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A computer algorithm for the solution of the state equations of descriptor fractional discrete-time linear systems

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Abstract

A method for finding of the solutions of the state equations of descriptor fractional discrete-time linear systems with regular pencils and a procedure for computation of transition matrices of the system are proposed. On the basis of this procedure a computer algorithm which calculates the transition matrices and solution of the system is presented. The effectiveness of the proposed procedure and algorithm is demonstrated on analytical and numerical examples.

Keywords: descriptor, fractional, linear, discrete-time system, regular pencil, solution.

Komputerowy algorytm wyznaczania rozwiązań singularnych liniowych układów dyskretnych niecałkowitych rzędów

Streszczenie

W pracy zaprezentowano metodę wyznaczania rozwiązań singularnych układów dyskretnych niecałkowitych rzędów o pęku regularnym. W rozdziale 2 przedstawiono rozwiązanie równania stanu tej klasy układów. Procedurę wyznaczania macierzy tranzycji tego rozwiązania zaprezentowano w rozdziale 3 oraz podano przykład numeryczny wyznaczania rozwiązania równania stanu (Example 1). W rozdziale 4 przedstawiono komputerowy algorytm wyznaczania macierzy tranzycji rozwiązania singularnych dyskretnych układów niecałkowitego rzędu (rys. 1). Działanie algorytmu zostało zilustrowane przykładami numerycznymi (Example 2 i Example 3). Dla otrzymanych rozwiązań wykreślono przebiegi składowych wektorów stanu (rys. 2 i rys. 3). W rozdziale 5 zamieszczono podsumowanie. W oparciu o rozważania z pracy można otrzymać analogiczną procedurę wyznaczania macierzy tranzycji dla singularnych układów ciągłych niecałkowitego rzędu o pęku regularnym. Problemem otwartym jest opracowanie metody rozwiązania równań stanu singularnych układów dwuwymiarowych ciągłych i dyskretnych niecałkowitych rzędów o regularnych pękach.

Słowa kluczowe: singularne, niecałkowitych rzędów, liniowe, dykretne, pęk regularny, rozwiązanie.

1. Introduction

Descriptor (singular) linear systems with regular pencils have been considered in many papers and books [1-4, 10-12, 15, 18, 19, 20]. The eigenvalues and invariants assignment by state and output feedbacks have been investigated in [10, 11] and the realization problem for singular positive continuous-time systems with delays in [15]. The computation of Kronecker's canonical form of a singular pencil has been analyzed in [22]. A delay dependent criterion for a class of descriptor systems with delays varying in intervals has been proposed in [2].

Fractional positive continuous-time linear systems have been addressed in [9] and positive linear systems with different fractional orders in [8, 13]. A new concept of the practical stability of the positive fractional 2D systems has been proposed in [14].

The reachability of the positive fractional linear systems has been considered in [9] and some selected problems in theory of fractional linear systems in the monograph [16].

A new class of descriptor fractional linear systems and electrical circuits has been introduced, their solution of state equations has been derived and a method for decomposition of the descriptor fractional linear systems with regular pencils into dynamic and static parts has been proposed in [6]. Positive fractional continuous-time linear systems with singular pencils has been considered in [7].

In [17] a new method finding of the solutions of the state equations of descriptor fractional discrete-time linear systems with regular pencils and a procedure for computation of the transition matrices of the system has been presented.

In this paper a computer algorithm for finding of the solutions of the state equations of descriptor fractional discrete-time linear systems with regular pencils will be proposed.

The paper is organized as follows. In Section 2 the solution to the state equation of the descriptor system is derived using the method based on the Z transform and the convolution theorem. A method and procedure for computation of transition matrix is proposed in Section 3. The proposed method is illustrated by a simple analytical example. In Section 4 a computer algorithm for computation of the transition matrices of the descriptor fractional discrete-time linear systems are presented. Calculations are performed in MATLAB environment using Symbolic Math Toolbox. The effectiveness of this algorithm is demonstrated on two numerical examples. Concluding remarks are given in Section 5.

The following notation will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices and $\Re^n = \Re^{n\times 1}$, Z_+ - the set of $n\times n$ nonnegative matrices, I_n - the $n\times n$ identity matrix.

