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MATHEMATICAL MODELING OF CARGO FLOW DISTRIBUTION IN A REGIONAL MULTIMODAL TRANSPORTATION SYSTEM

 Summary. An integrated approach is proposed in the study of rational schemes for the distribution of cargo flows at a regional transport loop for multimodal transportation, considered within the framework of an oligopolistic market. A technique has been developed for the parallel application of two approaches, differing in their mathematical nature, to the issues of increasing the economic efficiency of these transportations. The results obtained by the previously developed method of economic and geographical delimitation of «influence areas» of loading stations serve as a justification for the correctness of the results obtained by using an algorithm based on the Pareto optimization of the freight transportation process. Rational variants for organizing the freight transportation, taking into account time and cost indicators, have been obtained. The system of analytical calculations is used as a software tool to obtain a mathematically sound and transport–logistic diversified model of a regional oligopolistic freight market.

1. INTRODUCTION

1.1. Current state of the distribution cargo flows modeling

 A large number of studies carried out in recent years are related to the optimization of the distribution of cargo flows, including those considered within the framework of multimodal transportation systems [1–14]. This takes into account the drastic transformations that have taken place in the transport industry and associated with the development of the transport services market, as well as focused on a more complete fulfillment of the conditions put forward by customers. Attention is also drawn to the fact that in the cargo flows distribution, there are clashes of commercial interests of competing transport modes and individual transport enterprises.

 In [1], the problem of designing a queuing network in the system of railway freight transport is investigated, in which fixed charges and transportation costs are non-deterministic. For the purpose of designing a freight transportation system, models of indefinite programming are proposed. It is shown that a model with limited capabilities can be transformed into some equivalent deterministic transport model. As a result, an optimal transportation plan can be obtained with some restrictions, in particular, with a limited budget.

In [2], a combined tactical and operational two-stage model of food grain transportation with a linear formulation at the first stage and a mixed-integer nonlinear problem (MINLP) at the second stage is proposed. The application of the results obtained is shown for the example of the transport and technological system of India.

In [3], a decision support model was developed for a sustainable food grain supply chain, taking into account the entire network of purchasing centers. The aim is to simultaneously minimize the cost of transportation and carbon dioxide emissions. Examples of the functioning of the model based on two multipurpose algorithms that use the Pareto criterion are given.

In [4], the issues of reliability in the optimization of sustainable grain transportation with uncertain supplies and deliberate failures are investigated. A model of intermodal transportation has been developed, which provides for the facilitation of transportation for one of the types of grain, taking into account the uncertainty of procurement, greenhouse gas emissions and the deliberate destruction of nodal points. In [5], a generalized interval model of fuzzy mixed-integer programming is proposed for the study of multimodal freight transportation under uncertainty. At the same time, the optimal mode of transport is indicated and the optimal amount of each type of cargo transported along each route is found. Three methods are used to obtain equivalents of constraints and parameters: interval ranging, fuzzy linear programming and linear weighted summation. In order to implement the model, a corresponding heuristic algorithm has been developed.

In [6], the influence of demand uncertainty on the problem of cargo routing at the operational level in a powerful multimodal transport network, which consists of railroad transportation on the basis of a schedule and road transportation with flexible time frames, is studied. The fuzzy expected value model and fuzzy programming are used.

The possibilities of using digital technologies when choosing the most cost-effective freight transportation scheme based on a cybernetic model are considered in [7]. The «white box» model allows one to take into account the input parameters required by the consignor and select the optimal scheme from those parameters that are proposed for consideration.

In [8], methods for calculating the location of hubs are considered to minimize the economic costs of transporting goods from the manufacturer to the transshipment point. The problem belongs to a mixed integer programming problem, but due to the complexity of the problem being solved, the authors propose a heuristic approach.

Methods of mathematical programming are used in [9]. This solves a transport problem in which the parameters of supply and demand are stochastic, and the cost depends on their value. The aim is to minimize transportation costs depending on the volume of the available product.

The economic–geographical method was developed by the authors in [13] as a general approach in transport and logistics research. In this paper, this method is developed in combination with the numerical method developed by the authors in [14] for optimizing freight transportation with the considered time parameters. At the same time, the general theoretical basis for transport and logistics constructions is the egalitarian approach in the welfare theory [15].

1.2. Problem statements

The analysis of the above works provides an idea of the relevance of the issues discussed in them and the variety of methods used in the study of processes occurring in various transport and technological systems. If, in the context of these studies, we turn to raw materials exported from Russia, then in view of their bulk and relatively low cost, it seems natural to consider multimodal transportation carried out by rail and sea transport.

