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Circuit analysis of magnetic couplings between circular turn and spiral coil

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The paper deals with magnetic coupling between circular turns, modelling of spiral coil arranged concentrically on a plane and the single forcing turn. The turns are organized in axially symmetric arrangement. Circular turns are made of nonferromagnetic conductor. The equations of the magnetic field distribution around a single turn with current were derived from the Biot-Savart law, taking into account the turn wire radius Using the Biot-Savart law, equations describing the distribution of the magnetic field around a single turn with current, taking into account the radius of the turn wire, were derived. An original relation was obtained. It which allows to calculate self inductance of a single turn and mutual inductances relative to the other turns. The inductance matrix of the individual coil turns was derived from obtained formulas.From the formula inductance matrix of turns set of the spiral coil model was built. The spiral coil inductance was derived from inductance matrix of the adopted coil modelSpiral coil inductance has been described on the basis of the established coil model inductance matrix. The self inductance of the forcing coil, as well as the mutual inductance between forcing coil and the individual turns of the spiral coil were derived from the Biot-Savart lawSimilarly self inductance of the forcing coil and mutual inductance vector between the forcing turn and the each turn of spiral coil model were described.

KEYWORDS: spiral coil, magnetic coupling, circumferential equation

1. Introduction

The problems of electromagnetic couplings are usually modelled using by use of the finite element method. Usually the forcing current with specific frequency is assumed and the quasi-static state is considered. There is assumed the current force with a specific frequency and the quasistatic state is considered. The problem occurs when the non-sinusoidal power source is applied. In this case the system should be treated circumferentially. The basis for this approach is to determine self and mutual inductances of the coupled elements. For this purpose there can be applied the Biot-Savart law and on this basis the intensity of the magnetic field [1] can be calculated. Dimensionality of the problem is reduced to the symmetrical components of the system. The method of determining the distribution of single coil magnetic field of single coil has been described in [2]. Developed method was used to define inductance of the single-layer cylindrical coil.In [2] there has been developed a method determining the distribution of the magnetic field of a single turn and on this basis the single-layer cylindrical coil inductance specified. Inductance values calculated for the individual coils were compared with measurements.The inductance values for individual coils were compared with the results obtained using the multimeter. The differences between calculated values and measurements were less than 1%The values of these parameters were consistent with the calculated values to an accuracy of 1%. Very good accuracy was obtained for both the short and the long coils. It is much more difficult to determine the magnetic coupling occurring between the induced element and the forcing element. Such difficulty occurs in case of a flat circular plate where currents are induced by the currents flowing in the excitation winding. In [3] there has been studied the interaction of a single turn and circular plate, modelled as a set of concentric, magnetically coupled circular loops. After a proper transformation, the model can be used to investigate the flat spiral coil. Dependencies received in [3] were used to determine the self and mutual inductance for coaxial coils. It was assumed that the coils are made of material which is a conductor and is not ferromagnetic.

2. Magnetizing force and inductance of a single turn

The basis for determining the inductance of a cylindrical coil or a flat spiral coil is the knowledge of the value of the magnetizing force at any point of vicinity of the coil. Basing on the Biot-Savart law the magnetizing force surrounding the circular wire is calculated. The components of the magnetizing force at the point of cylindrical coordinates (r, φ, z) were determined from the Biot-Savart law. From the Biot-Savart law there are determined the components of the magnetizing force at the point of cylindrical coordinates (r, φ, z).

$$H_z(r,Rp,R,i)=\frac{i}{4\cdot\pi}\int_0^{2\pi}\frac{R\cdot(R-r\cdot cos(\varphi))}{\left(r^2+Rp^2+R^2-2\cdot r\cdot R\cdot cos(\varphi)\right)^{3/2}}d\varphi \tag{1}$$

$$H_r(r,z,R,i)=\frac{i}{4\cdot\pi}\int_0^{2\pi}\frac{z\cdot R\cdot cos(\varphi)}{\left(r^2+z^2+R^2-2\cdot r\cdot R\cdot cos(\varphi)\right)^{3/2}}d\varphi \tag{2}$$

where: i - current in the turn, r - the sliding radius, R - radius of the turn, Rp - radius of the turn wire, z - distance from the wire turn.