2. Solution of the state equation

Consider the descriptor fractional discrete-time linear system

$$E\Delta^{\alpha} x_{i+1} = Ax_i + Bu_i\,,\ i \in Z_+ = \{0,1,2,...\}\,,\ 0 < \alpha < 1\,, \qquad (1)$$

where α is fractional order, $x_i \in \Re^n$ is the state vector $u_i \in \Re^m$ is the input vector and $E, A \in \Re^{n\times n}$, $B \in \Re^{n\times m}$.

It is assumed that det E = 0 but the pencil (E, A) is regular, i.e.

$$\det[Ez - A] \neq 0 \qquad (2)$$

for some $z \in \mathrm{C}$ (the field of complex numbers).

Without loss of generality we may assume

$$E=\begin{bmatrix} E_1 & 0 \\ 0 & 0 \end{bmatrix} \in \Re^{n\times n},\ \ E_1 \in \Re^{r\times r} \tag{3a}$$

and

$$\operatorname{rank} E_1 = \operatorname{rank} E = r < n. \tag{3b}$$

Consistent initial conditions for (1) are given by x_0.

The fractional difference of the order $\alpha \in [0,1)$ is defined by [16, 21]

$$\Delta^{\alpha} x_i = \sum_{k=0}^{i} c_k x_{i-k}\ , \tag{4a}$$

where

$$c_k = (-1)^k \binom{\alpha}{k},\ \ k=0,1,... \tag{4b}$$

and

$$\binom{\alpha}{k} = \begin{cases} 1 & \text{for } \ k=0 \\ \dfrac{\alpha(\alpha-1)...(\alpha-k+1)}{k!} & \text{for } \ k=1,2,... \end{cases} \tag{4c}$$

Substitution of (4a) into (1) yields

$$Ex_{i+1} = Fx_i - \sum_{k=2}^{i+1} Ec_k x_{i-k+1} + Bu_i\ ,\ i \in Z_+\ , \tag{5}$$

where $F = A - Ec_1 = A + E\alpha$.

Applying to (5) the $\mathcal{Z}$ transform and taking into account that [11]

$$\mathcal{Z}[x_{i-p}] = z^{-p}X(z) + z^{-p}\sum_{j=-1}^{-p} x_j z^{-j},\ \ p=1,2,... \tag{6a}$$

$$\mathcal{Z}[x_{i+1}] = zX(z) - zx_0 \tag{6b}$$

we obtain

$$X(z) = [Ez-F]^{-1}\left[Ex_0 z - H(z) + BU(z)\right], \tag{7a}$$

where

$$X(z) = \mathcal{Z}[x_i] = \sum_{i=0}^{\infty} x_i z^{-i}\ ,\ U(z) = \mathcal{Z}[u_i] = \sum_{i=0}^{\infty} u_i z^{-i}\ ,$$

$$H(z) = \mathcal{Z}[h_i]\ ,\ h_i = \sum_{k=2}^{i+1} Ec_k x_{i-k+1}\ . \tag{7b}$$

Let

$$[Ez-F]^{-1} = \sum_{j=-\mu}^{\infty} \psi_j z^{-(j+1)}\ , \tag{8}$$

where μ is positive integer defined by the pair (E, A) and given by [11, 20]

$$\mu = p - r + 1\ , \tag{9a}$$

where

$$p = \deg \operatorname{adj}\left[Ez-F\right],\ r = \deg\det\left[Ez-F\right] \tag{9b}$$

and the degree of a matrix polynomial is defined as the largest power of z appearing.

Comparison of the coefficients at the same powers of z of the equation

$$[Ez-F]\left(\sum_{j=-\mu}^{\infty}\psi_j z^{-(j+1)}\right) = \left(\sum_{j=-\mu}^{\infty}\psi_j z^{-(j+1)}\right)[Ez-F] = I_n \tag{10a}$$

yields

$$E\psi_{-\mu} = \psi_{-\mu}E = 0 \tag{10b}$$

and

$$E\psi_k - F\psi_{k-1} = \psi_k E - \psi_{k-1}F = \begin{cases} I_n & \text{for} \quad k=0 \\ 0 & \text{for} \quad k = 1-\mu, 2-\mu, ..., -1, 1, 2, ... \end{cases} \tag{10c}$$