In general, when choosing schemes for multimodal transportation of goods exported through seaports, one should proceed from the geographic location, technical equipment and availability of these ports by railway. If we focus on the export of grain, then we should proceed from the presence of a large number of very scattered enterprises in the south of Russia, specializing in its production. Taking into account the ramified communication routes (including in the direction of ports) as well as the presence on the market of a large number of companies actively involved in the transportation of grain, the southwestern part of the North-Caucasian Railway is a platform for the functioning of

a fully developed market of transport services. As a result, for clients, there are very wide opportunities for choosing among a wide variety of freight transportation schemes that (in accordance with any indicators important for these consumers of services) are rational in terms of organizing the entire logistic chain.

This work is devoted to the study of the issues of organizing regional multimodal freight transportation on a qualitatively new basis in comparison with the studies carried out by the authors earlier. The main idea is to combine the approaches developed in [13, 14] and different in their mathematical nature in finding rational schemes for the implementation of the transportation process.

2. OPTIMIZATION MODEL OF REGIONAL FREIGHT TRANSPORTATION

From the point of view of the general idea, the ongoing research is in line with the egalitarian approach in the welfare theory [15]. We present the paradigm of cooperative decision-making, which is one of the key points of this approach, in the form of an optimization problem. The set of objective functions and restrictions in problems are such that, to one degree or another, the economic interests of a number of participants in the transportation process (that is, «agents», to use the terminology from [15]) are taken into account.

Let us clarify some general assumptions that are made when setting the optimization problems considered below. It is assumed that there are m loading stations and n unloading stations. Let a_i be the number of dispatch routes to be removed from the ith loading station ($i = 1, 2, \ldots, m$). Since the unloading stations considered in this work are such that their capacities for receiving grain are orders of magnitude higher than those volumes that are subject to export from each loading station, then, initially, the situation can be attributed to an open model.

Further, let x_{ij} be the number of forwarder routes that can be directed from the ith loading station to the jth unloading station ($j = 1, 2, \ldots, n$). At the first stage of the research, we do not make any assumptions about the set D of feasible transportation plans $\left(x_{ij}\right)$, except that everything located at the loading stations and planned for export to the addresses of the unloading stations under consideration should be exported. That is, the equalities must be satisfied:

$$\sum_{j=1}^{n} x_{ij} = a_i \,(i = 1, 2, \ldots, m). \tag{1}$$

In the process of further constructions, the choice of a set of D feasible transportation plans is made in accordance with the operational conditions of the transportation process under consideration.

It should be noted that the proposed model of the freight transportation process is not directly related to game theory. In particular, this model does not imply (fundamental for this theory) consideration of the strategies of the players. The key point in the process of optimizing the distribution of traffic flows is represented by the Pareto optimality criterion, and the main idea is to take into account the interests of all agents under consideration.

As characteristics of the economic feasibility and the level of organization of the transportation process, we will consider two time indicators and one cost indicator.

Time indicators are set by means of the objective functions of the optimization problem, which have the following form:

$$\tau_1 = \max_{i,j}\{t_{ij}\, sgn\, x_{ij}\} \tag{2}$$

and

$$\tau_2 = \sum_{i=1}^{m} \sum_{j=1}^{n} t_{ij} x_{ij} \tag{3}$$

Here, t_{ij} is the time spent by the forwarder route on the section between the ith loading station and the jth unloading station.

For each transportation plan $\left(x_{ij}\right)$, indicator τ_1 represents the time of the longest carriage of all carriages in that plan. Since the value of indicator τ_1 determines the implementation time of the entire

transportation plan $\left(x_{i_j}\right)$, this indicator is called the total time of this plan. For each transportation plan $\left(x_{i_j}\right)$, indicator 6 is the total time spent on the loop for all forwarder routes involved in the plan. Indicator τ_2 will be called the total time of the transportation plan $\left(x_{i_j}\right)$.

The cost indicator is set by means of an objective function of the following form:

$$c = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij} \ . \tag{4}$$

Here, c_{ij} is the cost of transportation of one route on the section between the ith loading station and the jth unloading station.

For each traffic plan $\left(x_{i_j}\right)$, indicator c is the total traffic cost of all forwarder routes that are part of the plan.

Moving on to agents, we will position as such at least three participants in the transportation process: JSCo «Russian Railways» (the carrier and owner of the infrastructure), some operator company (the owner of the rolling stock) and, for example, Universal Cargo Logistics Holding B.V. Note that UCL Holding implements supply chains almost independently.