To simplify the determination of the magnetizing force, the dimensionless variables were introduced in the forms:

$$\frac{r}{R}=\eta,\quad \frac{z}{R}=\zeta,\quad h=\frac{H}{H_{odn}},\quad H_{odn}=\frac{i}{R} \tag{3}$$

$$H(r,z,R,i) = H_{odn}(i,R)\cdot h(\eta,\zeta) = \frac{i}{R}\cdot h(\eta,\zeta) \tag{4}$$

The use of dimensionless variables reduces the number of variables and simplifies the formula for the approximated component (1) to the form:

$$h_z(\eta,\zeta) = \frac{1}{4\cdot\pi}\int_0^{2\pi}\frac{1-\eta\cdot\cos(\varphi)}{\left(1+\eta^2+\zeta^2-2\cdot\eta\cdot\cos(\varphi)\right)^{3/2}}\,d\varphi \tag{5}$$

Taking into account graphs shape of of the magnetizing force components presented in [2], its approximation is complicated. Therefore, authors decided to approximate the dimensionless *z*-component of the magnetizing flux ψ_z (6) in the form:

$$\psi_z(\zeta) = \int_0^{1-\zeta} h_z(\eta,\zeta)\cdot\eta\, d\eta \qquad dla \quad \zeta = \frac{z}{R}, \quad z = Rp \tag{6}$$

Figure 1 shows the component of the magnetizing flux ψ_z obtained for ratio - radius of the turn wire to radius turn equal 1e-4.

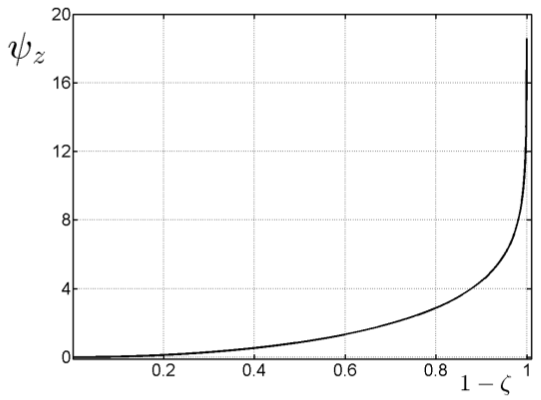


Fig. 1. The component of magnetic flux: ψ_z

Applying the approximation of the component of the magnetizing flux ψ_z there were obtained the analitycal relationships with a relative uncertainty less than 1%. There has been formulated function ψ_{zA} *(Rp/R)* (7) which approximates the component of magnetizing flux ψ_z *(Rp/R)* (7) for arguments *Rp/R*. The magnetiizing flux ψ_{zA} is a function of the radius of the turn wire and the radius coil ratio.

$$\psi_{zA}\left(\frac{Rp}{R}\right) = -2.174\cdot\ln\left(\frac{Rp}{R}\right) - 0.62 \tag{7}$$

Figure 2 shows a graph of equation (7) which approximates the component of the magnetic flux ψ_z. The relative error is within the range of 1e-4 < *Rp/R* < 0.5

4 and is less than 14%. Using(7) there can be determined the inductance of a single turn coil with an accuracy of approx. 1% from the following relationship:

$$L = \frac{\mu_o \cdot R}{2} \cdot \psi_{zA}\left(\frac{Rp}{R}\right) \tag{8}$$

where: μ_o magnetic permeability.