From (10b) and (10c) we have the matrix equation

$$G\begin{bmatrix} \psi_{0\mu} \\ \psi_{1N} \end{bmatrix} = \begin{bmatrix} V \\ 0 \end{bmatrix}, \tag{11a}$$

where

$$G = \begin{bmatrix} G_1 & 0 \\ G_{21} & G_2 \end{bmatrix} \in \Re^{(N+\mu+1)n\times(N+\mu+1)n},$$

$$G_{21} = \begin{bmatrix} 0 & ... & 0 & -F \\ 0 & ... & 0 & 0 \\ \vdots & ... & \vdots & \vdots \\ 0 & ... & 0 & 0 \end{bmatrix} \in \Re^{Nn\times(\mu+1)n},$$

$$G_1 = \begin{bmatrix} E & 0 & 0 & ... & 0 & 0 & 0 \\ -F & E & 0 & ... & 0 & 0 & 0 \\ 0 & -F & E & ... & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & ... & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & ... & -F & E & 0 \\ 0 & 0 & 0 & ... & 0 & -F & E \end{bmatrix} \in \Re^{(\mu+1)n\times(\mu+1)n}, \tag{11b}$$

$$G_2 = \begin{bmatrix} E & 0 & 0 & ... & 0 & 0 & 0 \\ -F & E & 0 & ... & 0 & 0 & 0 \\ 0 & -F & E & ... & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & ... & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & ... & -F & E & 0 \\ 0 & 0 & 0 & ... & 0 & -F & E \end{bmatrix} \in \Re^{Nn\times Nn},$$

$$\psi_{0\mu} = \begin{bmatrix} \psi_{-\mu} \\ \psi_{1-\mu} \\ \vdots \\ \psi_0 \end{bmatrix} \in \Re^{(\mu+1)n\times n},\ \psi_{1N} = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_N \end{bmatrix} \in \Re^{Nn\times n}, V = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ I_n \end{bmatrix} \in \Re^{(\mu+1)n\times n}.$$

The equation (11a) has the solution $\begin{bmatrix} \psi_{0\mu} \\ \psi_{1N} \end{bmatrix}$ for given G and V if and only if

$$\operatorname{rank}\left\{G, \begin{bmatrix} V \\ 0 \end{bmatrix}\right\} = \operatorname{rank} G\ . \tag{12}$$

It is easy to show that the condition (12) is satisfied if the condition (2) is met.

Substituting (8) into (7a) we obtain

$$X(z) = \left(\sum_{j=-\mu}^{\infty}\psi_j z^{-(j+1)}\right)[Ex_0 z - H(z) + BU(z)]\ . \tag{13}$$

Applying the inverse transform Z^{-1} and the convolution theorem to (13) we obtain

$$x_i = \psi_i E x_0 - \sum_{k=0}^{i+\mu-1} \psi_{i-k-1} \sum_{j=2}^{k+1} c_j x_{k-j+1} + \sum_{k=0}^{i+\mu-1} \psi_{i-k-1} B u_k \,. \quad (14)$$

To find the solution to the equation (1) first we compute the transition matrices ψ_j for $j = -\mu, 1-\mu, ..., 1, 2, ...$ and next using (14) the desired solution.

3. Computation of transition matrices

To compute the transition matrices ψ_k for $k = -\mu, 1-\mu, ..., N, ...$ of the system (1) with the matrices $\overline{E}, \overline{A}, \overline{B}$ and fractional (real) order α the following procedure is recommended.

Procedure 1.

Step 1. Performing elementary row and column operations [5, 11, 16] on the matrix $\overline{E}$ find the matrix E in the form (3). The matrices of a new system are given by

$$E = P\overline{E}Q\,,\ A = P\overline{A}Q\,,\ B = P\overline{B}\,, \quad (15)$$

where $P \in \Re^{n\times n}$ ($Q \in \Re^{n\times n}$) is the matrix of elementary row (column) operations.

Step 2. Find a solution $\psi_{0\mu}$ of the equation

$$G_1 \psi_{0\mu} = V\,, \quad (16)$$

where G_1, $\psi_{0\mu}$ and V are defined by (11b). Note that if the matrix E has the form (3) then the first r rows of the matrix $\psi_{0\mu}$ are zero and its last $n-r$ rows are arbitrary.