All the agents under consideration are directly interested in minimizing indicator τ_1. With respect to indicator τ_2, attention should be paid to the fact that it is directly related to the degree of exploitation of the track infrastructure of the loop, which occurs as a result of the implementation of the corresponding plan. Therefore, a controversial approach of the JSCo «Russian Railways» can be assumed to the value of this indicator. On the one hand, the source of the economic benefit of the specified agent is the operation of the infrastructure belonging to that person. On the other hand, one should take into account the fact that the increased exploitation of the said property entails an increase in the costs required for the corresponding depreciation. For the sake of completeness, it should be noted that minimization of indicator τ_2 is consistent with the principles of green logistics. Compliance with these principles is not directly related to anyone's commercial interests, but becomes increasingly more relevant and is closely monitored by various government and public structures.

Note also that indicator c acquires a dual character if we introduce into consideration one more agent, namely, a client paying for a transport service. This agent wants to minimize his or her expenses, and the above three agents want to maximize their profit.

Of course, indicators τ_1, τ_2, and c can be interpreted in more detail and in a different way from the point of view of the usefulness of the considered agents of the transportation process. From the results of the computational procedures given below, it can be seen that (in the literal sense) these indicators do not conflict with each other.

3. PRELIMINARY AND SUPPLEMENTARY RESULTS

3.1. Initial assumptions in building the model

Let us turn to the presentation of the foundations of the algorithm developed in this article for finding the optimal distribution of cargo flows on the transport range, as well as the implementation of this algorithm in accordance with the principles of the egalitarian approach in the welfare theory [15]. Let us pay attention to one of the fundamental rules of this approach, according to which the following condition is fulfilled for cooperative agents. An improvement in the utility of any of the agents should not occur together with deterioration in the utility of at least one of the other. With respect to the transportation process, the foregoing means that when searching for the optimal distribution of freight traffic, an increase in the effect of the implementation of a transportation plan for any agent is not allowed along with a decrease in the effect of the implementation of this plan for any other agent.

As a subject of research, we will consider the system of transportation of grain cargoes to the ports of the Azov-Black Sea basin, the versatility of which, in relation to the range of processed cargo, creates conditions for competition. We will assume that the grain cargo intended for transportation is

located at the loading stations Tacinskaya, Sal'sk, Zernograd, Remontnaya and Tihoreckaya, and the unloading stations are the port stations Novorossijsk, Tuapse, Taman', Ejsk and Azov (Fig. 1).
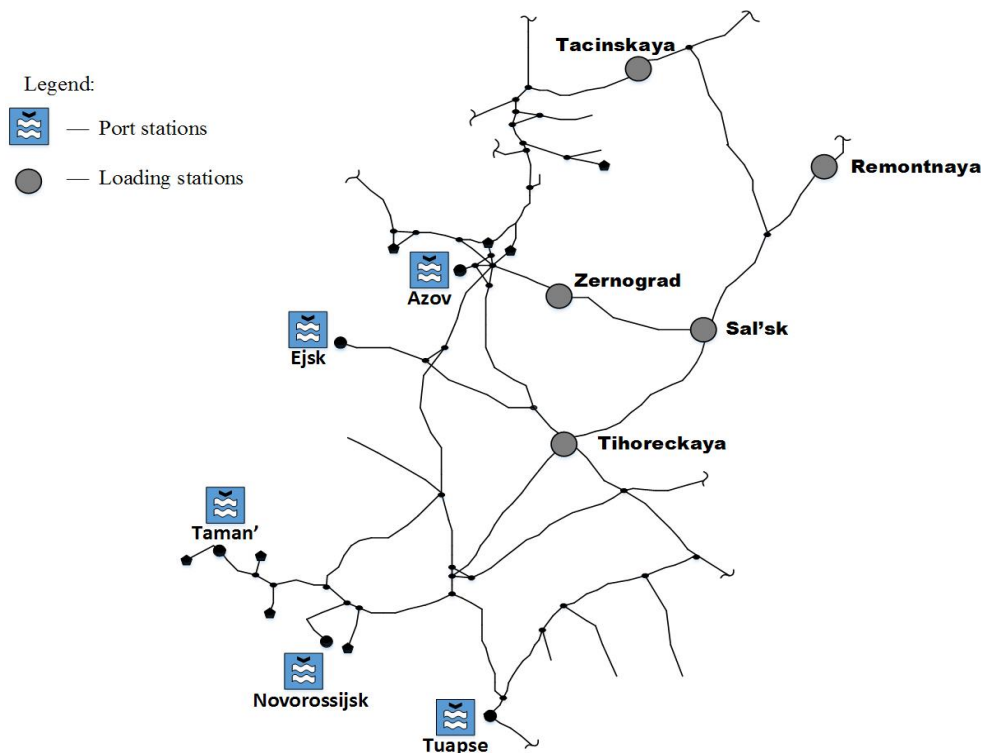
Fig. 1. Scheme of the considered part of the North-Caucasian Railway loop

Let us pay attention to the fact that the proposed algorithm for solving optimization transport and logistics problems is very general in nature and can be used, for example, in the study of freight transportation by road transport. In terms of the number of loading and unloading stations as well as the restrictions imposed on the set of feasible transportation plans, we note that they only affect the volume of computational procedures performed. We use Maxima (Free Ware) as software.