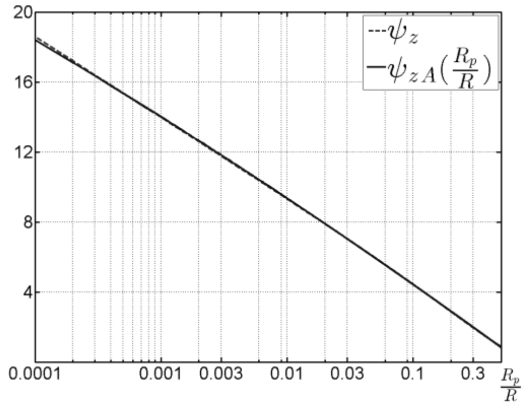


Fig. 3. Component of the magnetising flux ψ_z and its approximation ψ_{zA} as a function of *(Rp/R)*

Inductance of a single turn can also be determined numerically by using the magnetizing force component H_z (1) as follows:

$$L = \frac{\mu_o}{2 \cdot i} \int_0^R H_z\left(r, Rp, R, i\right) r\, dr \tag{9}$$

3. Self and mutual inductances of spiral coil turns

The spiral coil was modelled as a set of concentric turns made on a circuit printed plate. In the present configuration the radii of the spiral turns are changing in the range from r_k = 0.0255m to r_k = 0.0885m. The cross-section of the turn is a 1 x 0.035 mm a rectangle with dimensions of 1 mm to 0.035 mm. Turns are separated by a 1 mm distance. The spiral coil is prepared by a suitable connection of the turns on connectors. The connectors allows the short-circuiting turns (model of plate) or the connecting it in series (spiral coil). Using equation (10) there can be determined the matrix of mutual inductances between turns in a spiral coil.

$$\mathbf{M}_{pp}\left(r_j, r_k, Rp\right) \cdot i = \frac{\mu_0}{2} \int_0^{r_j} H_z\left(r, Rp, r_k, i\right) r\, dr \tag{10}$$

where: r_j, r_k – radii of subsequent turns of the spiral coil for k = 1..32.

The inductance matrix elements of the spiral coil turns were determined based on the values of their magnetic flux and fluxes associated from other turns of the plate.

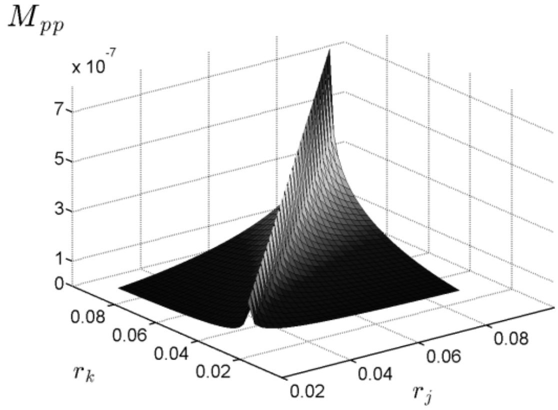


Fig. 3. Surface chart of the matrix elements of inductances turns

Mutual inductance between the turns *j*-th and *k*-th, calculated on a height of the radius of the spiral coil wire *Rp* is determined by equation (10). Inductance matrix is symmetric. A surface chart in Figure 3 shows the matrix elements. For coils with a small number of turns it is not a big computational problem, but when we considered a thin plate with a large disc radius, the numerical calculations take a lot of time. The use of equation (7) with suitable input parameters significantly reduces the computation time. The analytical model relation (11) allow to determine the self and mutual inductances for the spiral coils with any physical parameters.

$$\lambda = \frac{2\left(Rp^2 + \left(r_j - r_k\right)^2\right)}{\left(r_j + r_k\right)^2}$$
$$L_{sc}\left(r_j, r_k\right) \cdot i = \frac{\mu_o \cdot \left(r_j + r_k\right)}{2}\left(-1.087 \cdot \ln\left(\lambda\right) - 0.62\right) \tag{11}$$

After substituting (11) it is obtained:

$$L = \sum_{j=1}^{n} \sum_{k=1}^{n} L_{sc} \tag{12}$$

Using equation (12) the self inductance of spiral coil is determined with an accuracy of 3%. In the formula (13) [5], we obtain the value of the inductance [H], and the input values are given in [m].

$$L = 31.33\mu_o \frac{n^2 \cdot a^2}{8a + 11c} \tag{13}$$

where: n – number of turns, a – the mean radius of the coil, c - width of coil turns (the difference between the outer radius and inner radius of the coil).