Step 3. Choose $n-r$ arbitrary rows of the matrix ψ_0 so that the equation

$$\begin{bmatrix} E & 0 \\ -F & E \end{bmatrix} \begin{bmatrix} \psi_0 \\ \psi_1 \end{bmatrix} = \begin{bmatrix} I_n + F\psi_{-1} \\ 0 \end{bmatrix} \quad (17)$$

where $F = A + \alpha E$, has a solution with arbitrary last $n-r$ rows of the matrix ψ_1.

Step 4. Knowing $\psi_{0\mu}$ choose the last $n-r$ rows of the matrix ψ_1 so that the equation

$$\begin{bmatrix} E & 0 \\ -F & E \end{bmatrix} \begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \begin{bmatrix} F \\ 0 \end{bmatrix} \psi_0 \quad (18)$$

has a solution with arbitrary last $n-r$ rows of the matrix ψ_2.

Repeating the last step for $\begin{bmatrix} \psi_2 \\ \psi_3 \end{bmatrix}, \begin{bmatrix} \psi_3 \\ \psi_4 \end{bmatrix}, \ldots$ we may compute the desired matrices ψ_k for $k = -\mu, 1-\mu, ..., N$.

The state vector of the system (1) with the matrices $\overline{E}, \overline{A}, \overline{B}$ is given by

$$\overline{x}_i = Q x_i \quad (19)$$

where x_i is obtained using (14).

The details of the procedure will be shown on the following example.

Example 1.

Find the solution to the equation (1) for α = 0.5 with the matrices

$$E = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix},\ A = \begin{bmatrix} 0 & 0 \\ 1 & -2 \end{bmatrix},\ B = \begin{bmatrix} 1 \\ 2 \end{bmatrix} \quad (20)$$

and the initial condition $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

In this case the pencil (2) of (20) is regular since

$$\det[Ez - A] = \begin{vmatrix} z & 0 \\ -1 & 2 \end{vmatrix} = 2z \quad (21)$$

$\mu = 1$ and

$$F = [A + E\alpha] = \begin{bmatrix} \alpha & 0 \\ 1 & -2 \end{bmatrix} = \begin{bmatrix} 0.5 & 0 \\ 1 & -2 \end{bmatrix}. \quad (22)$$

Using Procedure 1 we obtain the following.

Step 1. The matrix $\overline{E}$ is in the desired form (3).

Step 2. In this case the equation (16) has the form

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha & 0 & 1 & 0 \\ -1 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_{-1} \\ \psi_0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (23)$$

and its solution with the arbitrary second row $[\psi_{21}^0 \ \ \psi_{22}^0]$ of ψ_0 is given by

$$\begin{bmatrix} \psi_{-1} \\ \psi_0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \\ 1 & 0 \\ \psi_{21}^0 & \psi_{22}^0 \end{bmatrix}. \quad (24)$$

Step 3. We choose the row $[\psi_{21}^0 \ \ \psi_{22}^0]$ of ψ_0 so that the equation

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -\alpha & 0 & 1 & 0 \\ -1 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \psi_0 \\ \psi_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (25)$$

has the solution

$$\begin{bmatrix} \psi_0 \\ \psi_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.5 & 0 \\ \alpha & 0 \\ \psi_{21}^1 & \psi_{22}^1 \end{bmatrix} \quad (26)$$

with the second arbitrary row $[\psi_{21}^1 \ \ \psi_{22}^1]$ of ψ_1.

Step 4. We choose $[\psi_{21}^1 \ \ \psi_{22}^1]$ so that the equation

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ \alpha & 0 & 1 & 0 \\ -1 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} \alpha & 0 \\ \psi_{21}^1 & \psi_{22}^1 \\ -\alpha^2 & 0 \\ \psi_{21}^2 & \psi_{22}^2 \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad (27)$$

has the solution

$$\begin{bmatrix} \psi_1 \\ \psi_2 \end{bmatrix} = \begin{bmatrix} \alpha & 0 \\ 0.5\alpha & 0 \\ \alpha^2 & 0 \\ \psi_{21}^2 & \psi_{22}^2 \end{bmatrix} \quad (28)$$

with arbitrary $[\psi_{21}^2 \quad \psi_{22}^2]$.

