and internal heat exchange of the working fluid under these conditions to be efficient enough, the pressure in the heat engine must be significantly higher than the atmospheric pressure. In this case, the working fluid must have a low viscosity and the highest possible thermal conductivity.

The conducted thermodynamic analysis showed that this cycle is a direct cycle, as a result of which heat turns into work. The advantage of this cycle is the fact that it allows operation in a wide temperature range of hot and cold sources at relatively small values of the ratio of the compression and expansion pressures.

5. CONCLUSION

The proposed rotary heat engine with external heat supply will facilitate:

– a reduction in the dimensions and weight of the power plant due to the small number of interacting units; and
– an increase in the efficiency of the engine by reducing the friction of the rotor against the case and shaft gears.

The use of solid roller wheels will facilitate:
– an increase in the specific power of the rotary external combustion engine; and
– an increase in the performance of the power unit due to the possibility of using the power plant at increased rotor speed.

To reduce the forces of inertia and ensure constant meshing of the gears with the rotor gear rim, both shafts are displaced in opposite directions. The design feature of the presented power plant is the ability to replace two power take-off shafts without dismantling the rotor.

The carried out thermodynamic calculation made it possible to determine the main condition parameters at the characteristic points of the ideal Rallis cycle and to describe the operation of a heat engine with external combustion of fuel fairly accurately. The overall thermal efficiency of the power plant for one full revolution of the rotor was 75%, which indicates the possibility of using this engine as a power plant for driving motor vehicles.

The practical significance of the proposed rotary heat engine with an external heat supply lies in the fact that it can be used not only to obtain mechanical and electrical energy but also to generate heat energy in heaters in air heating systems and forced ventilation systems with mechanical air induction. Maintaining permissible microclimate conditions in the working area of large buildings can be ensured using the proposed heat engine.

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