Table 1. The spiral coil inductance values - measured and calculated

Number of turns	32	40
Wire radius [mm]	0.1	0.45
The inner coil radius [mm]	25.5	40
The outer coil radius [mm]	88.5	76
Inductance measured [μH]	118	246
Inductance by the formula (13) [μH]	113	246
Error of the formula (13) [%]	4.23	0
Inductance by the formula (12) [μH]	120120	254254
Error of the formula (12) [%]	1.691.69	3.253.25

It should be pointed that (13) allows the calculation of only self inductance of the spiaral coil.

4. Magnetic couplings between a forcing turn and a spiral coil

The conductive turns with resistivity ρ and cross-section of wire with the circle shape and radius of turn r_k are considered. A single turn with radius R_w and radius wire r_{pw} with a forcing current induces currents in the spiral coil turns. The forcing turn is set concentrically and axially relative to turns of the spiral coil. The symmetry of the system shows that the current flowing in the coils has only an angular component. These wires are mutually magnetically coupled with each other and with the turn of the forcing coil. Figure 4 shows the considered model: a forcing turn – conductive turns.

The concentricity of the spiral coil planes and the excitation coil and use of the H_z magnetic intensity component allowed to obtain the relations between the mutual inductances of the considered system (12). Basing on the concentric surfaces of the spiral coil and the forcing turn and using the component H_z of the magnetizing force, the equation (12) allows to calculate the mutual inductance of the model tested.

Similarly to (10) the magnetic flux generated by the forcing turn, penetrating through the subsequent turns of the spiral coil with the radius r_k may be determined. An expression for configuration is identical for the reverse coil.

$$\mathbf{M}_{po}\left(r_k, R_w, z\right) i = \mathbf{M}_{op}\left(r_k, R_w, z\right) i = \frac{\mu_0}{2} \int_0^{r_k} H_z\left(r, z, R_w, i\right) r \, dr \tag{14}$$

where: r_k – subsequent radius of spiral coil turns for k = 1..32, R_w – radius of the forcing turn, z - distance between the forcing turn and the spiral coil.

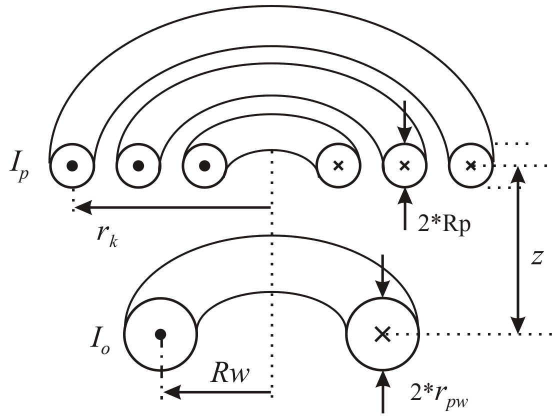
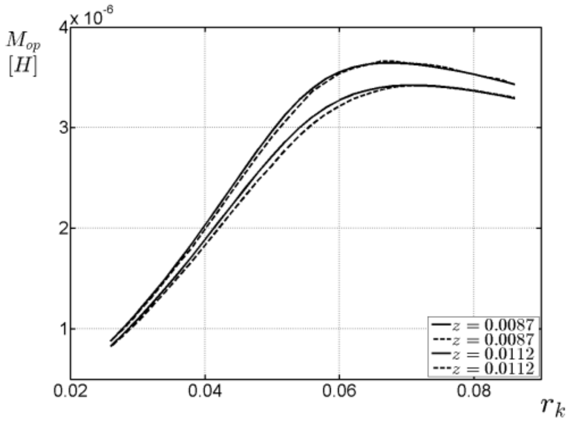


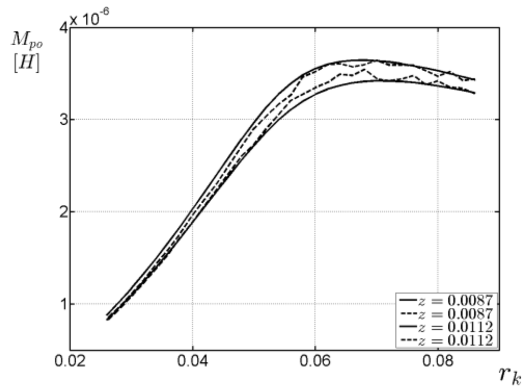
Fig. 4. System forcing turn – spiral coil