3.2. Model of the freight transportation process with time indicators

Let us start by researching the process of multimodal transportation of cargo within the framework of an optimization problem with two objective functions (2) and (3).

Each transportation plan $\left(x_{i_j}\right) \in D$ is assigned a vector $\{\tau_1, \tau_2\}$, called a utility vector. An optimal transportation plan is $\left(x_{i_j}^*\right) \in D$ with a utility vector $\{\tau_1^*, \tau_2^*\}$ such that there is no transportation plan $\left(x_{i_j}\right) \in D$ for which the coordinates of the utility vector $\{\tau_1, \tau_2\}$ satisfy condition ($\tau_1 < \tau_1^*$ and $\tau_2 \leq \tau_2^*$) or condition ($\tau_1 \leq \tau_1^*$ and $\tau_2 < \tau_2^*$). In this formulation of the problem, the (only) controlled variable is utility, which is expressed by the time. From the point of view of minimizing, this indicator is useful for each of the agents specified in section 2.

We begin the computational procedures by considering the situation when at each of the loading stations, there is only one forwarder route with grain. First, we do not impose any restrictions on transportation plans (except for those expressed by equalities (1)). Let us make a general comment that is computationally relevant. According to the well-known combinatorial formula, the set D of all feasible transportation plans $\left(x_{i_j}\right)$ consists of n^m elements. It turns out that in the case under consideration (when $n = m = 5$), there are 3125 transportation plans.

In the process of implementing the optimization algorithm, two sequential transportation plans are obtained, which are presented in Table 1.

Table 1

Transportation plans when implementing the optimization algorithm

№	Transportation plan	Number of routes arriving at port stations					τ_1	τ_2
		Novorossijsk	Tuapse	Taman'	Ejsk	Azov		
1	0,0,0,0,1,0,0,0,0,1,0,0,0,0,1,0,0,0,0,1,0,0,0,0,1	0	0	0	0	5	1,12	3,55
2	0,0,0,0,1,0,0,0,0,1,0,0,0,0,1,0,0,0,0,1,0,0,0,1,0	0	0	0	1	4	1,12	3,50

In the optimal transportation plan, all forwarder routes are directed only to two port stations: Ejsk and Azov. A route from the Tihoreckaya loading station is sent to the address of the Ejsk station, and routes from all the other four loading stations are directed to the Azov station. This result is expected from a geographical point of view (see Fig. 1). However, it is not entirely obvious that exactly one route turns out to be directed to the Ejsk station.

In what follows, we will assume that at least one forwarder route should be directed to each of the port stations. Note that with this formulation of the problem in the example under consideration, the set of feasible transportation plans decreases to $5! = 120$. In the process of implementing the optimization algorithm, 6 sequential transportation plans are obtained, which are presented in Table 2.

Table 2

Transportation plans when implementing the optimization algorithm

№	Transportation plan	Number of routes arriving at port stations					τ_1	τ_2
		Novorossijsk	Tuapse	Taman'	Ejsk	Azov		
1	0,1,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,1,0,0,0,1,0,0	1	1	1	1	1	2,11	6,65
2	0,1,0,0,0,0,0,0,0,1,0,0,0,0,1,0,1,0,0,0,0,0,1,0,0	1	1	1	1	1	2,11	6,28
3	1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,1,0,0,0,0,0,1,0,0	1	1	1	1	1	1,95	6,15
4	0,0,0,1,0,0,0,0,0,1,1,0,0,0,0,0,1,0,0,0,0,0,1,0,0	1	1	1	1	1	1,81	6,15
5	0,0,0,1,0,0,0,0,0,1,0,0,1,0,0,1,0,0,0,0,0,1,0,0,0	1	1	1	1	1	1,78	6,11
6	0,0,0,0,1,0,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,0,0	1	1	1	1	1	1,78	5,75

In the transition from the 1st received plan to the 2nd, indicator τ_1 does not change and only indicator τ_2 decreases. When moving from the 2nd to the 3rd plan, both indicators decrease.