Continuing the procedure we obtain

$$\psi_{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}, \ \psi_k = \begin{bmatrix} \alpha^k & 0 \\ 0.5\alpha^k & 0 \end{bmatrix} \text{ for } k = 0,1,...,N. \quad (29)$$

Using (14), (20) and (29) we obtain the desired solution of the form

$$x_i = \psi_i \begin{bmatrix} 1 \\ 0 \end{bmatrix} - \sum_{k=0}^{i} \psi_{i-k-1} \sum_{j=2}^{k+1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} c_j x_{k-j+1} + \sum_{k=0}^{i} \psi_{i-k-1} \begin{bmatrix} 1 \\ 2 \end{bmatrix} u_k, \quad (30)$$

where c_j are defined by (4b).

4. Computer algorithm for computation of the transition matrices

On the basis of Procedure 1 the computer algorithm which calculates the transition matrices of the state equations of descriptor fractional discrete-time linear systems is proposed. Calculations are performed in MATLAB environment using Symbolic Math Toolbox.

The diagram of this algorithm is shown in Fig. 1.

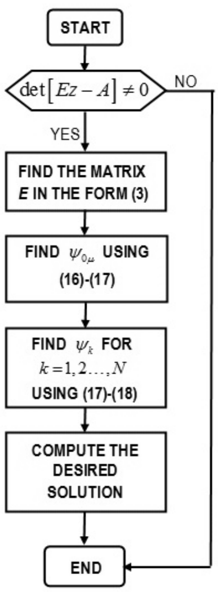

Fig. 1. Algorithm for the computation of transition matrices and solution of the state equations for descriptor fractional discrete-time systems
Rys. 1. Algorytm wyznaczania macierzy tranzycji rozwiązania singularnego układu dyskretnego

First, using the condition (2), we check if the pencil (E, A) is regular. If this pair of the matrices is not regular the algorithm stops. If the pencil is regular the program finds the matrix E in the desired form (3) and the matrices of elementary row and column operations P, Q. Then according to Procedure 1 we calculate the transition matrices of the system. Using (30) and (19) we obtain the state variables of the descriptor fractional discrete-time systems for given matrices $\overline{E}, \overline{A}, \overline{B}$, fractional order α for initial condition x_0.

Example 2. (continuation of Example 1)

Using the computer algorithm we obtain what follows.
The matrices (20) are in the desired form (3) and

$$P = Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. \quad (31)$$

Computing of $\psi_{0\mu}$ we obtain

$$\psi_{-1} = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}, \ \psi_0 = \begin{bmatrix} 1 & 0 \\ 0.5 & 0 \end{bmatrix}. \quad (32)$$

The transition matrices

$$\psi_1 = \begin{bmatrix} 0.5 & 0 \\ 0.25 & 0 \end{bmatrix}, \ \psi_2 = \begin{bmatrix} 0.25 & 0 \\ 0.125 & 0 \end{bmatrix}$$
$$\psi_3 = \begin{bmatrix} 0.125 & 0 \\ 0.0625 & 0 \end{bmatrix}, \ \psi_4 = \begin{bmatrix} 0.0625 & 0 \\ 0.03125 & 0 \end{bmatrix}. \quad (33)$$

The solution of the state equation of the system (1) with the matrices (20), fractional order $\alpha = 0.5$, initial condition $x_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and zero input is obtained using (14) and shown in Fig. 2.

Example 3.

Find the solution to the equation (1) for $\alpha = 0.7$ with the matrices

$$\overline{E} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 2 & 0 \end{bmatrix}, \ \overline{A} = \begin{bmatrix} 0.1 & 0.5 & 0 \\ 0.2 & 0.1 & 0.9 \\ 0.3 & 0.1 & 0 \end{bmatrix}, \ \overline{B} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \quad (34)$$

and the initial condition $\overline{x}_0 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

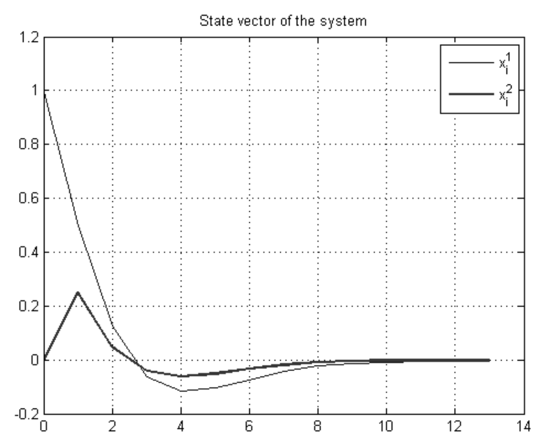

Fig. 2. State variables of the system (20)
Rys. 2. Wykres zmiennych stanu układu (20)