According to the Neumann formula inductances M_{op} and M_{po} are equal. To identify the parameters of the turn-coil interactions and vice versa the there has been built a measuring system was built. Voltages and currents were discretized and recorded on an oscilloscope. These values were analyzed using with use of MATLAB. Using the least squares method (LSM) parameters the coil parameters were calculated. In order to reduce interference the Savitzky-Golay filter was applied. Figure 5 shows the mutual inductances between the forcing turn and the *k*-th spiral coil turn. Numerical results were compared with the results obtained during from the measurements. Calculations were carried out for the forcing turn R_w = 0.055 m and the distance between the forcing turn and plate turns z = [0.0087, 0.0112] m. Leading non-sinusoidal voltage to the terminals and forming separated spiral coil windings the characteristics of magnetic coupling between the turns of the spiral inductor modelling and the single turn has been collected. The resulting values of the mutual inductance are identical as in Figure 5. There are only minor errors caused by the electromagnetic field generated by the forcing element.

Based on these results it can be concluded that the maximum value of the mutual inductances is shifted outside the radius of the forcing turn. This is caused by the electromagnetic flux distribution around the forcing coil. Three cases were analyzed: a power source separately adjusted to (*1*, *17*, *32*) wire and five magnetic coupling of adjacent turns were tested. Using LSM the mutual inductance was determined and compared with the results which were calculated numerically using equation (14), (15) - Figure 6. Mutual inductances calculated and measured are close to each other with an accuracy of approx. 1%.



a)



b)

Fig. 5. Mutual inductance between a) the forcing coil and subsequent turns of the model of spiral coil, b) the turns of the model of spiral coil and forcing turn, from formula (14) and measured values

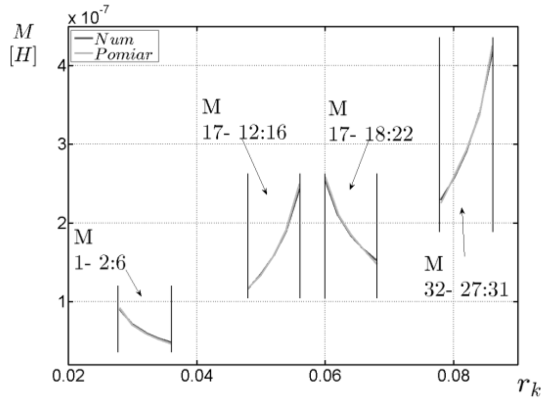


Fig. 6. Mutual inductance between the turns of the model spiral coil

5. System equations

For the system presented at Figure 4 the equations in the following form are valid:

$$\begin{aligned} L_o \cdot \dot{I}_o + \mathbf{M}_{op} \cdot \dot{\mathbf{I}}_p + R_o \cdot I_o &= U_o(t) \\ \mathbf{M}_{po} \cdot \dot{I}_o + \mathbf{L}_{pp} \cdot \dot{\mathbf{I}}_p + \mathbf{R}_{pp} \cdot \mathbf{I}_p &= \mathbf{U}_p(t) \end{aligned} \tag{15}$$

where: $U_o(t)$, $\mathbf{U}_p(t)$ voltage supplied to the forcing turn and voltage across the spiral coil, I_o, $\mathbf{I}_p$ the current in the forcing turn and vector of spiral coil currents, L_o, $\mathbf{L}_{pp}$, $\mathbf{M}_{po}$, $\mathbf{M}_{op}$, self and mutual inductances and R_o, $\mathbf{R}_{pp}$ resistances of forcing turn and spiral coil.