3.3. *GEM* of the freight market in the region

The mathematical model of the process of regional transportation developed in this article can be developed in different directions. First, it is the expansion of the set of indicators of the transportation process that characterize the economic efficiency and quality of the organization of this process. Second, one can turn to other forms of optimization criteria, by means of which the selection of more

rational transportation plans is algorithmized. It is also possible to vary the set of feasible transport plans according to the existing operational situation at the transport range.

Let us first turn to the restrictions that we will impose on the set D of feasible transportation plans $\left(x_{ij}\right)$. The idea is that as one of the sources for formulating such restrictions, we will consider the previously developed *geometric Euclidean model* (*GEM*) of the territorial oligopolistic freight market [13]. A condition that ensures the correctness of the construction of such a model is the presence of a sufficiently dense network of railways in the region under consideration. In this case, the «idealization» of the logistic situation is admissible, which consists of the fact that Euclidean distances on a flat geographical map between their beginnings and ends are considered as the lengths of road sections.

«Influence areas» of loading stations, obtained by the method of economic–geographical delimitation, make it possible to formulate a number of preferred directions in the distribution of cargo flows. Let us recall that the cost of transportation of cargo from these stations to destinations serves as an indicator, according to which the specified areas are determined in the *GEM*.

3.4. Finding the cost indicators of the transportation process

To construct a *GEM* of the freight traffic market in the region, for each of the loading stations under consideration, it is required to find an expression for the dependence of the cost c (thousand rubles) of transportation of one car on the length l (km) of the route traveled. The source for finding the corresponding formulas are the data that characterize the transport flow of grain cargo between loading stations and unloading stations, presented in table 3.

Table 3

Characteristics of the investigated traffic flow

Loading stations	Port stations									
	Novorossijsk		Tuapse		Taman'		Ejsk		Azov	
	t, days	c, thousand roubles	t, days	c, thousand roubles	t, days	c, thousand roubles	t, days	c, thousand roubles	t, days	c, thousand roubles
Tacinskaya	1,95	1810,56	2,11	2117,82	2,12	1965,78	1,3	1397,16	0,89	1179,3
Sal'sk	1,3	1397,16	1,33	1565,94	1,52	1565,94	1,02	1285,14	0,64	1026,36
Zernograd	1,34	1459,8	1,5	1748,4	1,52	1565,94	0,69	1062,6	0,28	775,56
Remontnaya	1,78	1748,4	1,81	1895,28	2,0	1895,28	1,5	1565,94	1,12	1341,54
Tihoreckaya	0,84	1141,26	0,87	1341,54	1,06	1285,14	0,57	1285,14	0,62	984,66

After processing the given numerical data by the least-squares method, we find that for each loading station, the sought dependence has the following form: $c = p + ql$. Here, p and q are cost indicators, which express the costs of initial–final and movement operations spent on the carriage of one wagon (per 1 km of route). The obtained formulas are presented in table 4. The linear form of the considered dependence is very clearly manifested, as in [13]. Let us pay attention to the fact that the difference between loading stations is expressed both in the cost of initial and final operations and in the cost of motion operations. Consequently, the territorial picture of the freight transport market is described by algebraic curves of the fourth order, namely, Descartes' ovals.

3.5. Territorial picture of the freight market

Note that in this section of the article, the concepts of duopoly and oligopoly are applied in relation to loading stations, which, in economic terms, can be positioned as oligopolists. Competition between

these stations can be expressed, for example, in the desire to make the most efficient use of the wagon fleet of the operator in question.

Table 4

Expressions of the dependence of the cost of transportation

№	Loading stations	$c = p + ql$
1	Tacinskaya	$c = 524{,}14 + 2{,}16l$
2	Sal'sk	$c = 623{,}06 + 1{,}98l$
3	Zernograd	$c = 572{,}72 + 2{,}18l$
4	Remontnaya	$c = 574{,}95 + 2{,}14l$
5	Tihoreckaya	$c = 965{,}74 + 0{,}94l$

«Influence areas» of loading stations Zernograd and Tihoreckaya in a duopolistic situation are shown in Fig. 2. The «influence area» of Tihoreckaya station includes the port stations Novorossijsk, Tuapse and Taman', and the «influence area» of Zernograd station includes the port stations Ejsk and Azov. Note that the loading station Tihoreckaya falls into the «influence area» of the loading station Zernograd. The explanation is that the costs of the initial and final operations at Tihoreckaya station are almost twice as high as those at Zernograd station (see Table 4). However, since the costs of movement operations at Tihoreckaya station are significantly lower than at all other loading stations, this loading station is more competitive in terms of destinations that could be located at sufficiently large distances from all loading stations.
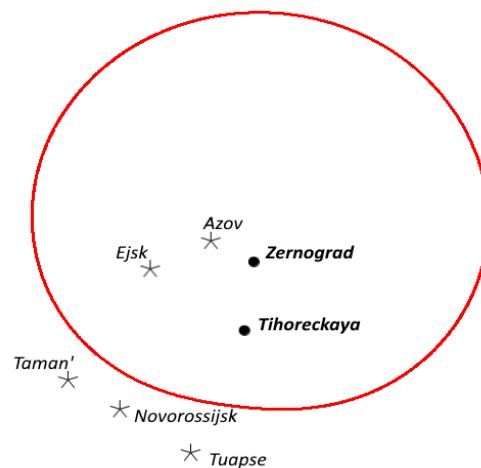