Using the algorithm we obtain

$$E=\begin{bmatrix}1&0&0\\0&1&0\\0&0&0\end{bmatrix},\ A=\begin{bmatrix}0.5&0.1&0\\-0.9&0.1&0\\0.1&0.2&0.9\end{bmatrix},\ B=\begin{bmatrix}1\\-1\\0\end{bmatrix},\ x_0=\begin{bmatrix}2\\1\\4\end{bmatrix} \quad (35)$$

and matrices of elementary operations

$$P=\begin{bmatrix}1&0&0\\-2&0&1\\0&1&0\end{bmatrix},\ Q=\begin{bmatrix}0&1&0\\1&0&0\\0&0&1\end{bmatrix}. \quad (36)$$

In this case $\mu=1$ and

$$\psi_{-1}=\begin{bmatrix}0&0&0\\0&0&0\\0&0&0.111(1)\end{bmatrix},\ \psi_0=\begin{bmatrix}1&0&0\\0&1&0\\-0.111(1)&-0.222(2)&0\end{bmatrix}. \quad (37)$$

The transition matrices

$$\psi_1=\begin{bmatrix}1.2&0.1&0\\-0.9&0.8&0\\0.0666&-0.1888&0\end{bmatrix},\ \psi_2=\begin{bmatrix}1.35&0.2&0\\-1.8&0.55&0\\0.25&-0.1444&0\end{bmatrix},$$
$$\psi_3=\begin{bmatrix}1.44&0.295&0\\-2.655&0.26&0\\0.43&-0.0905&0\end{bmatrix},\ \psi_4=\begin{bmatrix}1.4625&0.38&0\\-3.42&-0.0575&0\\0.5975&-0.0294&0\end{bmatrix},\dots \quad (38)$$

The solution of the state equation of the system (1) with the matrices (34), fractional order α = 0.7, initial condition $\bar{x}_0=\begin{bmatrix}2\\0\\1\end{bmatrix}$ and zero input is obtained using (14) and shown in Fig. 3.

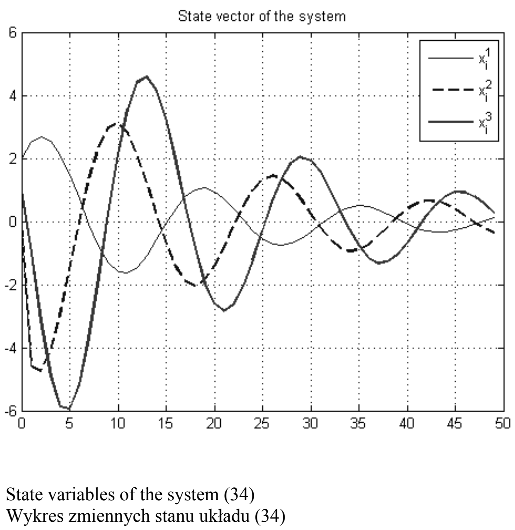

Fig. 3. State variables of the system (34)
Rys. 3. Wykres zmiennych stanu układu (34)

5. Conclusions

A method for computing of the solution of the state equation of descriptor fractional discrete-time linear systems with regular pencils has been presented. A procedure for computation of the transition matrices has been proposed and its application has been demonstrated on a simple numerical example. On the basis of this procedure the computer algorithm which calculates the transition matrices and solution of the system has been proposed. The effectiveness of the proposed algorithm has been demonstrated on numerical examples.

The presented approach can be easily extended to continuous-time descriptor fractional linear system with regular pencils. An open problem is an extension of the method for 2D descriptor fractional discrete and continuous-discrete linear systems.

This work was supported by National Science Centre in Poland under work No N N514 6389 40.

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otrzymano / received: 12.04.2013
przyjęto do druku / accepted: 03.06.2013 *artykuł recenzowany / revised paper*