For the spiral coil model:

$$U_{sp} = \mathbf{1}_h \cdot \mathbf{U}_p(t), \quad \mathbf{I}_p(t) = \mathbf{1}_v \cdot I_{sp} \tag{16}$$

where: $\mathbf{1}_h$ and $\mathbf{1}_v$ are horizontal and vertical vectors respectively. Multiplying left-side of the second equation (15) by $\mathbf{1}_h$ and using the substitution (16) the system of equations (15) can be written as:

$$\begin{aligned} L_o \cdot \dot{I}_o + \mathbf{M}_{op} \cdot \mathbf{1}_v \cdot \dot{I}_{sp} + R_o \cdot I_o &= U_o(t) \\ \mathbf{1}_h \cdot \mathbf{M}_{po} \cdot \dot{I}_o + \mathbf{1}_h \cdot \mathbf{L}_{pp} \cdot \mathbf{1}_v \cdot \dot{I}_{sp} + \mathbf{1}_h \cdot \mathbf{R}_{pp} \cdot \mathbf{1}_v \cdot I_{sp} &= U_{sp}(t) \end{aligned} \tag{17}$$

After transforming the system of equations (17) can be rewritten as:

$$\begin{aligned} L_o \cdot \dot{I}_o + M_{osp} \cdot \dot{I}_{sp} + R_o \cdot I_o &= U_o(t) \\ M_{spo} \cdot \dot{I}_o + L_{sp} \cdot \dot{I}_{sp} + R_{sp} \cdot I_{sp} &= U_{sp}(t) \end{aligned} \tag{18}$$

where it was determined as follows: $U_o(t)$, $U_{sp}(t)$ forcing turn voltage and the spiral coil voltage, I_o, I_{sp} current in the forcing turn and spiral coil currents vector, L_o, L_{sp}, M_{spo}, M_{osp}, self and mutual inductances and R_o, R_{sp} resistances of forcing turn and spiral coil.

To determine the system response to non-nonsinusoidal forcing, the Laplace transformation may be used. At zero initial conditions and spiral coil loaded with a resistance with a value of $R_m = 0.55\ \Omega$ the following formulas may by derived from equation (18):

$$I_o(s) = \frac{s \cdot L_{sp} + R_{sp} + R_m}{(s \cdot L_o + R_o)(s \cdot L_{sp} + R_{sp} + R_m) - s^2 \cdot M_{osp} \cdot M_{spo}} U_o(s) \tag{19}$$

$$I_{sp}(s) = \frac{-s \cdot M_{spo}}{(s \cdot L_o + R_o)(s \cdot L_{sp} + R_{sp} + R_m) - s^2 \cdot M_{osp} \cdot M_{spo}} U_o(s) \tag{20}$$

All the parameters of self and mutual inductance between the coils and resistances of individual coils can be identified by applying the LSM to the system of equations (18). Assuming that the mutual inductances are equal, the system of equations (18) can be written in a matrix form. For numerical calculations we perform concatenation of input values along the columns which allows to determine the magnitude of the researched quantities describing the magnetic coupling of the tested system:

$$\begin{bmatrix} \dot{\mathbf{I}}_{\mathbf{o}} & \dot{\mathbf{I}}_{\mathbf{sp}} & \mathbf{0} & \mathbf{I}_{\mathbf{o}} & \mathbf{0} \\ \mathbf{0} & \dot{\mathbf{I}}_{\mathbf{o}} & \dot{\mathbf{I}}_{\mathbf{sp}} & \mathbf{0} & \mathbf{I}_{\mathbf{sp}} \end{bmatrix} \cdot \begin{bmatrix} L_o & M_{osp} & L_{sp} & R_o & R_{sp} \end{bmatrix}^T = \begin{bmatrix} \mathbf{U}_{\mathbf{o}} \\ \mathbf{U}_{\mathbf{sp}} \end{bmatrix} \tag{21}$$

6. Final remarks

A spiral coil can be modelled as a set of circular turns connected in series, but the radius of the turn wire has to be taken into account. The fact that the results of calculated and measured parameters are close, shows that the model is correct and provides a basis for modelling more complex configurations of systems such as: turn - conductive plate. Models of a single turn and a spiral coil allow the creation of a multi-layered cylindrical coil model.

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