Fig. 2. «Influence areas» of Zernograd and Tihoreckaya stations in the case of a duopoly

The description of the complete territorial picture of the oligopolistic market created by five loading stations involves 10 algebraic curves that are Descartes' ovals. Therefore, the boundary of the «influence area» of each loading station is a broken line, generally speaking, of four curved parts. Each of these parts is some fragment of Descartes' oval (in more complex cases, it may also turn out that this fragment consists of several parts), which, in the case of a duopoly, delimits the «influence areas» of the station in question and any one of the other four loading stations.

The territorial picture of the freight market, created by five loading stations, obtained using a system of analytical calculations, is shown in Fig. 3.

3.6. Formulation of constraints in the optimization problem

The territorial picture of the regional freight transportation market makes it possible to formulate a number of restrictions on the set D of feasible transportation plans in the considered optimization problems. We see that all port stations are «split» between two loading stations. The stations Novorossijsk, Tuapse and Taman' are in the «influence area» of Tihoreckaya station, and the stations

Ejsk and Azov are in the «influence area» of Zernograd station. Let us use the indicated «preferences» in the distribution of cargo flows as follows. We will assume that the forwarder route from the Tihoreckaya loading station can be directed only to the address of the Novorossijsk, Tuapse or Taman' station, and the forwarder route from the Zernograd loading station can only be directed to the Ejsk or Azov station.
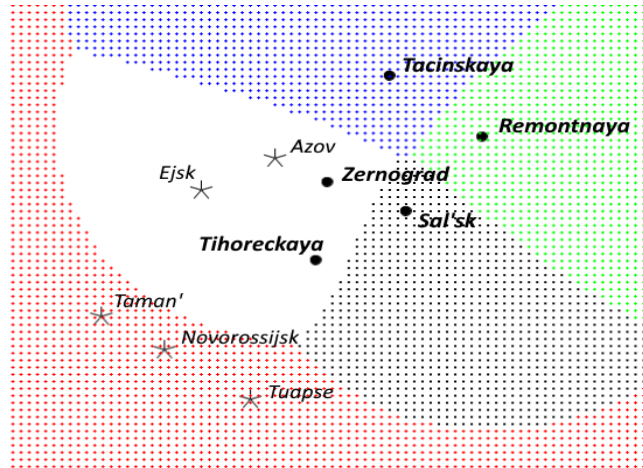

Fig. 3. Territorial picture of the regional oligopolistic freight traffic market in *GEM*

Sequential transportation plans obtained in the process of implementing the optimization algorithm, in the design of which the two indicated restrictions on the set of feasible transportation plans, are introduced, are presented in Table 5.

Table 5

Transportation plans when implementing the optimization algorithm

№	Transportation plan	Number of routes arriving at port stations					τ_1	τ_2
		Novorossijsk	Tuapse	Taman'	Ejsk	Azov		
1	0,1,0,0,0,0,0,0,0,1,0,0,0,1,0,1,0,0,0,0,0,1,0,0	1	1	1	1	1	2,11	6,28
2	1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,1,0,0,0,0,1,0,0	1	1	1	1	1	1,95	6,15
3	0,0,0,0,1,0,0,1,0,0,0,0,0,1,0,1,0,0,0,0,0,1,0,0	1	1	1	1	1	1,78	5,75

The optimal transportation plan coincided with the plan obtained in subsection 3.2. (see Table 2). However, in this case, the optimization «ladder» turned out to consist of only three «steps» (instead of six as in the previous case).

Let us formulate two conclusions. First, the results obtained by the methods developed by the authors for finding rational transportation plans do not contradict each other. Second, the absence of «unnecessary» intermediate plans in the process of applying the optimization algorithm using the constraints given by the method of economic–geographical delimitation of the «influence areas» of loading stations can significantly reduce the amount of computing resources spent.

4. RESULTS

4.1. Restrictions in the optimization problem for shallow water ports

Let us single out several objective factors that characterize the port stations considered in the article, namely, their geographical location and export potential.

Located on the coast of the Taganrog Bay of the Azov Sea and at the mouth of the Don, the port stations of Ejsk and Azov are available for ships with a draft of up to 4.5 m and 4 m, respectively. When an ice cover is established in the indicated area on the surface of waterways, navigation is carried out using icebreaker assistance (which, it should be noted, entails additional costs). The ports of these two cities can be visited by sea vessels with deadweight of up to 7000 tons. If we consider the dispatch routes, consisting on average of 50 wagons with a carrying capacity of 64 tons each, then each vessel is practically filled with two such routes. Further, the grain cargo is transported to the Black Sea and on the roadstead is reloaded into Panamax-type vessels. Due to the low throughput of railway approaches to these ports and the lack of warehouses of sufficiently large capacity, it is necessary to carefully plan the time of transport of cargo to them. Note also that the bulk of the grain is delivered to ports by an active competitor – road transport. As we can see, a well-grounded and high-quality organization of the entire multimodal transportation process is highly relevant. Such an organization makes it possible to exclude excess idle time of rolling stock pending unloading.

In accordance with the remarks made in relation to the port stations of Ejsk and Azov, we introduce restrictions in the optimization problem considered below in addition to those made in subsection 3.6. We will assume that 2 or 4 routes can be directed to each of these stations. We proceed from the fact that the volume of grain contained in them practically corresponds to the carrying capacity of one or two vessels with deadweight of up to 7000 tons.

In terms of the deep-water ports of the Black Sea, we note that their loading capacity is much higher than the volumes of freight transportation that we are considering. Therefore, we will not introduce any additional restrictions with respect to the three port stations considered in the article.

4.2. Multi-criteria optimization in terms of time and cost indicators

Let us turn to the study of the freight transportation process within the framework of the optimization problem with three objective functions (2), (3) and (4). In setting the problem, we will proceed from the following definition.

Each transportation plan $\left(x_{ij}\right) \in D$ is assigned a vector $\{\tau_1, \tau_2, c\}$, called a utility vector. An optimal transportation plan is $\left(\overset{*}{x}_{ij}\right) \in D$ with a utility vector $\left\{\tau_1^*, \tau_2^*, c^*\right\}$ such that there is no transportation plan $\left(x_{ij}\right) \in D$, for which the coordinates of the utility vector $\{\tau_1, \tau_2, c\}$ satisfy the following condition:

$$\left(\tau_1 < \tau_1^* \wedge \tau_2 \leq \tau_2^* \wedge c \leq c^*\right) \vee \left(\tau_1 \leq \tau_1^* \wedge \tau_2 < \tau_2^* \wedge c \leq c^*\right) \vee \left(\tau_1 \leq \tau_1^* \wedge \tau_2 \leq \tau_2^* \wedge c < c^*\right). \quad (5)$$

From the expression of the logical connective (5), it can be seen that in the process of implementing the corresponding algorithm, transportation plans are improved in the sense of the Pareto optimization with three objective functions τ_1, τ_2 and c.

When carrying out computational procedures, we will consider a situation when, at each loading station, there are 3 routes with grain intended for dispatch to the specified port stations. Note that in this case (according to the remark from the field of combinatorics given in subsection 3.2), the set D of all feasible transportation plans $\left(x_{ij}\right) (i, j = 1,2,3,4,5)$ consists of $\left(5^5\right)^3 = 5^{15} = 30517578125$ plans.

The restrictions made above make it possible to significantly reduce the volume of feasible transportation plans that are analyzed in the optimization process, without compromising the quality of the results obtained (see the remarks in subsection 3.6).

Figs. 4 and 5 show images of sequential transportation plans obtained in two cases of implementation of the optimization algorithm with indicators τ_1, τ_2 and c.

Table 6

Distribution of forwarder routes by port stations (1st case)

№	Number of routes arriving at port stations				
	Novorossijsk	Tuapse	Taman'	Ejsk	Azov
1	4	2	1	4	4
2	4	2	1	4	4
3	4	2	1	4	4
4	4	2	1	4	4
5	3	3	1	4	4
6	3	3	1	4	4
7	4	2	1	4	4
8	4	2	1	4	4
9	4	2	1	4	4

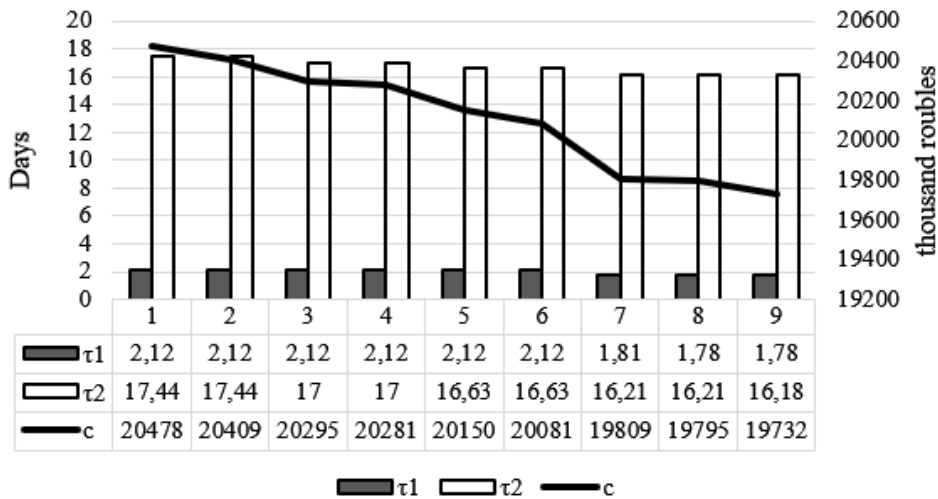|	1	2	3	4	5	6	7	8	9
τ1	2,12	2,12	2,12	2,12	2,12	2,12	1,81	1,78	1,78
τ2	17,44	17,44	17	17	16,63	16,63	16,21	16,21	16,18
c	20478	20409	20295	20281	20150	20081	19809	19795	19732

Fig. 4. Indicators obtained in the optimization process (1st case)

Table 7

Distribution of forwarder routes by port stations (2nd case)

№	Number of routes arriving at port stations				
	Novorossijsk	Tuapse	Taman'	Ejsk	Azov
1	3	3	1	4	4
2	4	2	1	4	4
3	3	2	2	4	4
4	4	2	1	4	4
5	4	2	1	4	4
6	4	2	1	4	4
7	3	3	1	4	4
8	3	3	1	4	4

4.3. Discussion of the results

Let us pay attention to the nature of changes in the values of these indicators. The change in indicator τ_1 (for obvious reasons) occurs abruptly, that is, it has a pronounced discrete character. The nature of the change in the values of indicators τ_2 and C, in a sense, turns out to be close to continual.
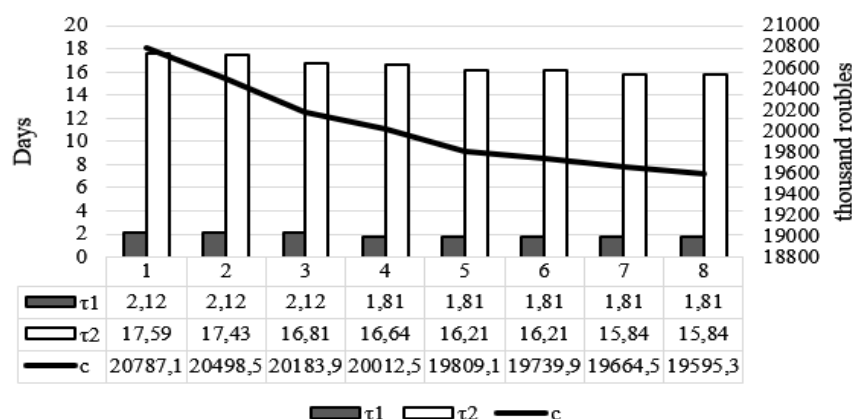
Fig. 5. Indicators obtained in the optimization process (2nd case)

The optimal transportation plans found by the Maxima system are such that in the first plan, the value of indicator τ_1 is less than that in the second plan (Table 6 and Fig. 4). However, in the second optimal transportation plan, the values of each of the indicators τ_2 and C are less than the corresponding values in the first plan (Table 7 and Fig. 5).

5. CONCLUSIONS

Modern research of multimodal transportation involves not just the development of existing approaches and the proposal of new approaches in organizing the distribution of cargo flows and finding rational schemes for transporting cargo.

A no less urgent problem is a versatile comparison of the methods used and their complex use. When combining previously developed approaches and their joint implementation, the interests of various participants of the transportation process it should be taken into account.

In this paper, we propose a method for researching the process of freight transportation at a regional transport range, which consists of the parallel application of two approaches, different in their mathematical essence, to the issues of increasing the economic efficiency of these transportation processes. The economic–geographical method of delimitation the «influence areas» of loading stations, developed earlier by the authors, confirms the correctness of the results obtained using the algorithm, the basis of which is the Pareto optimization of the freight transportation process with consideration of time and cost indicators.

The analytical, computational and graphic capabilities of specialized software products allow obtaining theoretically grounded and diversified transport and logistics models of the regional oligopolistic freight market.

Acknowledgments

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